MA4825

Robotics

Xie Ming, PhD (France)

mmxie@ntu.edu.sg

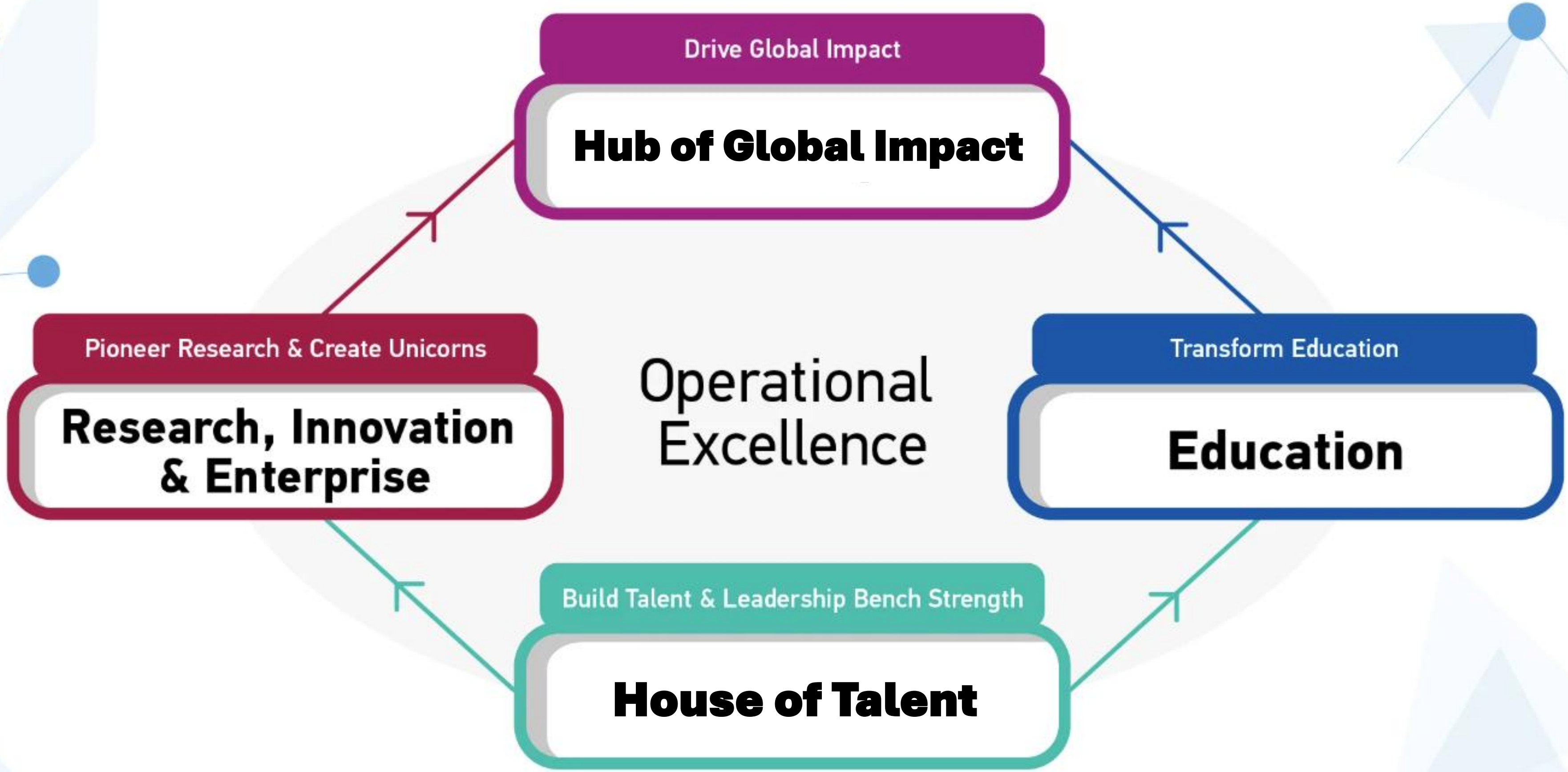
http://personal.ntu.edu.sg/mmxie

Outline

- Module 1: Robot’s Advanced Body
- Module 2: Robot’s Advanced Perception
- Module 3: Robot’s Advanced Planning
- Module 4: Robot’s Advanced Control

About NTU

Remember NTU's Vision …



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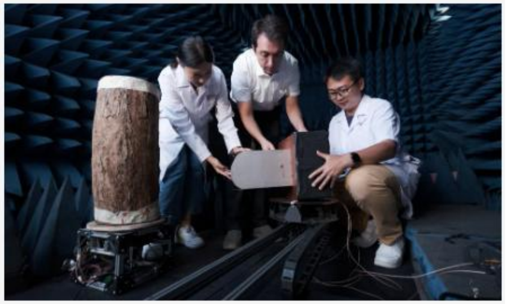
Education

We deliver transformative educational experiences that make our students both future- and AI-ready, so they are sought after by employers.



House of Talent

We attract, develop, and retain the very best people to drive excellence across the University.

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李总理：排名不应是办大学终极考量

黄伟曼 报道
ngwaimun@sph.com.sg

本地大专学府的主要服务对象是新加坡学生，大学除了追求学术表现之外，也扮演重要的国家与社会角色，李显龙总理提醒本地大学不应为追求名次而忽略所肩负的重任，必须持续改善本地教育，为新加坡人提供精益求精的教育。

李总理昨晚主持新加坡国立大学大学城（University Town，简称UTown）开幕仪式时指出，本地大学的世界排名一直受到关注，但在衡量大学的表现时，不应只看名次先后，排名也不应是办大学的终极考量。

他向现场近1000名国大学生、教师与嘉宾解释说，许多在世界大专教育排行榜上名列前茅的大国顶尖大学收生广泛，例如美国哈佛大学、斯坦福大学、英国牛津大学、剑桥大学、中国北京大学和清华大学等每年录取的学生中，只有一小部分是当地学生，而本地的情况却不同，大专学府主要收本地生，必须履行更广泛的国家任务。

他说："本地大学的责任在于发展学生的品格与社会良知、鼓励他们在求学时期建立长久的友谊与默契。大学要以灌输新加坡价值观和精神特质为出发点，教导学生取之社会用之社会的精神，让他们知道自己有责任推动新加坡前进。"

"要达到这个宗旨，每一所大学都应该建立自己独特的教育模式，不要一味复制其他大学。"

占地19公顷的大学城是国大搬到肯特岗校园现址后，最大型的一次扩展计划。设于大学城的四所寄宿型学院和为研究生所设的大学城宿舍，将为4000多名学生提供住宿，将教学融入住宿环境里，让来自不同背景和科系的学生一起生活、一起学习。

在大学城学院课程中，学生需在所住的学院里修读五个单元，包括两个写作单元和三个研讨课单元。他们也有机会聆听外国学者发表专题演讲，并参加各种不同的课外活动。

李总理昨天肯定国大为提升学生学习体验所做的努力，他认为大学城为学生营造了一个有利于相互学习的空间。他指出，大学城是国大的"新聚集点"，它将国大社群缩小，进一步拉近学生与同学和教授的关系。

他也以陈爱丽丝与彼德学院（College of Alice & Peter Tan)学生之前在"整合式课程"（Capstone Programme）下为乐龄人士制定健康饮食计划为例，表示希望看到大学城继续推行类似社会计划，鼓励学生积极参与这些关怀社会的活动。

国大校长陈祝全教授致辞时指出，除了设施与空间设计吸引人之外，大学城为国大社群以及国大的运作和成长带来了深刻且积极的影响。"大学城可说已成为国大校园的核心，在一些方面补足了之前肯特岗校园所缺少的生气。"

相关新闻刊第6页

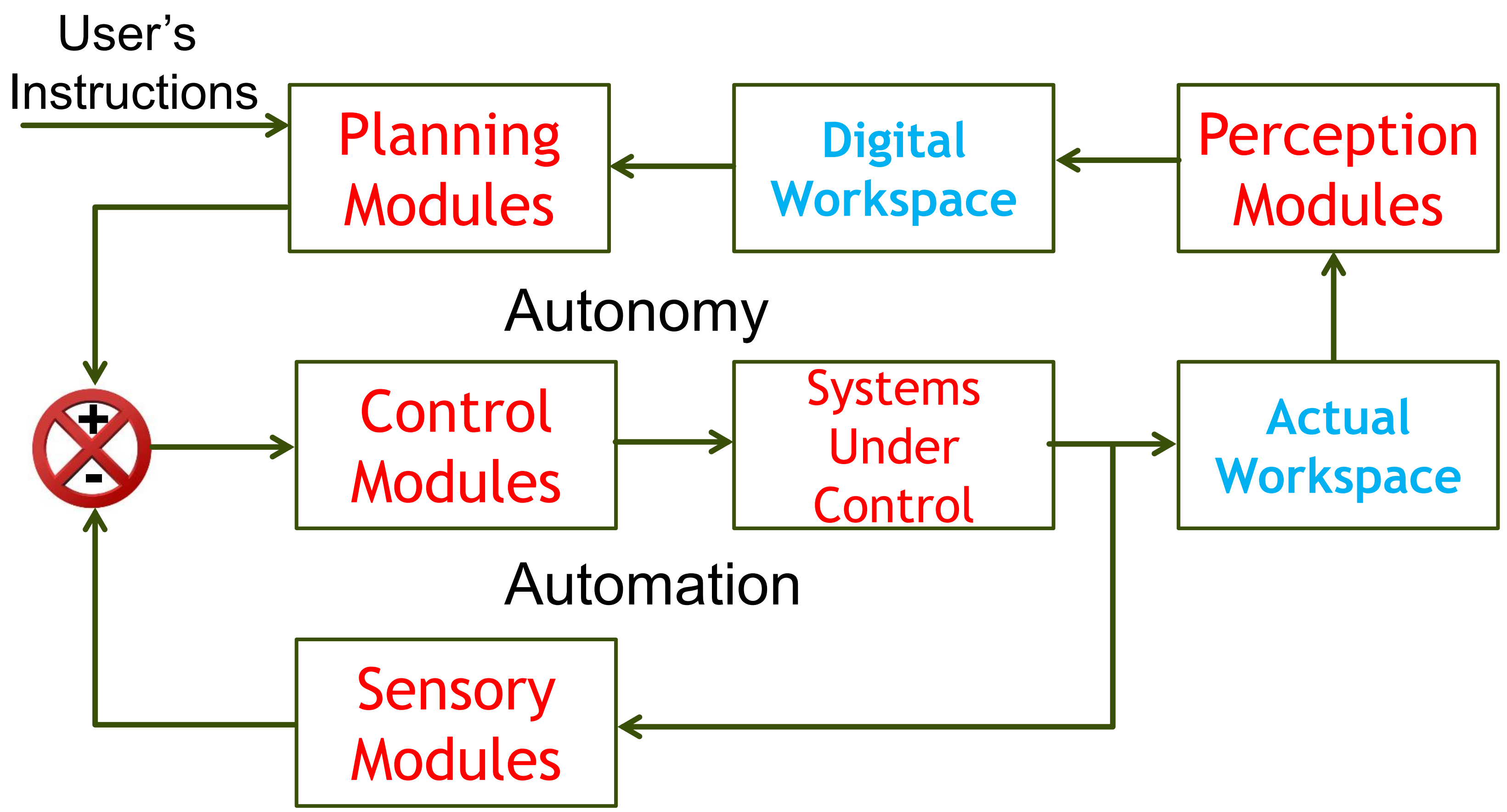
About You

Remember your mission as MAE undergraduates …

- You are here to grow your knowledge and skills so as to be able to design machines with controllable behaviors and hopefully in some intelligent ways.

How to fulfill your mission?

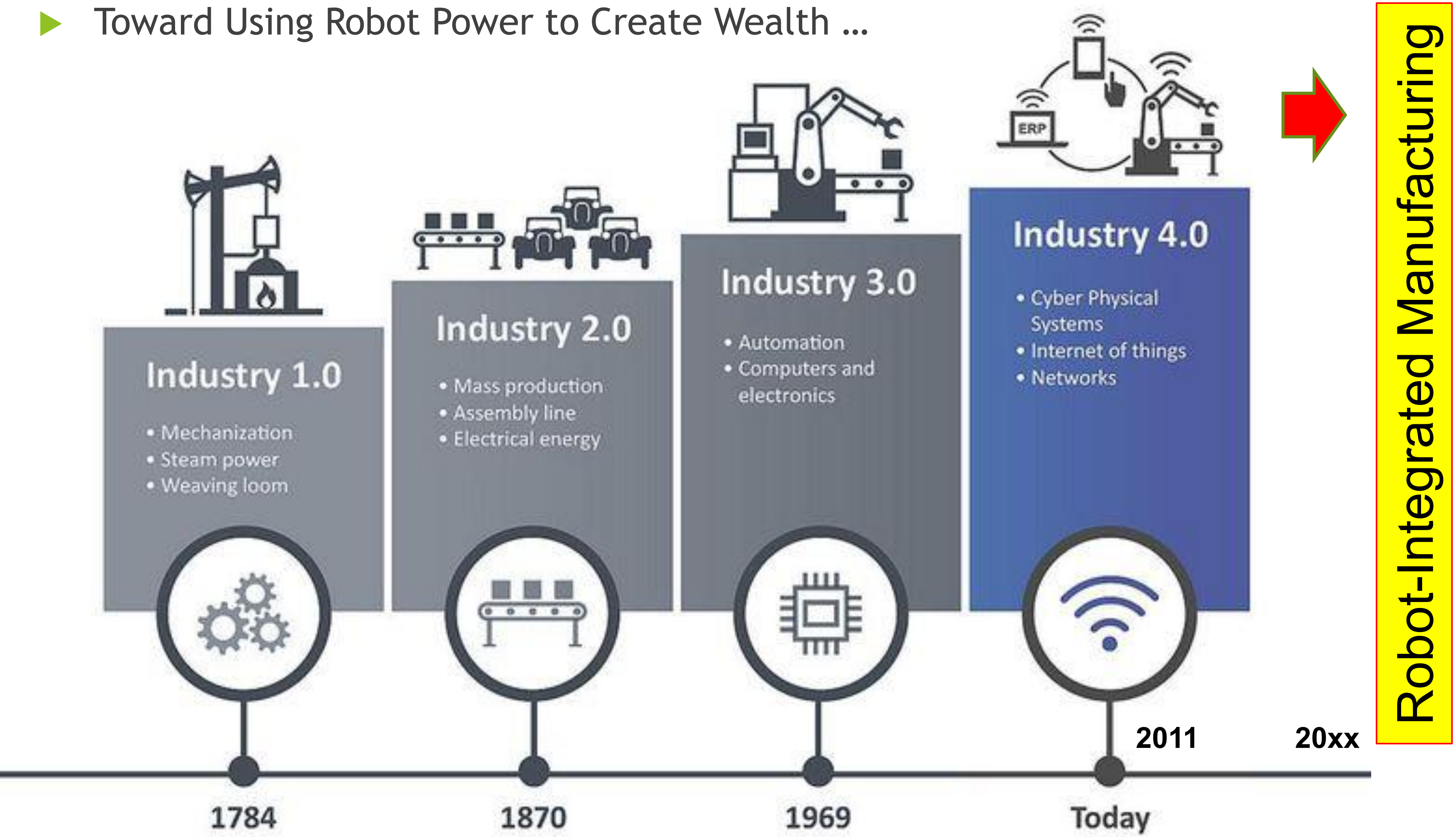
- To apply learnt knowledge and skills into the implementation of the following universal blueprint underlying all the intelligent machines or systems.



About Course

Why to study this course?

- Toward Using Robot Power to Create Wealth …



How to study this course?

- To put yourselves into the mindset of designers of robots as products:
  - Who are the users?
  - What are the needs of users?
  - What are your robots which could meet the needs of your users or buyers?
  - What are the solutions behind the design of your robots?

Market Demands or Needs

↓

Product Specifications

↓

Design Specifications

↓

Conceptual Design

↓

Selection of Materials/Components/Devices

↓

Embodiment Design

↓

Prototyping

↓

Optimizing

↓

Production

↓

Marketing

What to Learn?

Q1: What is the energy flow?

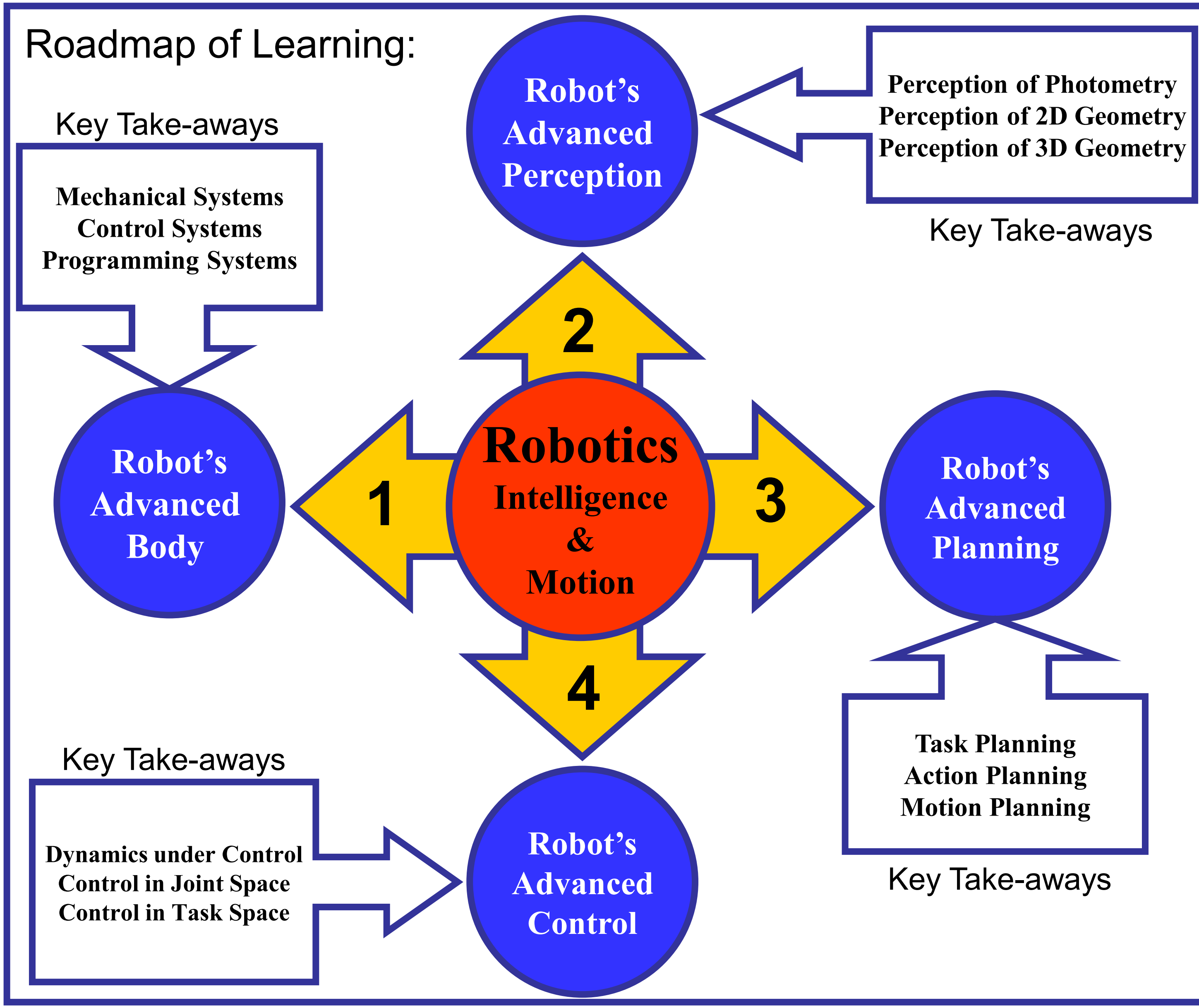
Q2: What is the signal flow?

Q3: What is the knowledge flow?

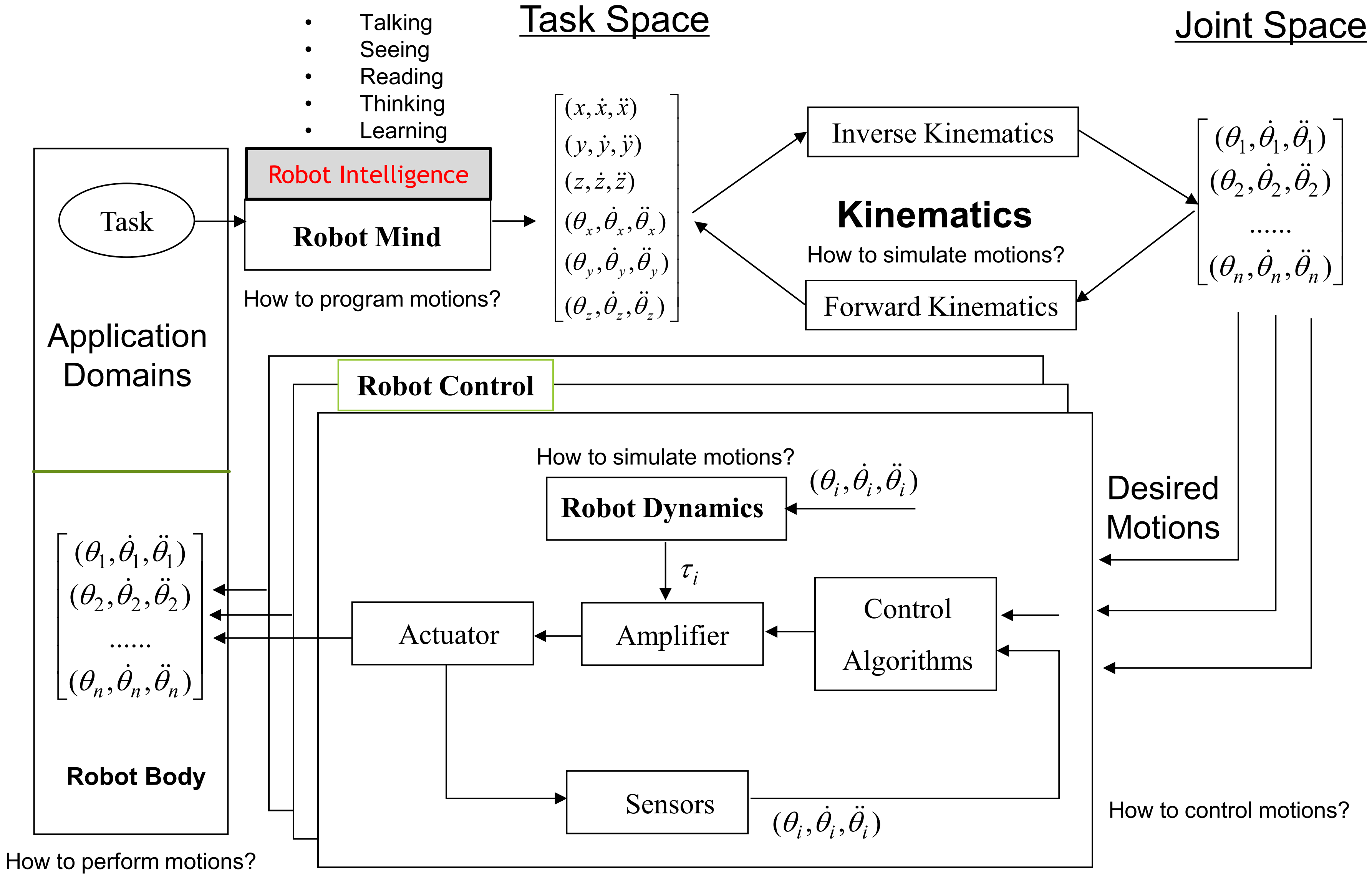
Q4: What is the relationship between energy flow and signal flow?

Q5: What is the relationship between signal flow and knowledge flow?

1. One Machine
2. Two Capabilities
3. Three Benefits
4. Four Pillars



How to Apply?


Talking
Seeing
Reading
Thinking
Learning
Task Space
Joint Space
Task
Robot Intelligence
Robot Mind
How to program motions?
$(x,\dot{x},\ddot{x})$ $(y,\dot{y},\ddot{y})$ $(z,\dot{z},\ddot{z})$ $(\theta_x,\dot{\theta}_x,\ddot{\theta}_x)$ $(\theta_y,\dot{\theta}_y,\ddot{\theta}_y)$ $(\theta_z,\dot{\theta}_z,\ddot{\theta}_z)$
Inverse Kinematics
Kinematics
How to simulate motions?
Forward Kinematics
$(\theta_1,\dot{\theta}_1,\ddot{\theta}_1)$ $(\theta_2,\dot{\theta}_2,\ddot{\theta}_2)$ ...... $(\theta_n,\dot{\theta}_n,\ddot{\theta}_n)$
Application Domains
Robot Control
How to simulate motions?
$(\theta_i,\dot{\theta}_i,\ddot{\theta}_i)$
Robot Dynamics
τ_i
Desired Motions
Control Algorithms
Actuator
Amplifier
$(\theta_1,\dot{\theta}_1,\ddot{\theta}_1)$ $(\theta_2,\dot{\theta}_2,\ddot{\theta}_2)$ ...... $(\theta_n,\dot{\theta}_n,\ddot{\theta}_n)$
Robot Body
Sensors
$(\theta_i,\dot{\theta}_i,\ddot{\theta}_i)$
How to control motions?
How to perform motions?

Terminology Alert

- Advanced Robotics is about the study of advanced robots which could perform tasks in some intelligent ways.

- Advanced Robot is a machine which has
  - two capabilities (automatic control and autonomous control),
  - three benefits and
  - four pillars.

Today's Lectures …

- Module 1: Robot's Advanced Body
- Module 2: Robot's Advanced Perception
- Module 3: Robot's Advanced Planning
- Module 4: Robot's Advanced Control

Module 3

MA4825 Robotics

Robot's Advanced Planning

Xie Ming, PhD (France)
http://personal.ntu.edu.sg/mmxie

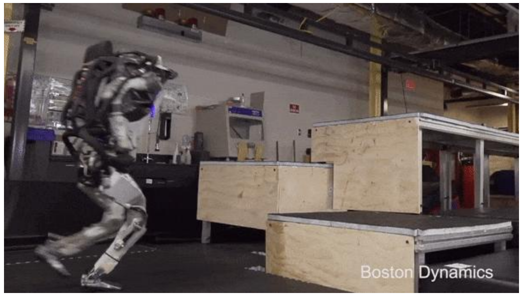
Outline of Module 3

Conceptual World

- Task Planning
- Action Planning
- Motion Planning

Physical World

- Path Planning
- Trajectory Planning



Outline of Module 3

Conceptual World

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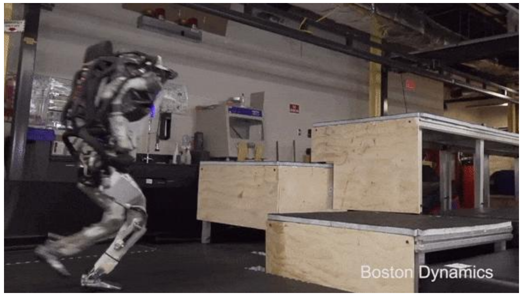

School of Mechanical & Aerospace Engineering

Design, Machine, Control, Intelligence

Module 3

MA4825 Robotics

Lecture 1

Task Planning

Xie Ming, PhD (France)

http://personal.ntu.edu.sg/mmxie

Outline of Lecture 1

- Purpose of Task Planning
- Input to Task Planning
- Output from Task Planning
- Process of Task Planning

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- Purpose of Task Planning
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Definition of Task Planning

- Task planning is a process which takes job descriptions as input and produces a sequence of tasks as output, which are to be performed by robots.

What are tasks which could be performed by robots?

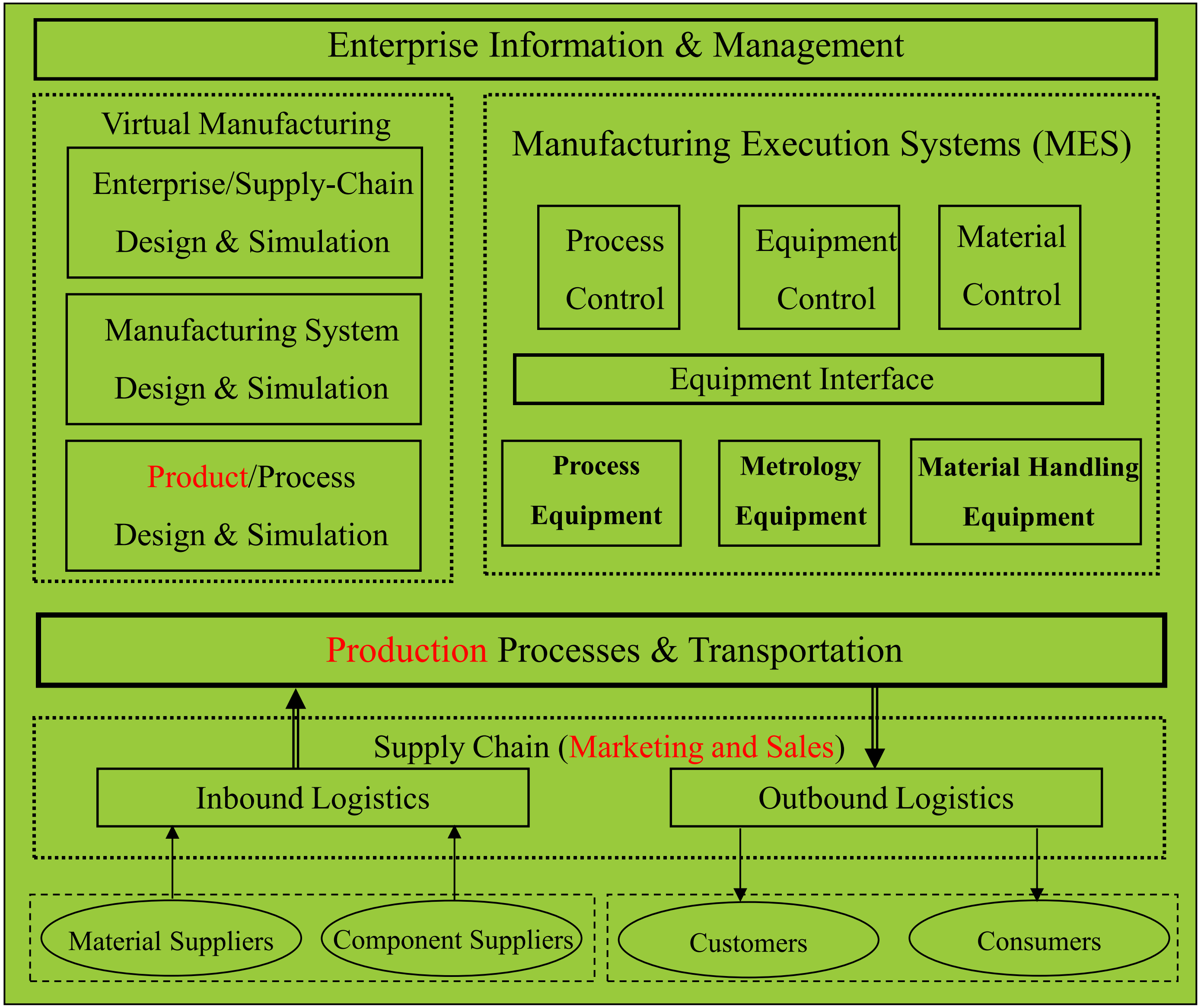
Where to find jobs for robots to do?

- Jobs in Manufacturing
- Jobs in Transportation
- Jobs in Services
- Jobs in Agriculture
- Jobs in Defence
- Jobs in Education
- Others

Scenario of Task Planning in Manufacturing

Many Jobs

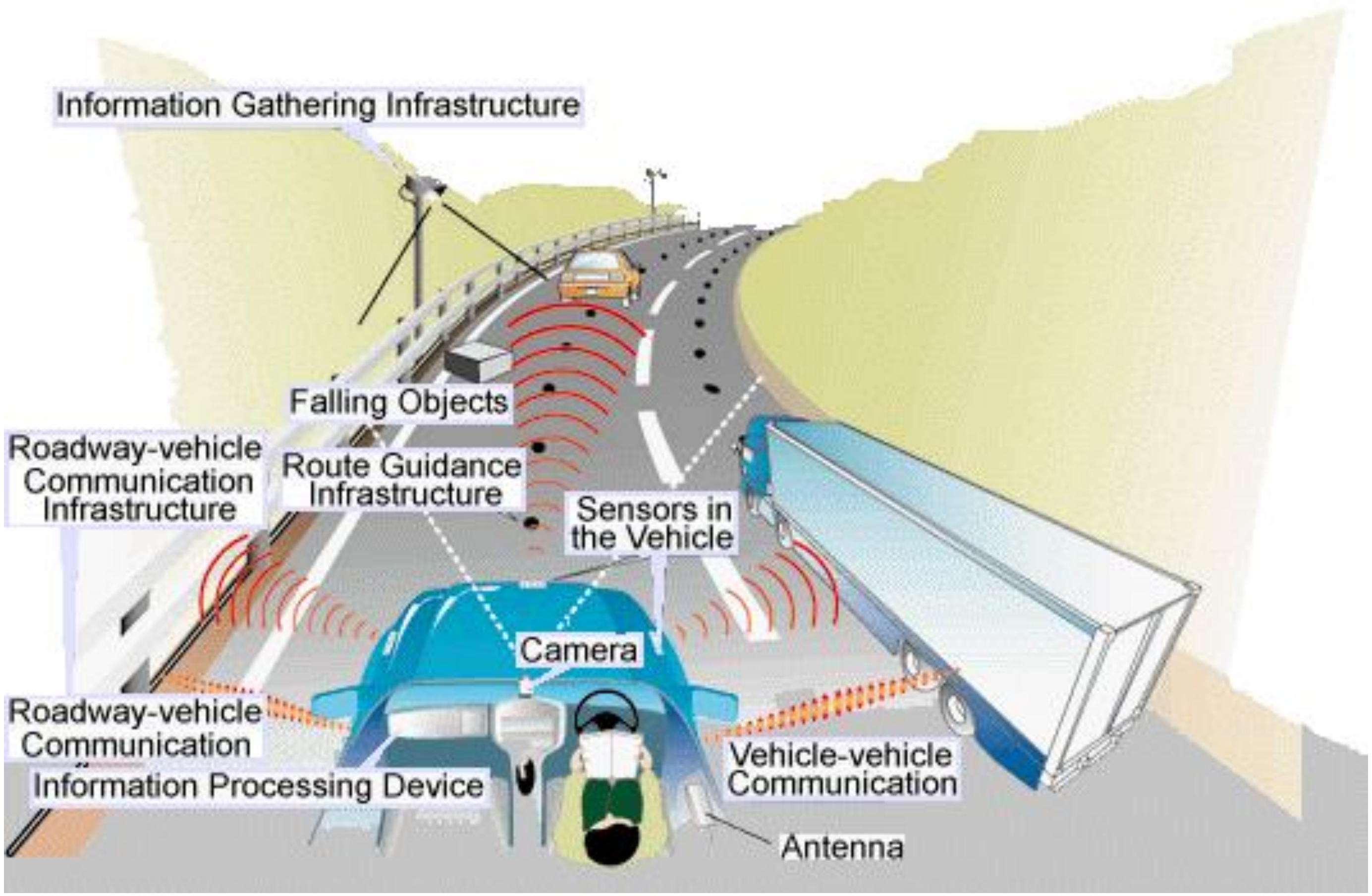
Many Tasks



Example of Job in Smart Factory

Example of Job of Producing Cars

Example of Job of Transporting People and Goods

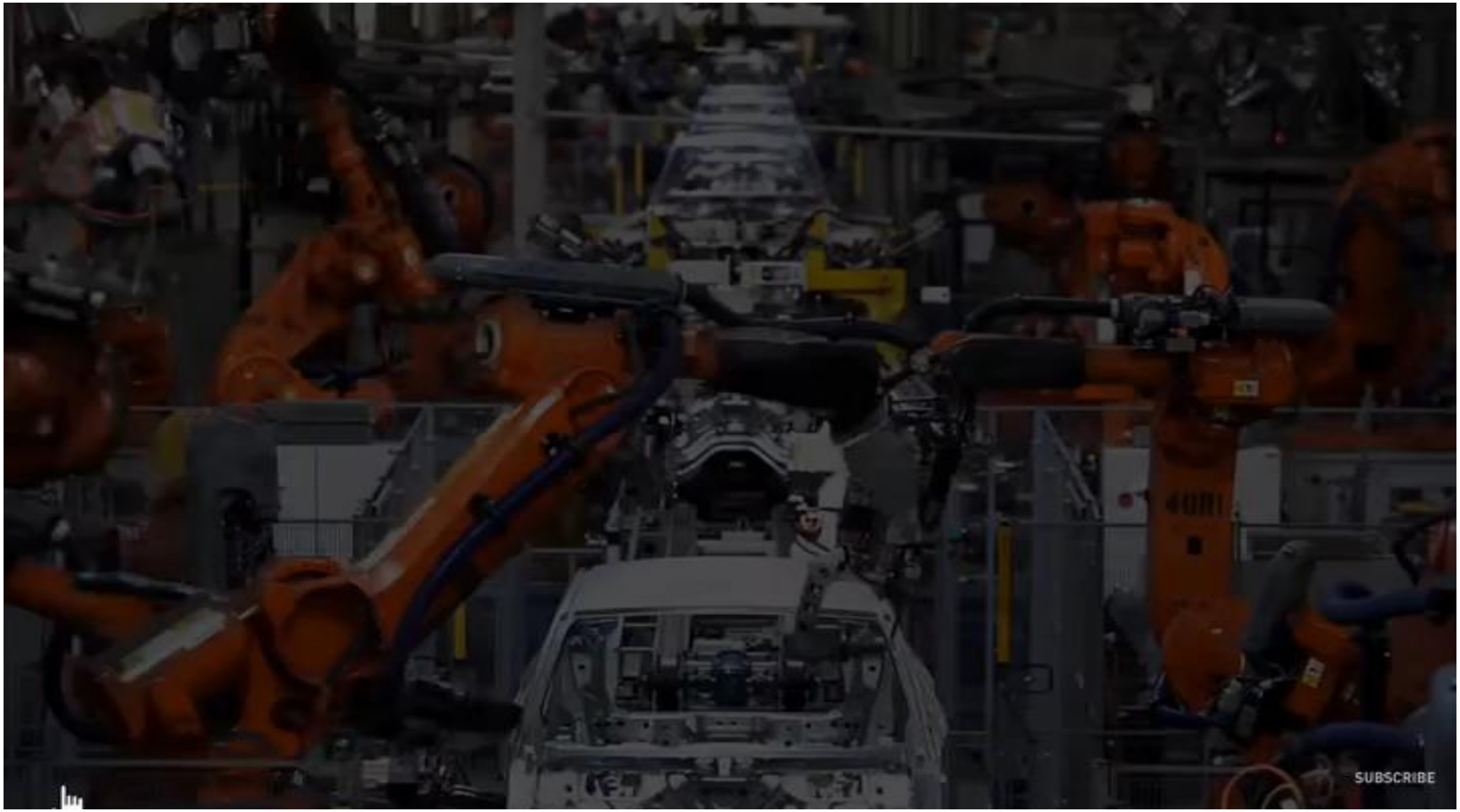


Example of Job in Agriculture

Example of Job in Hospital

Example of Job in Restaurant

Example of Job in School



Example of Job in Smart City

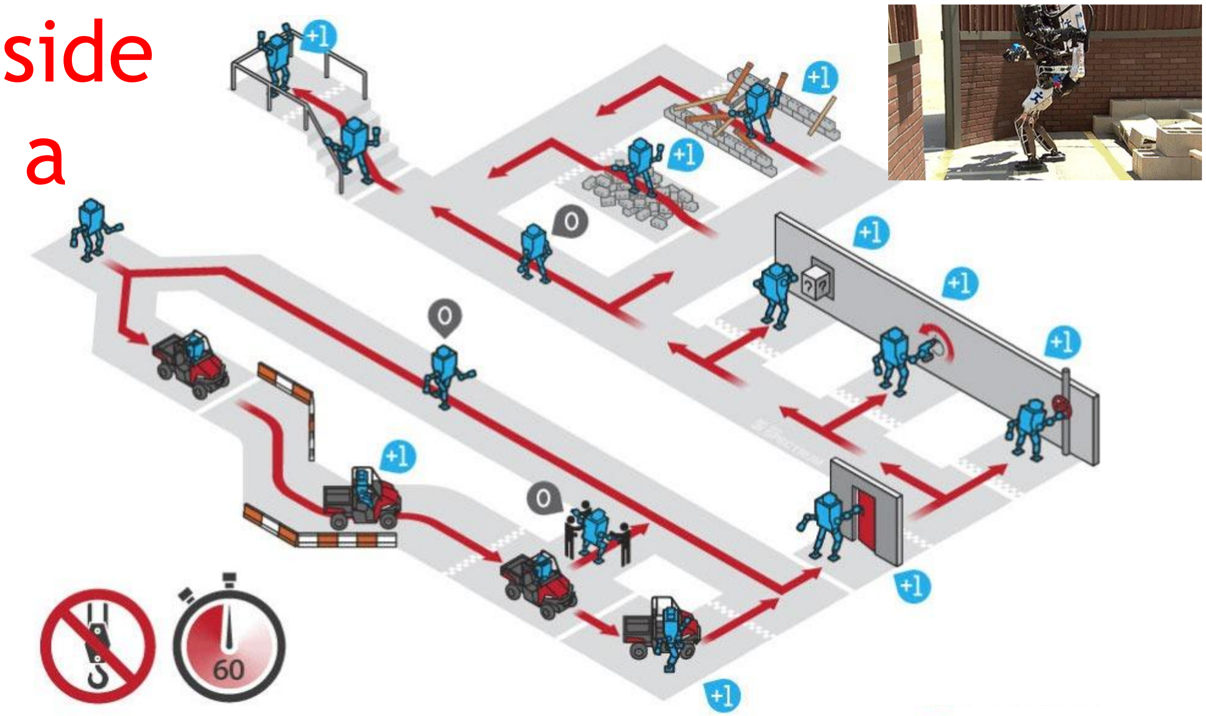
Outline of Lecture 1

- Purpose of Task Planning
- Input to Task Planning
- Output from Task Planning
- Process of Task Planning

Input to Task Planning

- The input to task planning is the description of a given job in the form of natural languages.

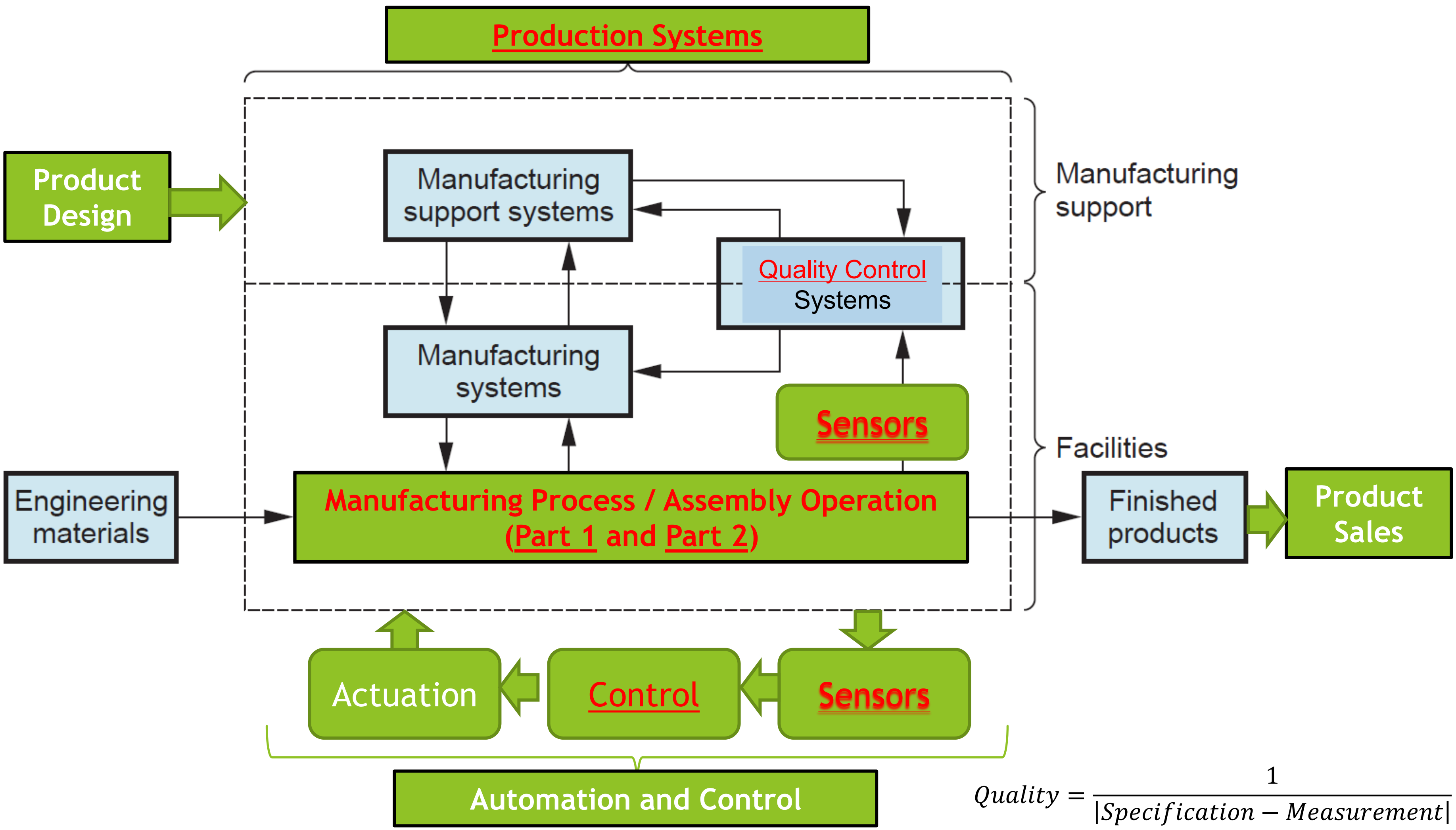
What should be inside the description of a given job?



Answer:

- 1. The outcome of job to be achieved
- 2. The space in which the job is to be undertaken
- 3. The time within which the job is to be undertaken

Jobs in Manufacturing

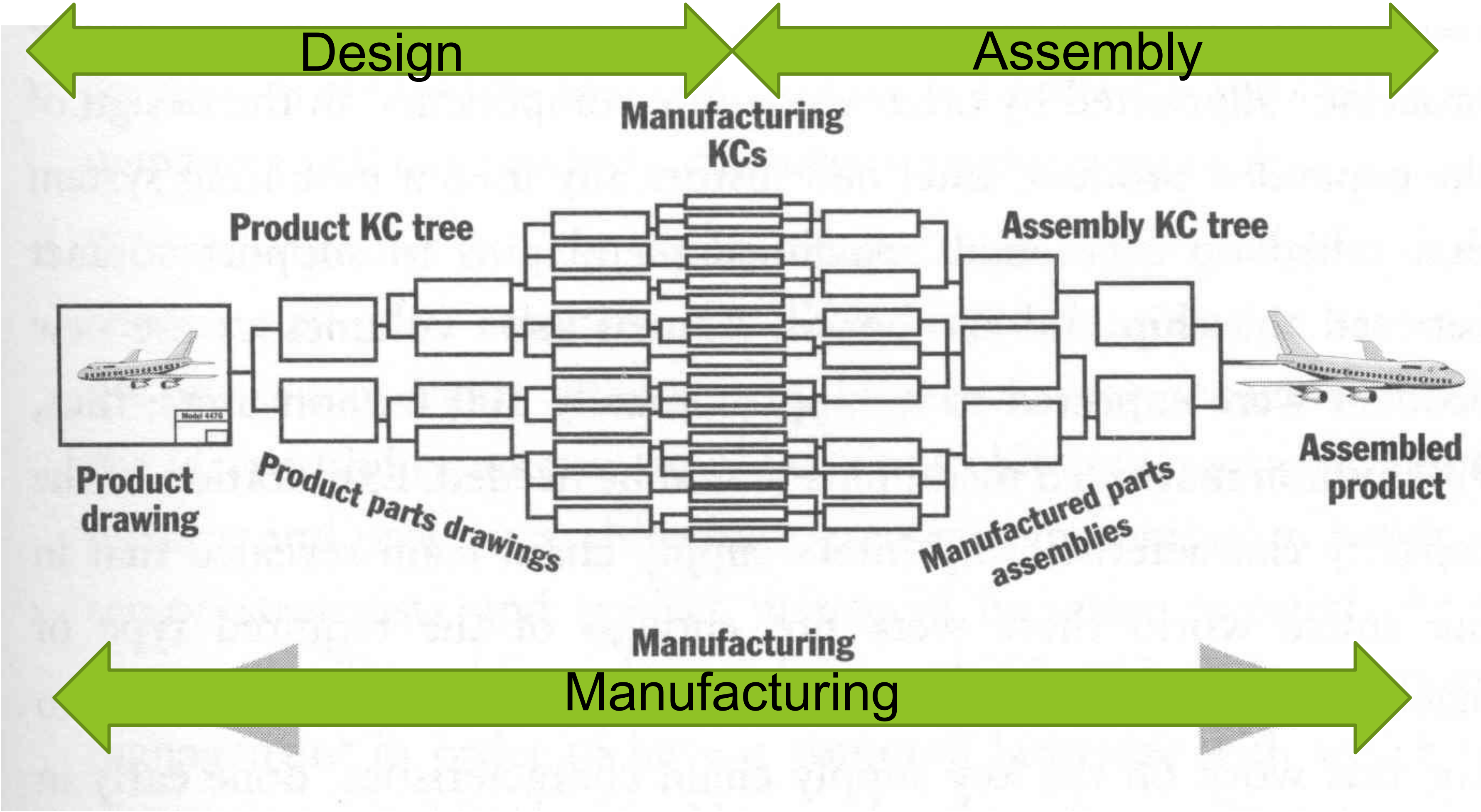


$$Quality = \frac{1}{|Specification - Measurement|}$$

Example of Manufacturing Jobs

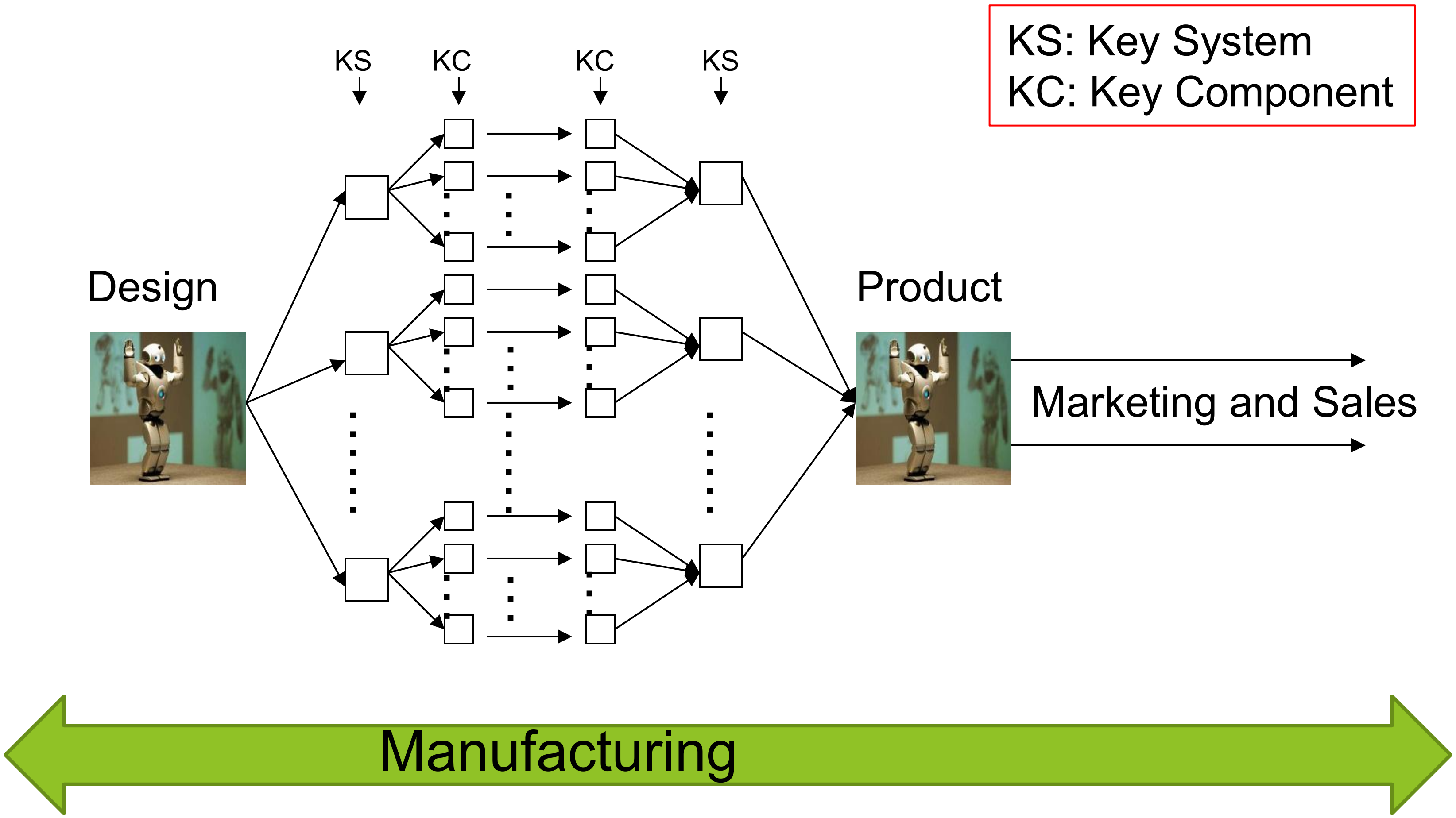
KS: Key System
KC: Key Component

- From Design to Marketing/Sales



Another Example of Manufacturing Job

- From Design to Marketing/Sales



Outline of Lecture 1

- Purpose of Task Planning
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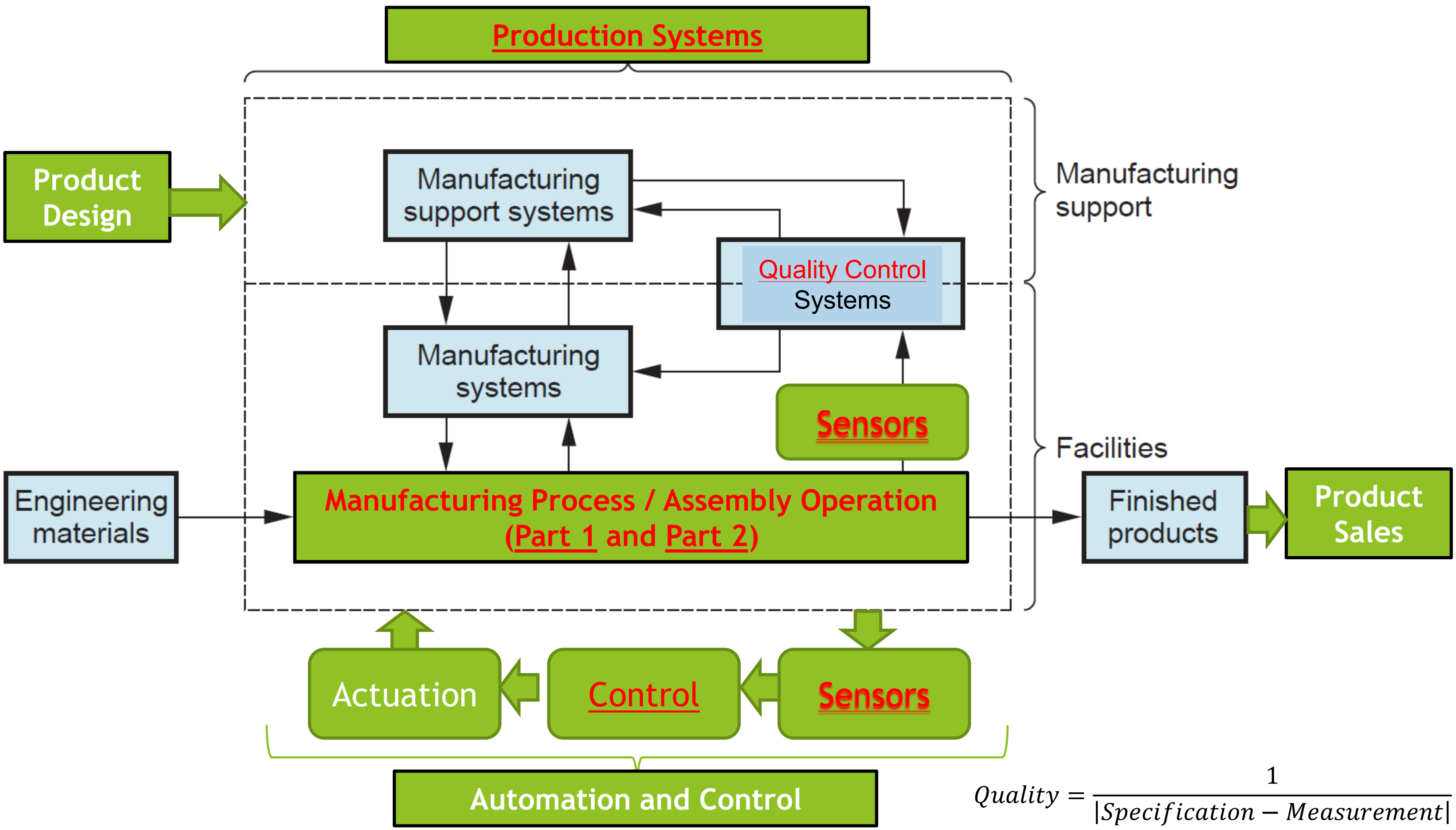
- Process of Task Planning

Output of Task Planning

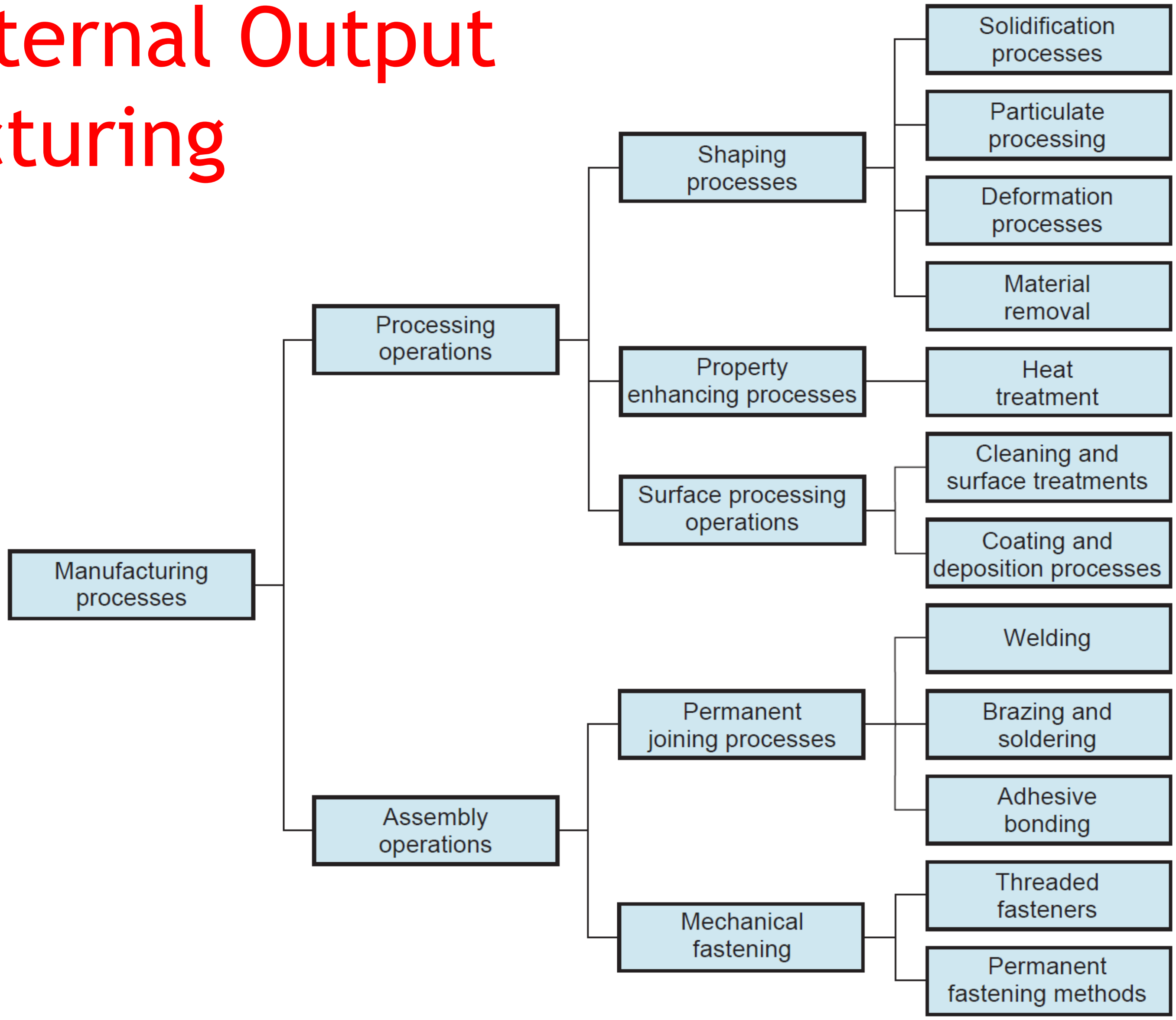
- The output of task planning is a sequence of tasks which could lead to the accomplishment of intended outcome of a given job.

Internal Output of Manufacturing Systems



$$Quality = \frac{1}{|Specification - Measurement|}$$

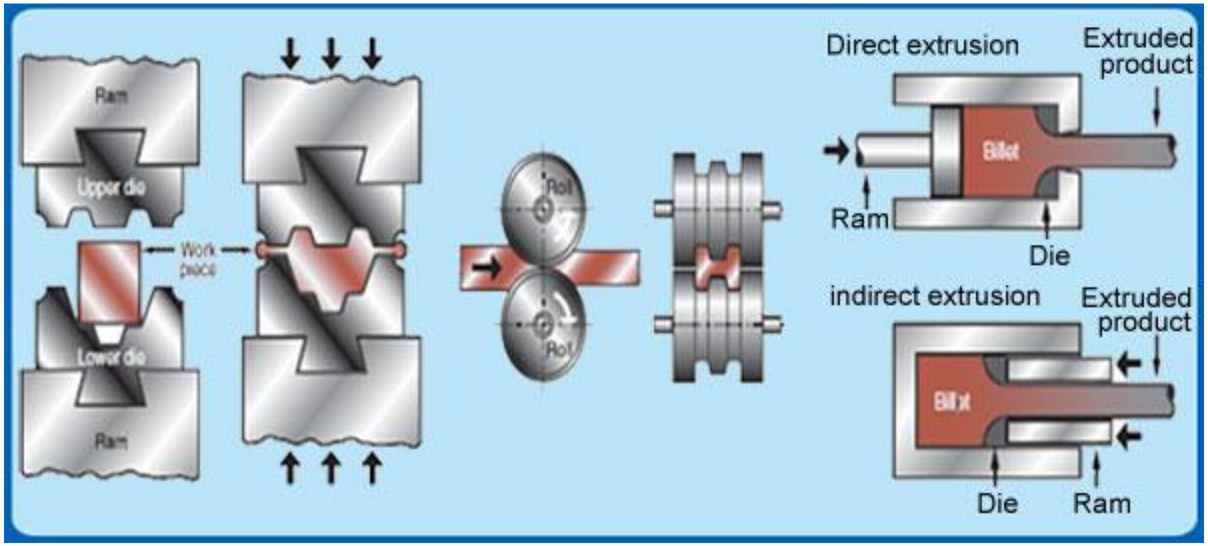
Tasks as Internal Output in Manufacturing

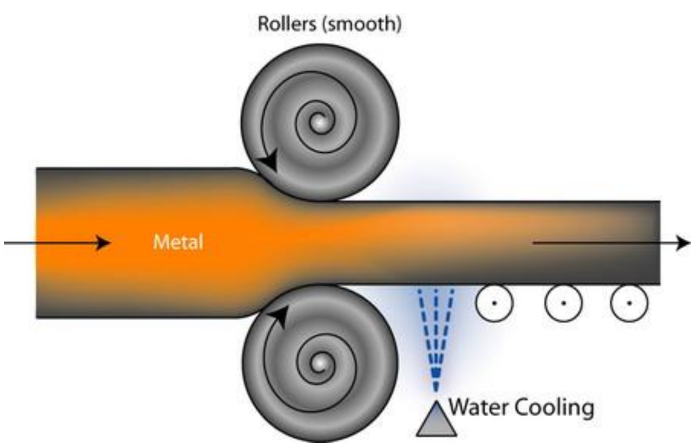


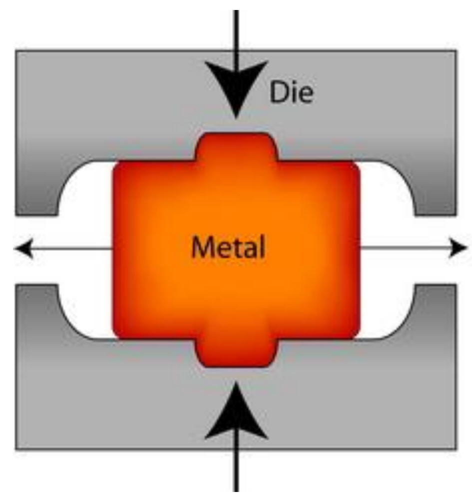
In general, there are five typical types of jobs in manufacturing:

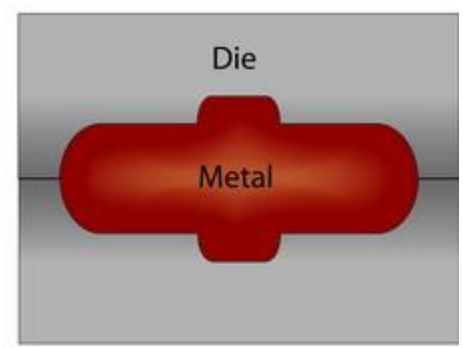
- 1. Doing deformation processes
- 2. Doing material removal processes
- 3. Doing solidification processes
- 4. Doing assembly processes
- 5. Doing material handling processes

Examples of Deformation Processes

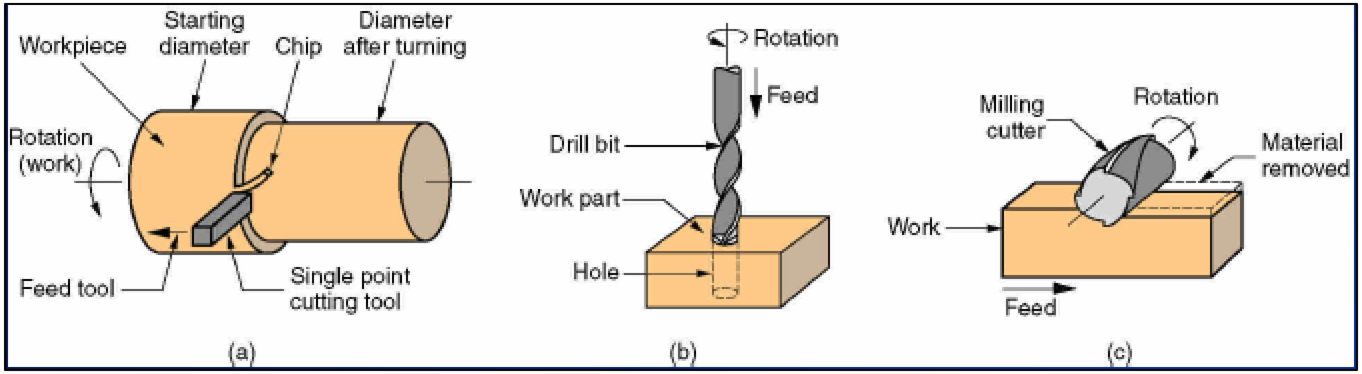


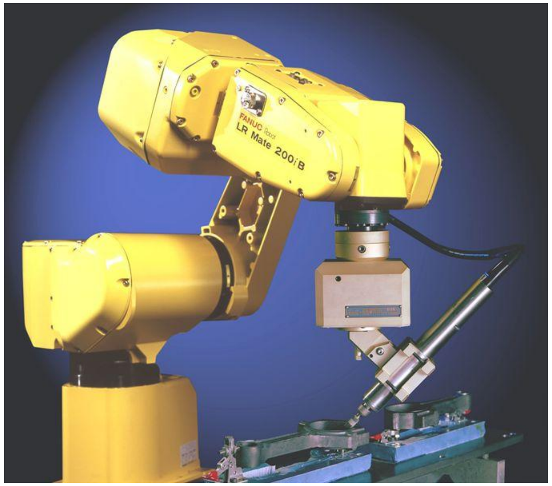






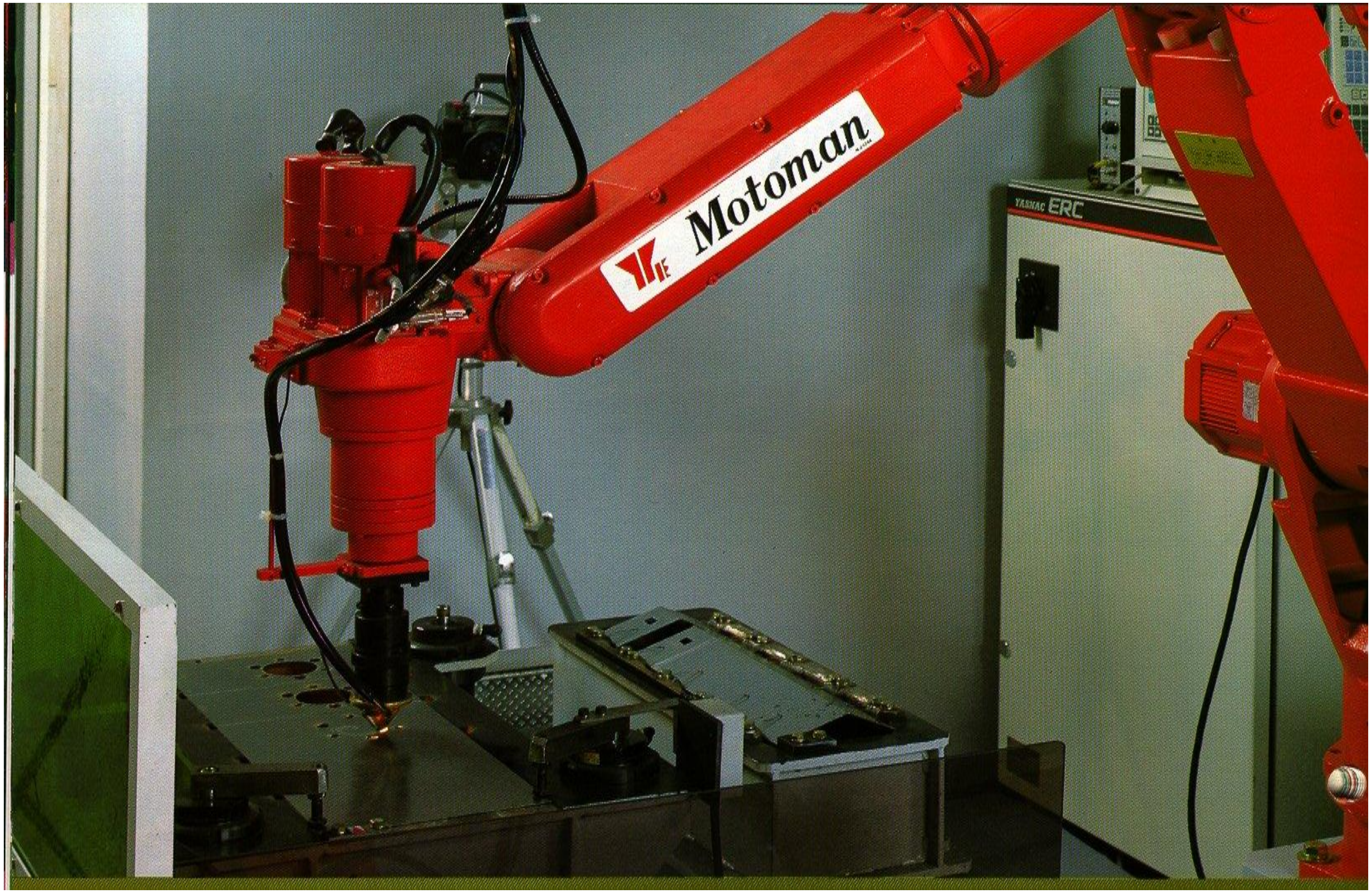
Examples of Material Removal Processes



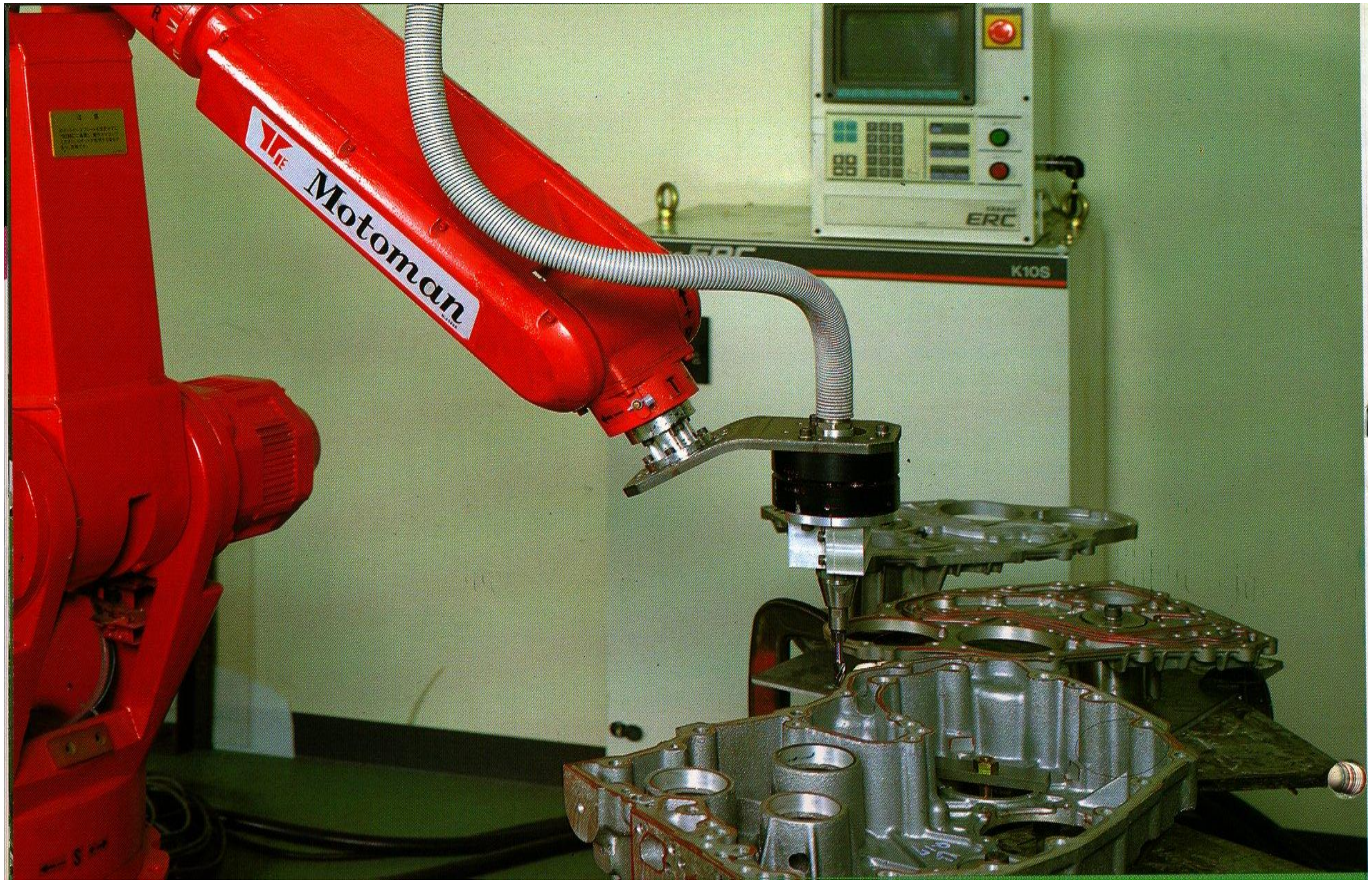


Sculpturing robots

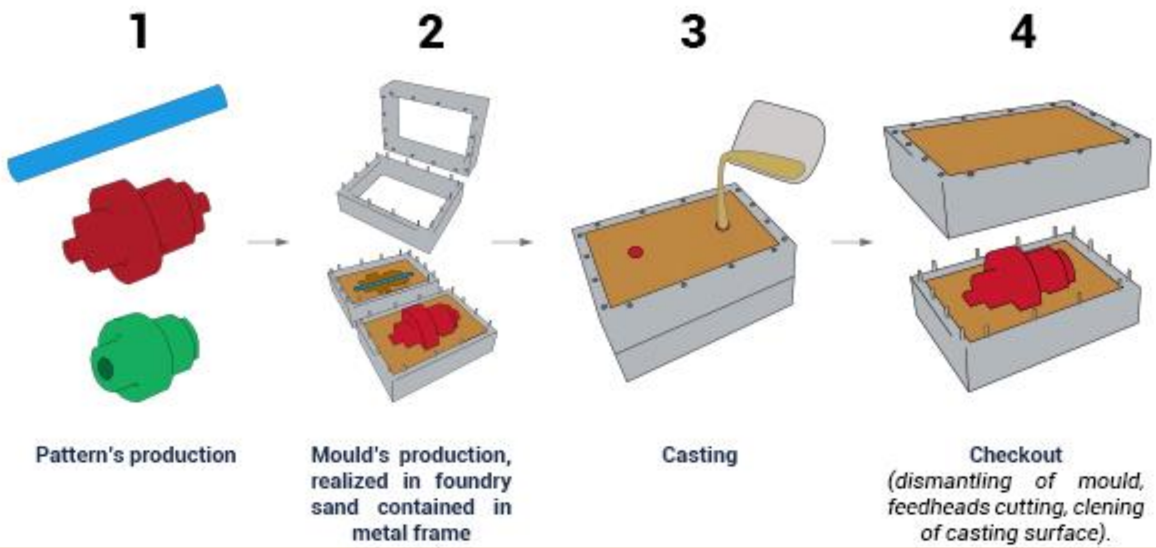
Example of Material Removal Processes

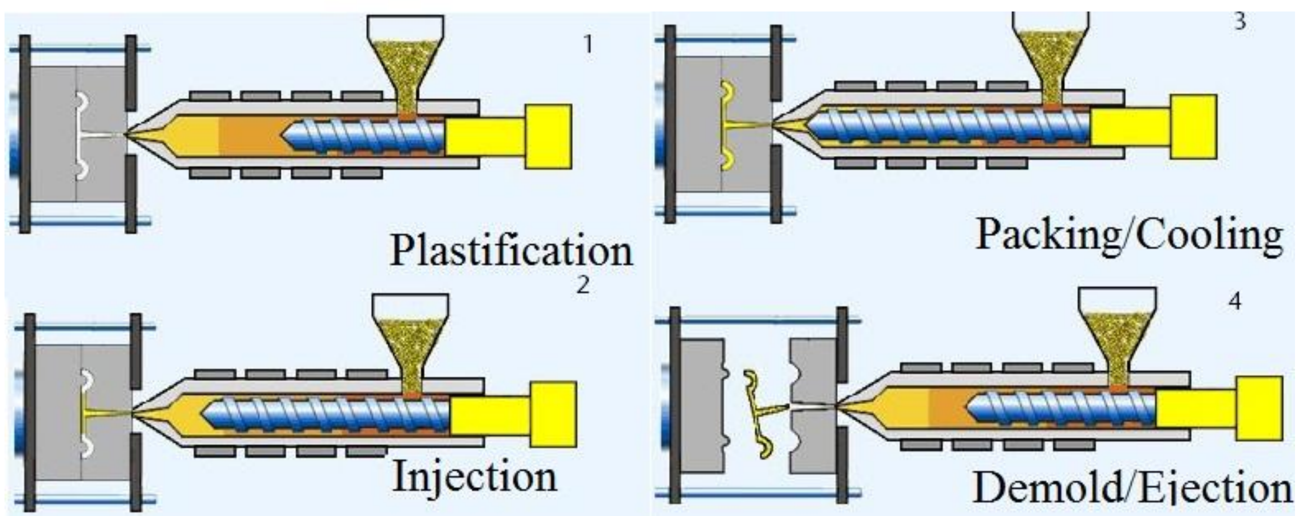


Example of Material Removal Processes



Examples of Solidification Processes





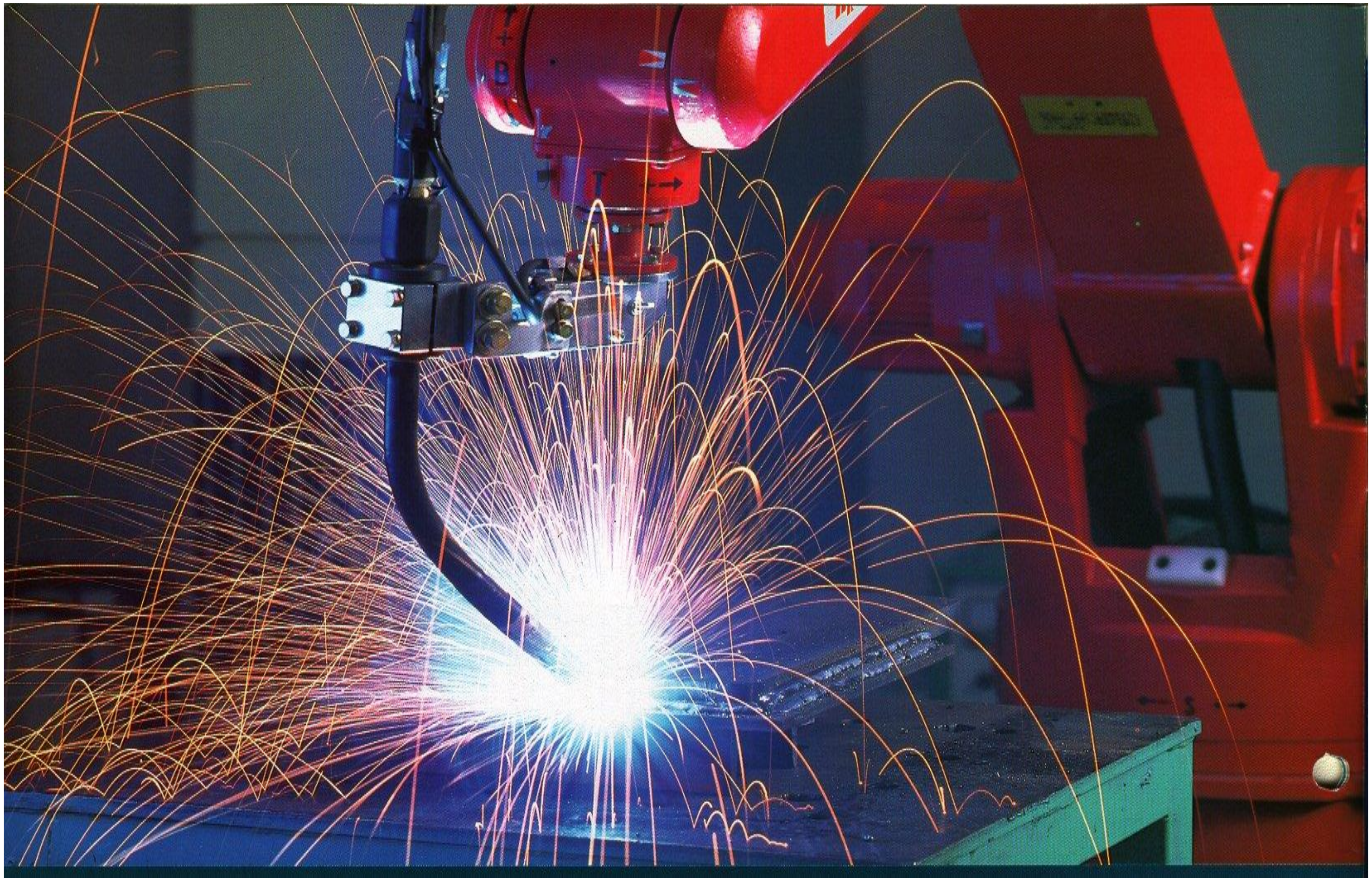
Example of Solidification Processes

Example of Solidification Processes

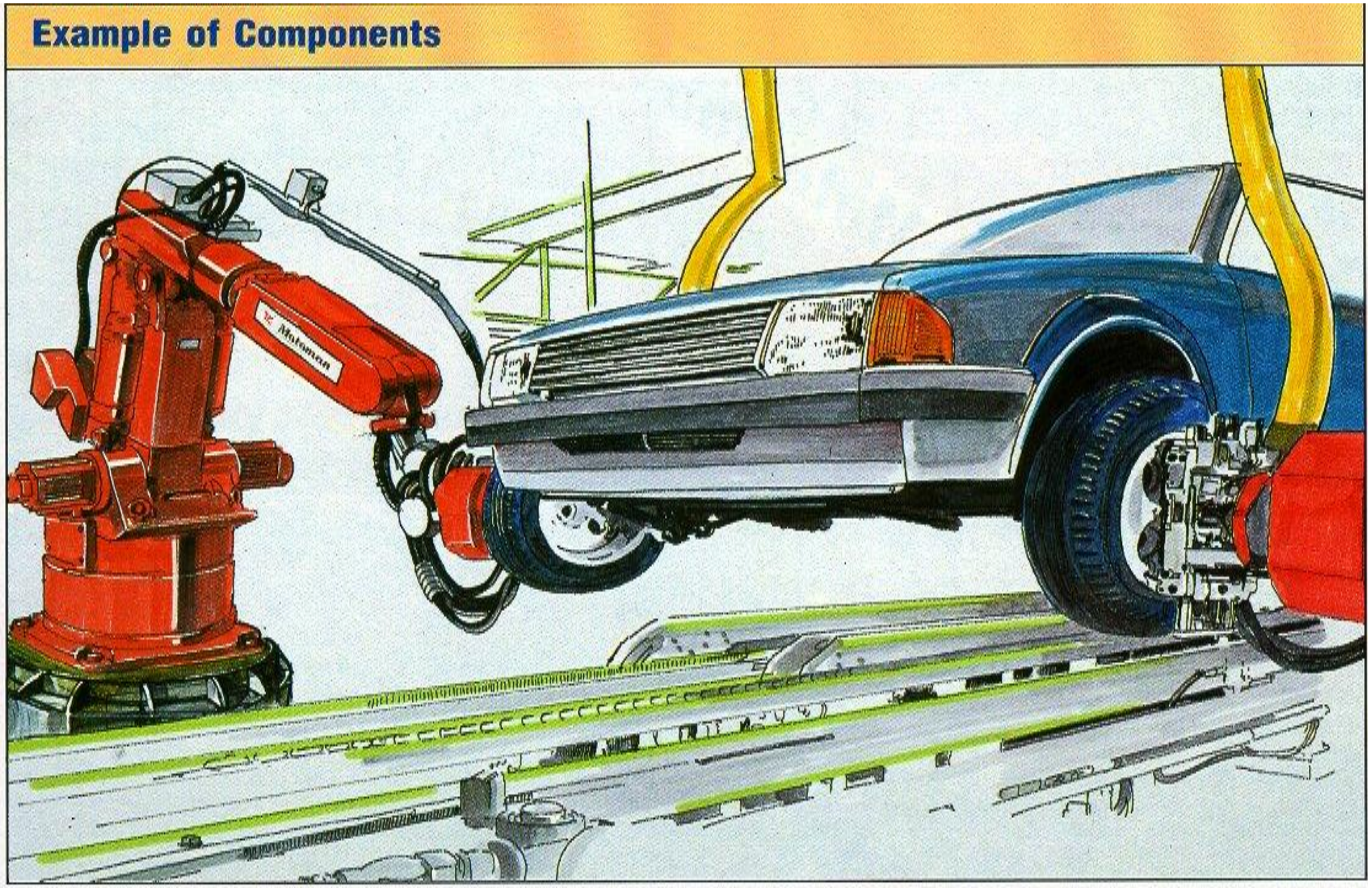
Examples of Assembly Processes

Example of Assembly Processes

Example of Assembly Processes

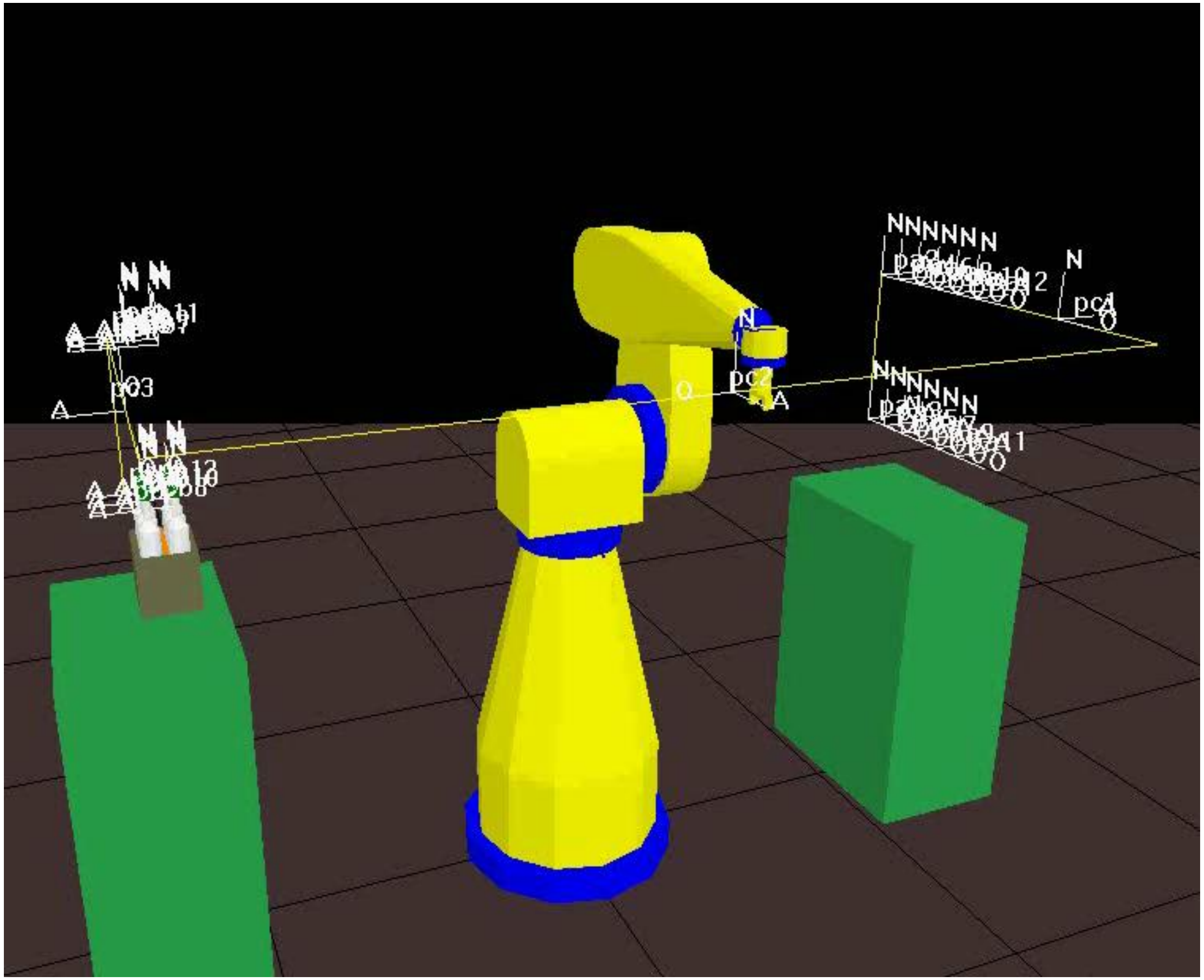


Example of Assembly Processes



Example of Material Handling Processes

Example of Material Handling Processes



Example of Material Handling Processes

Example of Material Handling Processes

Outline of Lecture 1

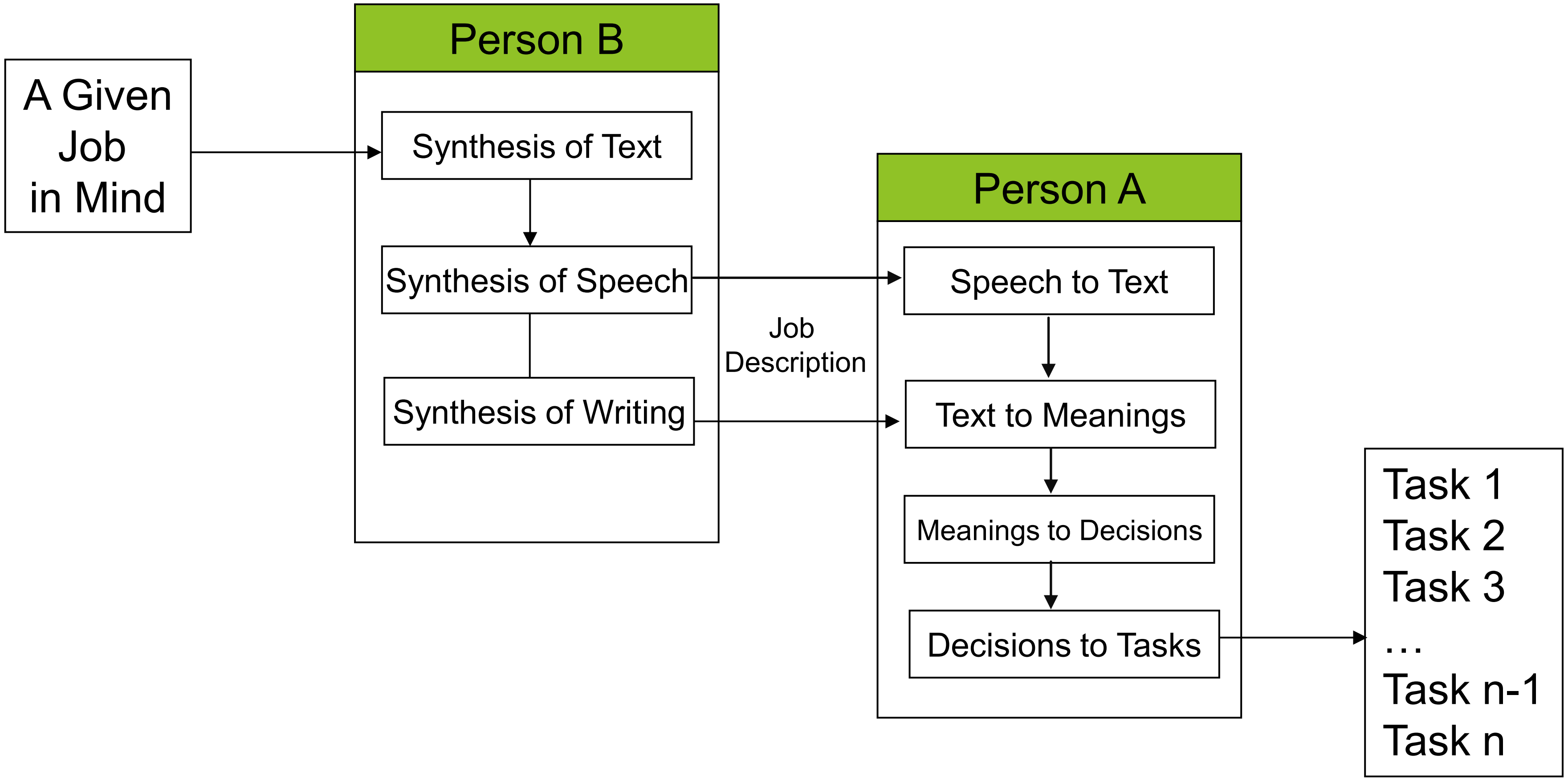
- Purpose of Task Planning
- Input to Task Planning
- Output from Task Planning
- Process of Task Planning

How to do task planning?

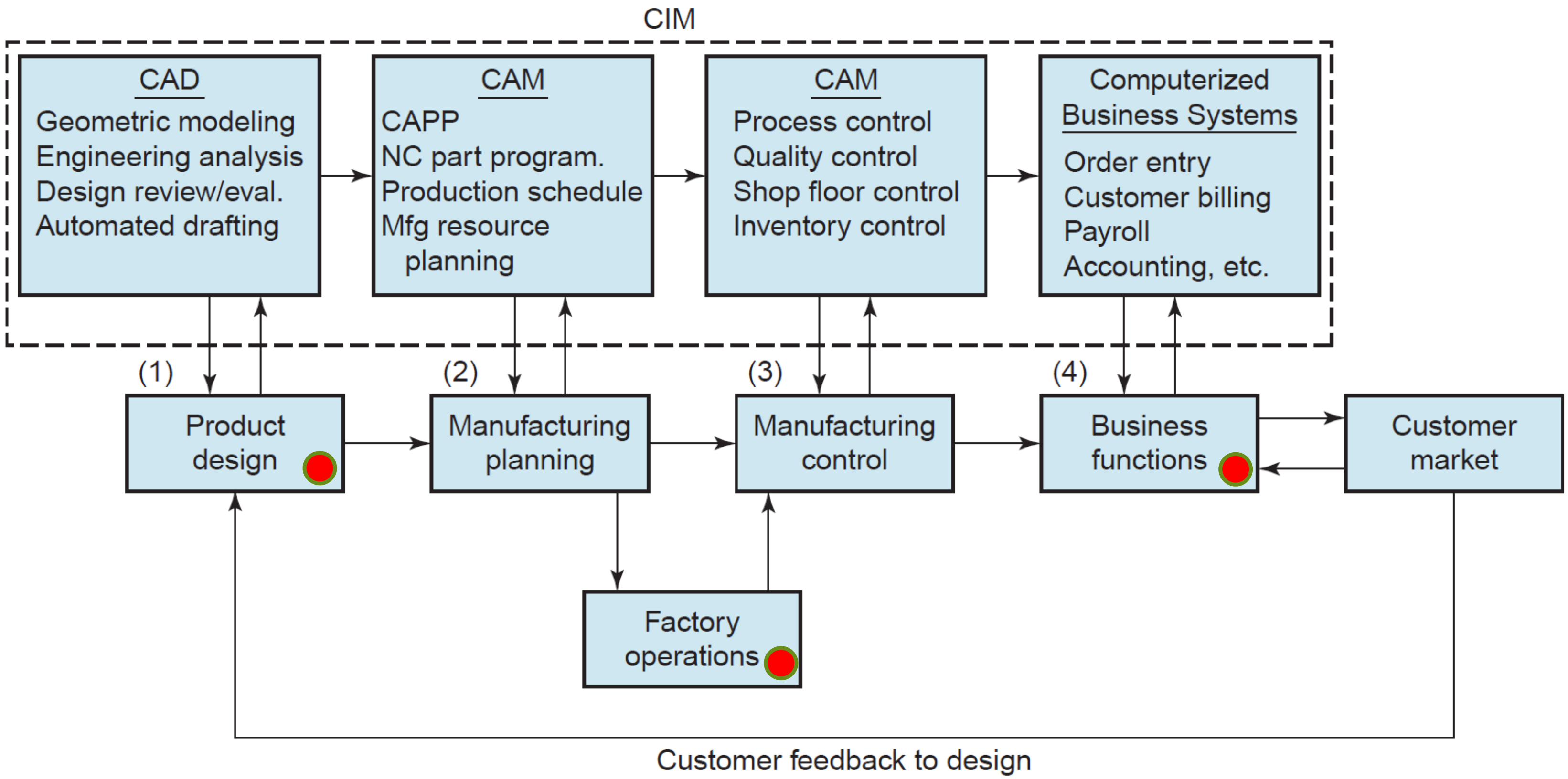
- Scenario 1: You are the buyers or users of robots
  - Human-Asisted Task Planning

- Scenario 2: You are the designers of robots
  - Task Planning by Robot

Scenario of Human-Asisted Task Planning

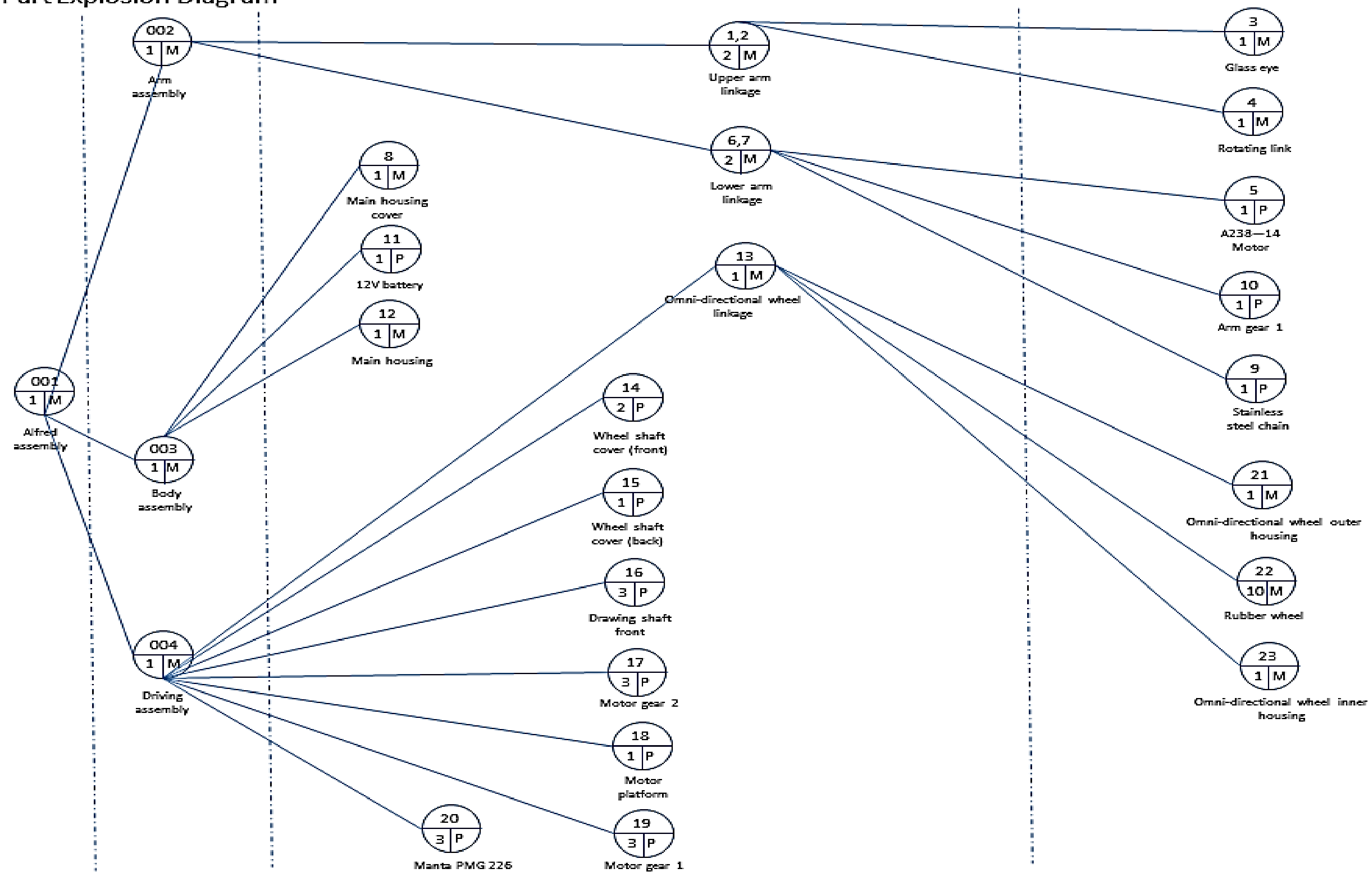


Example of Activities Related to Jobs in Manufacturing



From Marketing/Sales Activities to Product Design …

Part Explosion Diagram



From Product Design to Product-Making Planning

Yearly Quantity

Purchased Parts	Produced Parts

↓

Monthly Quantity

Purchased Parts	Produced Parts

↓

Weekly Quantity

Purchased Parts	Produced Parts

↓

Daily Quantity

Purchased Parts	Produced Parts

From Product-Making Planning to Process-Deployment Planning

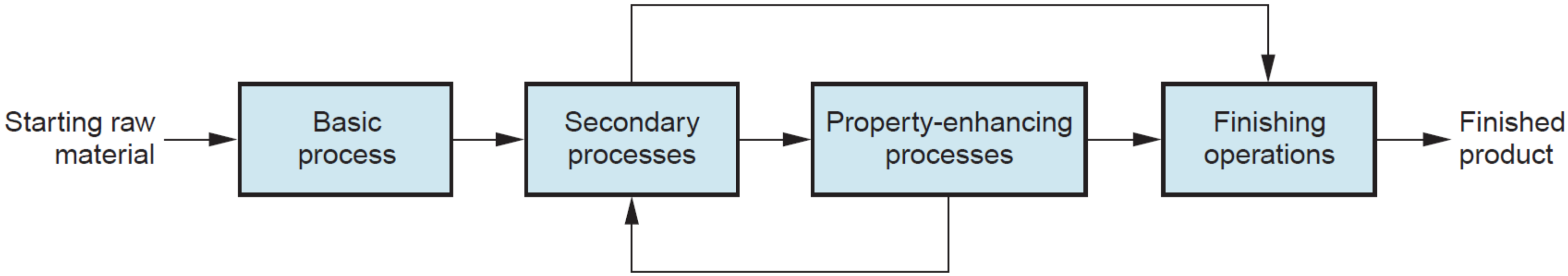
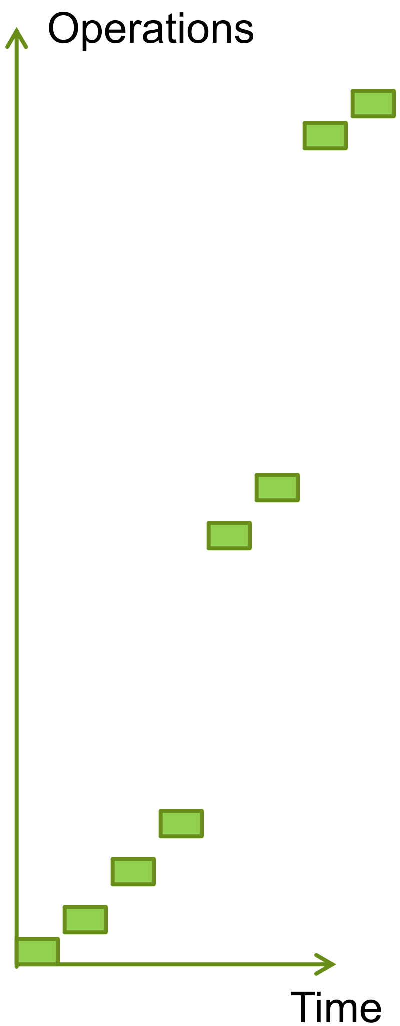


TABLE 40.2 Some typical process sequences.

Basic Process	Secondary Process(es)	Property-Enhancing Processes	Finishing Operations
Sand casting	Machining	(none)	Painting
Die casting	(none, net shape)	(none)	Painting
Casting of glass	Pressing, blow molding	(none)	(none)
Injection molding	(none, net shape)	(none)	(none)
Rolling of bar stock	Machining	Heat treatment (optional)	Electroplating
Rolling of sheet metal	Blanking, bending, drawing	(none)	Electroplating
Forging	Machining (near net shape)	(none)	Painting
Extrusion of aluminum	Cut to length	(none)	Anodize
Atomize metal powders	Pressing of powder metal part	Sintering	Painting

From Process-Deployment Planning to Work-Schedule Planning

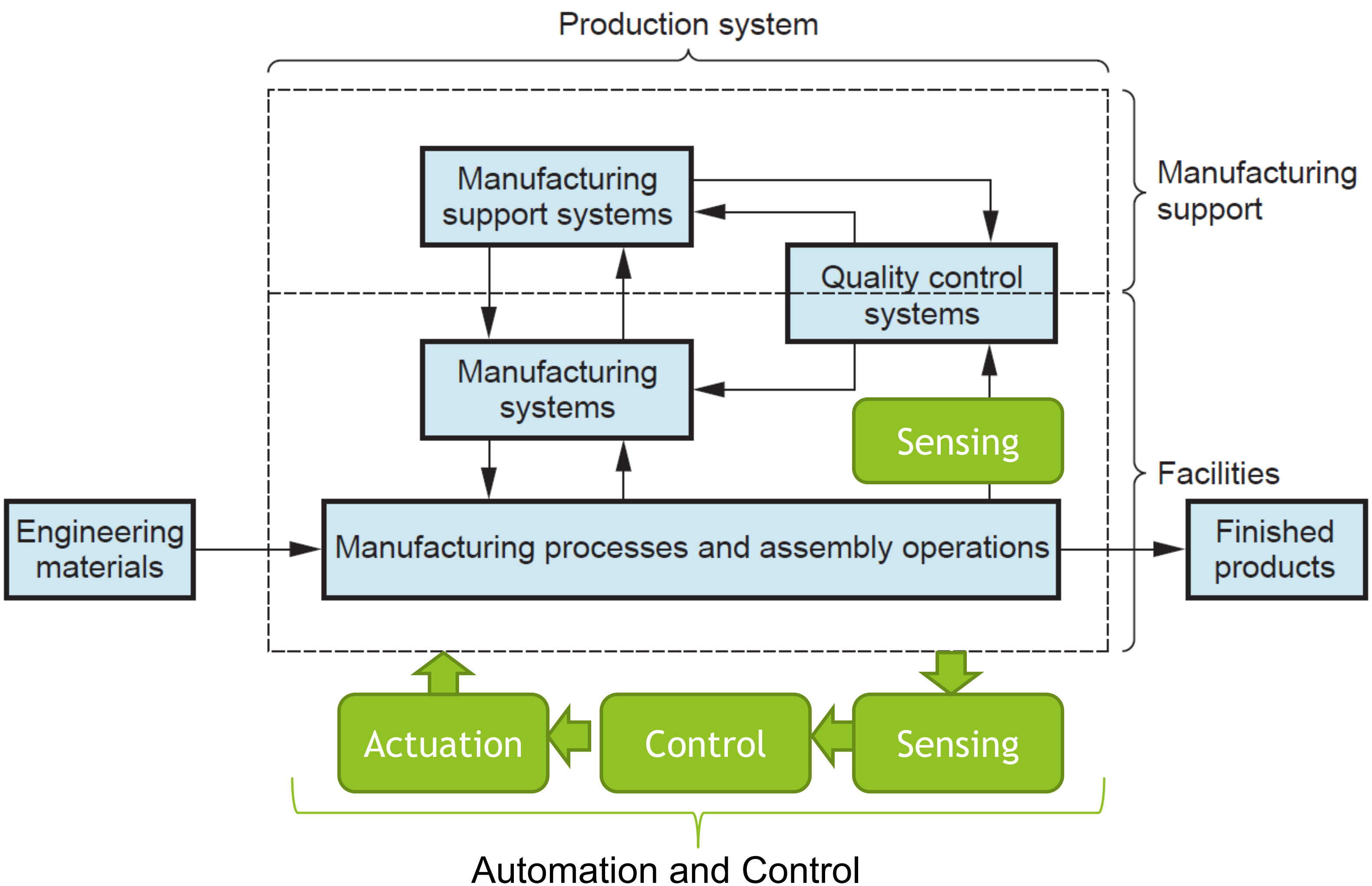


Part No:	Part Name:	Rev.	
031393	Housing, valve	2	Page 1 of 2

Matl:	Size:	Planner:	Date:
416 Stainless	2.0 dia × 5. long	MPG	3/13/XX

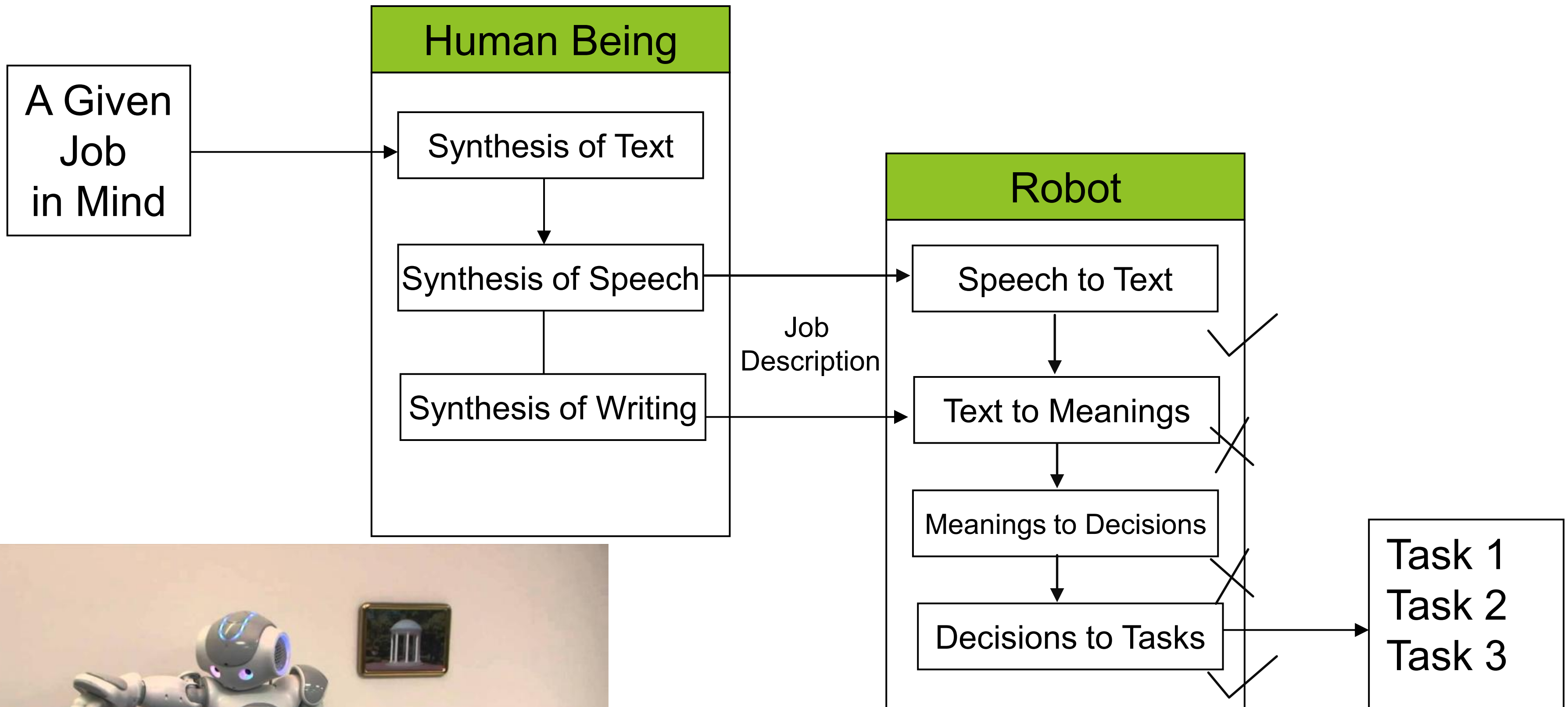
No.	Operation	Dept.	Machine	Tooling, gages	Setup time	Cycle time
10	Face; rough & finish turn to 1.473 ± 0.003 dia. × 1.250 ± 0.003 length; face shoulder to 0.313 ± 0.002; finish turn to 1.875 ± 0.002 dia.; form 3 grooves at 0.125 width × 0.063 deep.	L	325	G857	1.0 h	8.22 m
20	Reverse; face to 4.750 ± 0.005 length; finish turn to 1.875 ± 0.002 dia.; drill 1.000 + 0.006, −0.002 dia. axial hole.	L	325		0.5 h	3.10 m
30	Drill & ream 3 radial holes at 0.375 ± 0.002 dia.	D	114	F511	0.3 h	2.50 m
40	Mill 0.500 ± 0.004 wide × 0.375 ± 0.003 deep slot.	M	240	F332	0.3 h	1.75 m
50	Mill 0.750 ± 0.004 wide × 0.375 ± 0.003 deep flat.	M	240	F333	0.3 h	1.60 m

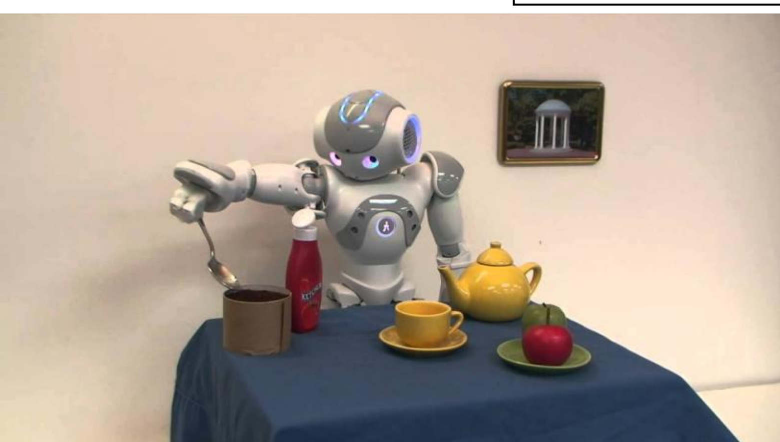
From Work-Schedule Planning to Factory Automation



Example of Computer-Aided Factory Operations

Scenario of Task Planning by Robots

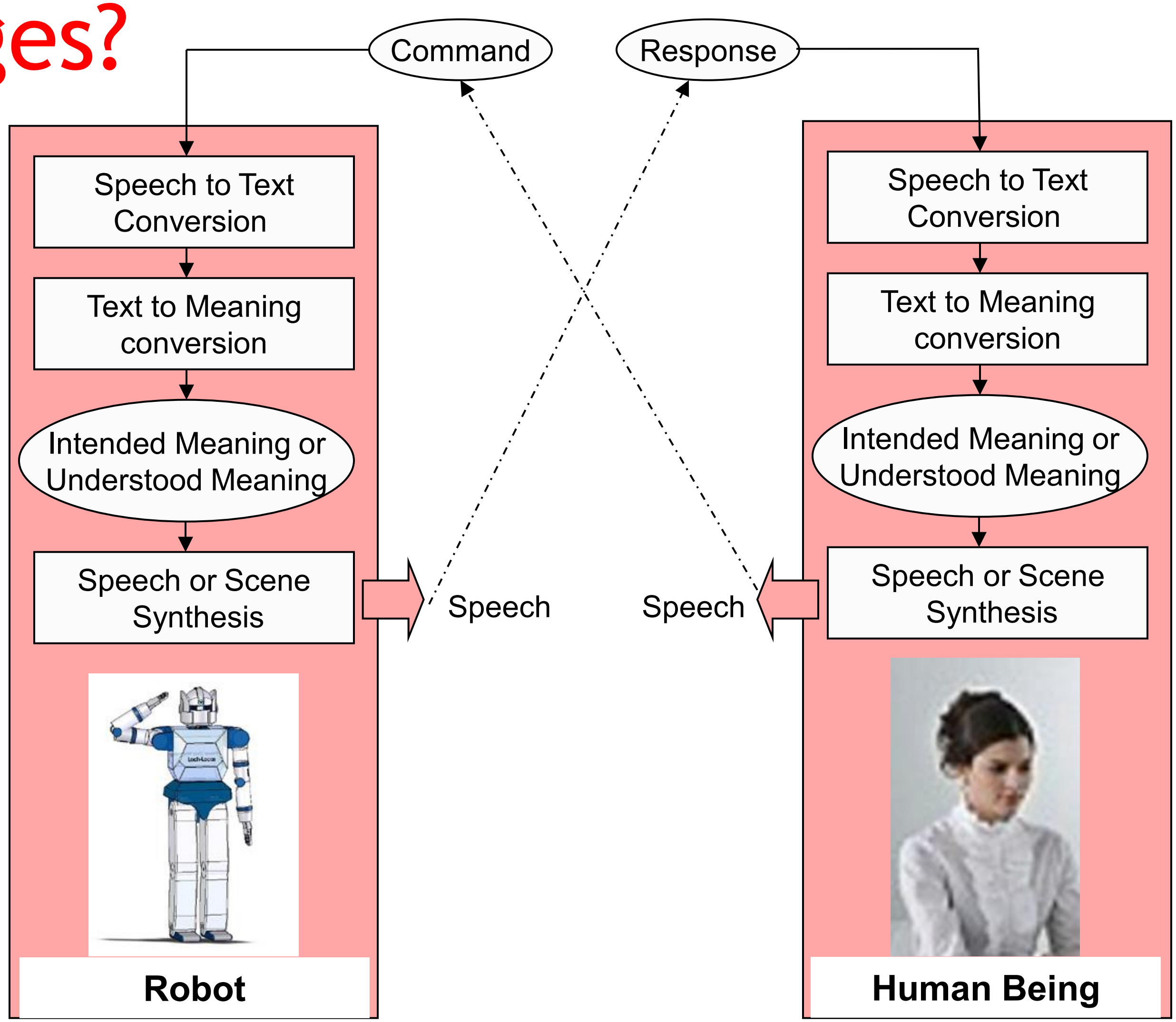




Opportunities
For AI Research

Could today’s robots understand human languages?

Opportunities
For AI Research



Challenges and Opportunities …

Summary of Lecture 1

- Purpose of Task Planning
- Input to Task Planning
- Output from Task Planning
- Process of Task Planning

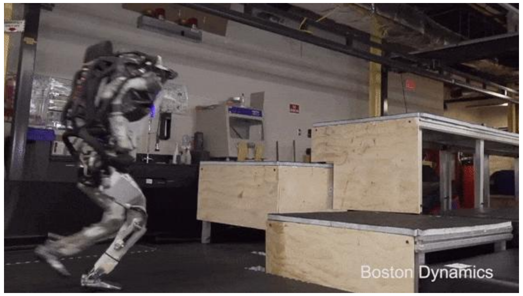
Outline of Module 3

Conceptual World

- Task Planning
- Action Planning
- Motion Planning

Physical World

- Path Planning
- Trajectory Planning



School of Mechanical & Aerospace Engineering

Design, Machine, Control, Intelligence

Module 3

MA4825 Robotics

Lecture 2

Action Planning

Xie Ming, PhD (France)

http://personal.ntu.edu.sg/mmxie

(Learning, Teaching) <o> (Research, Innovation) <o> (Leadership, Service)

Outline of Lecture 2

- Purpose of Action Planning
- Input to Action Planning
- Output from Action Planning
- Process of Action Planning
- Task Models in Manufacturing

Outline of Lecture 2

- Purpose of Action Planning
- Input to Action Planning
- Output from Action Planning
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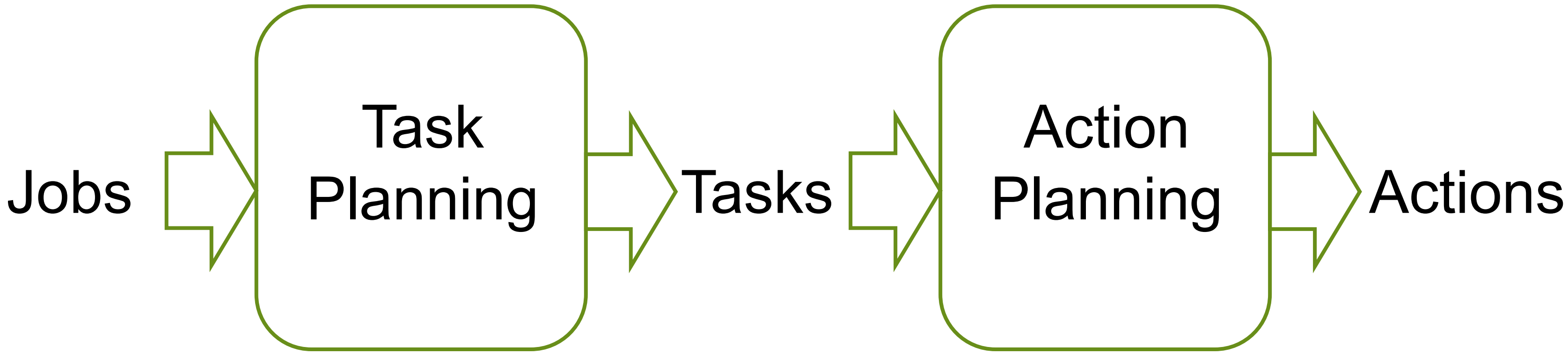
Definition of Action Planning

- Action planning is a process which takes the description of a task as input and produces a sequence of actions as output, which are to be performed by robots.

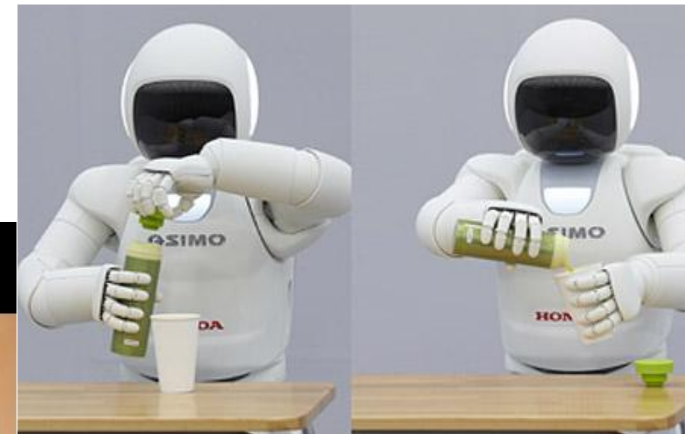


What are actions that robots can perform?

- Actions by Robot Arms
  - To push / To pull / To Follow / To Throw
- Actions by Robot Hands:
  - To open / To close / To perform complex actions
- Actions by Robot Legs:
  - To support / To carry / To push / To swing
- Actions by Robot Wheels:
  - To move forward / To move backward / To turn

Actions in Manipulation (1)

- Open/Close/Move/Pick/Place

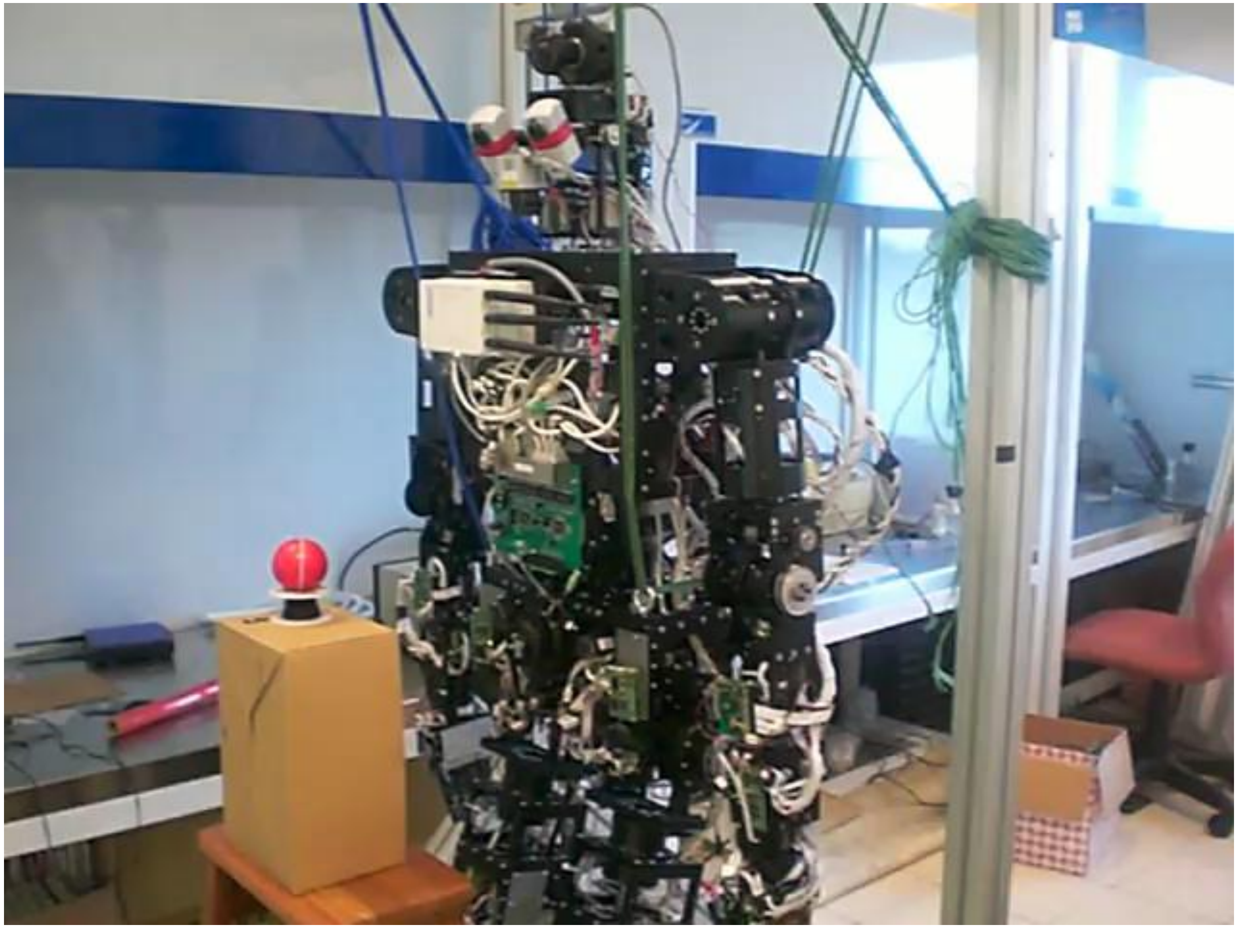


Actions in Manipulation (2)

- Move To/Move With/Move Along

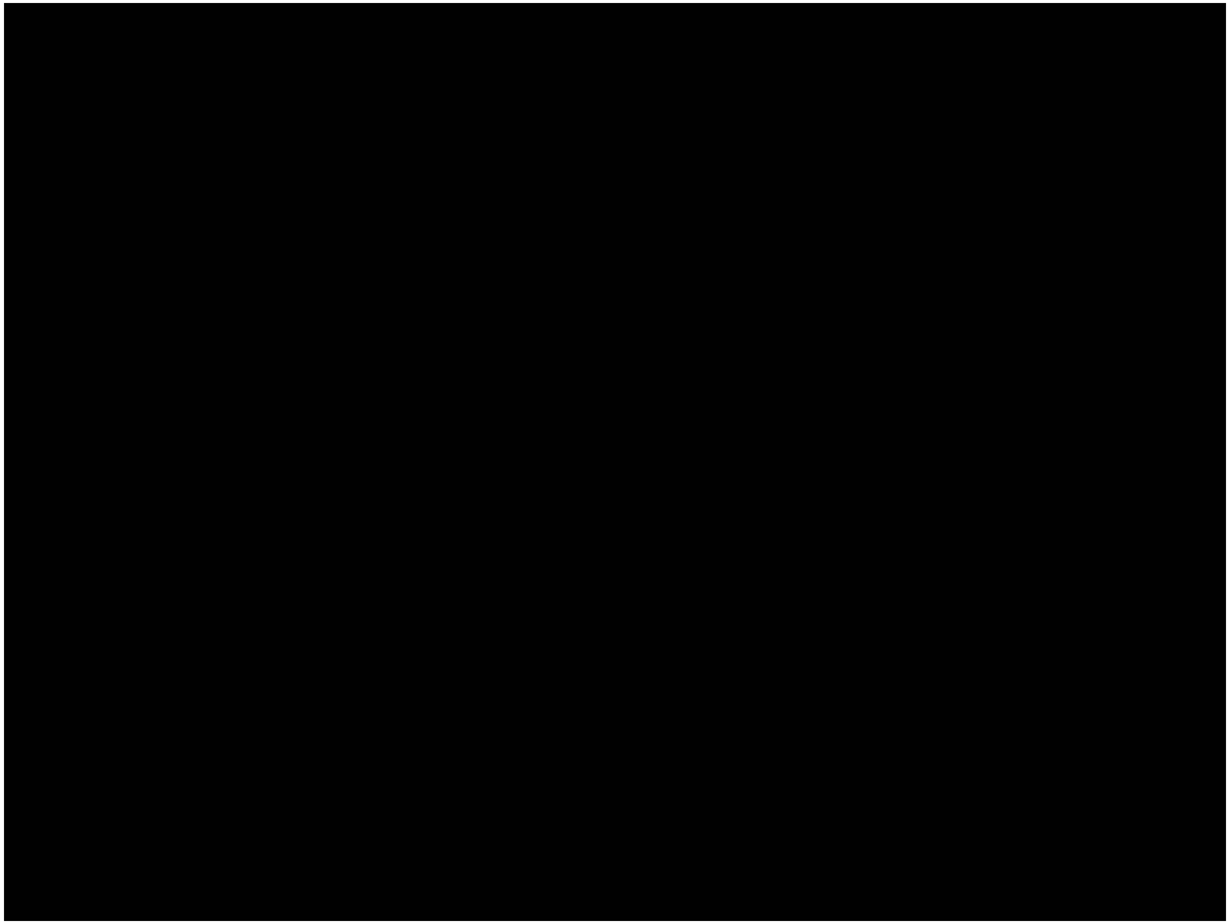
Actions in Manipulation (3)

- Open/Close/Move To/Move With/Move Along



Actions in Wheeled Locomotion

- Move Forward/Move Backward/Turn Left/Turn Right

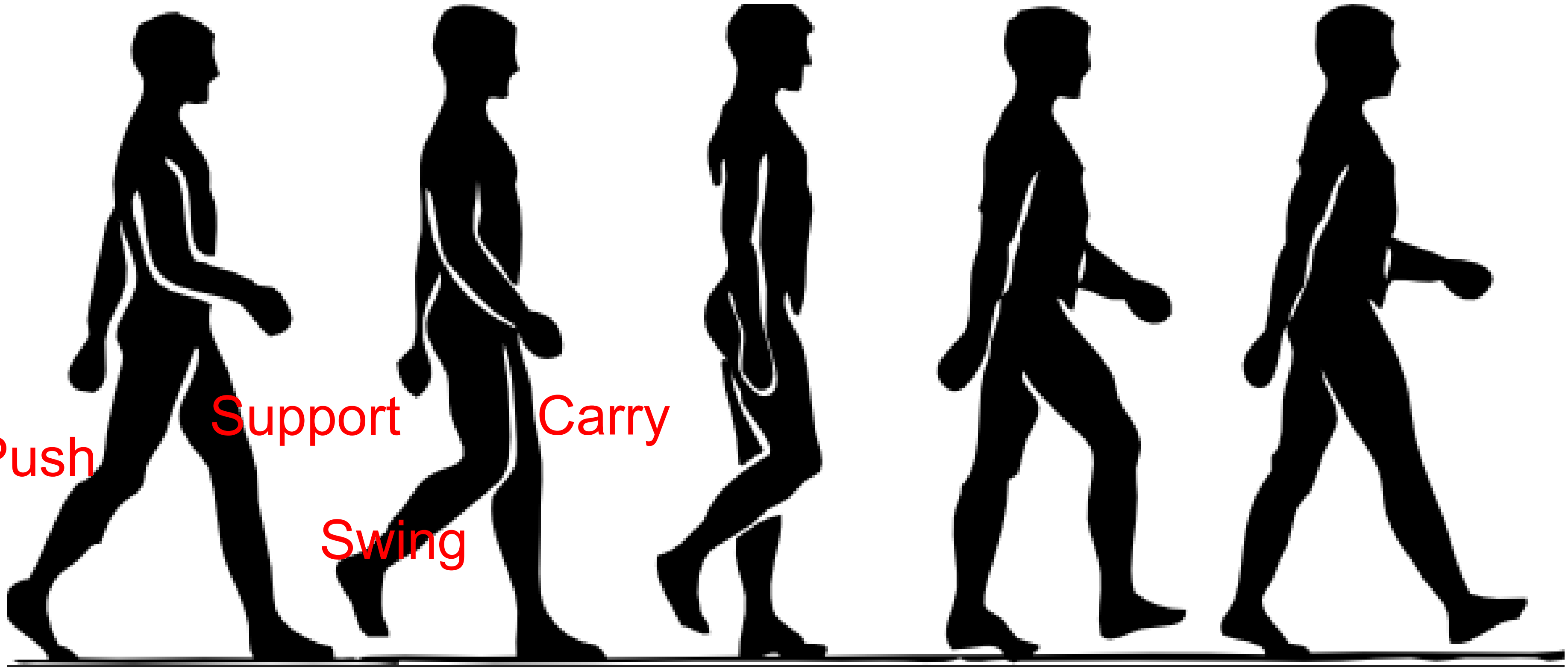


Actions in Legged Locomotion (1)

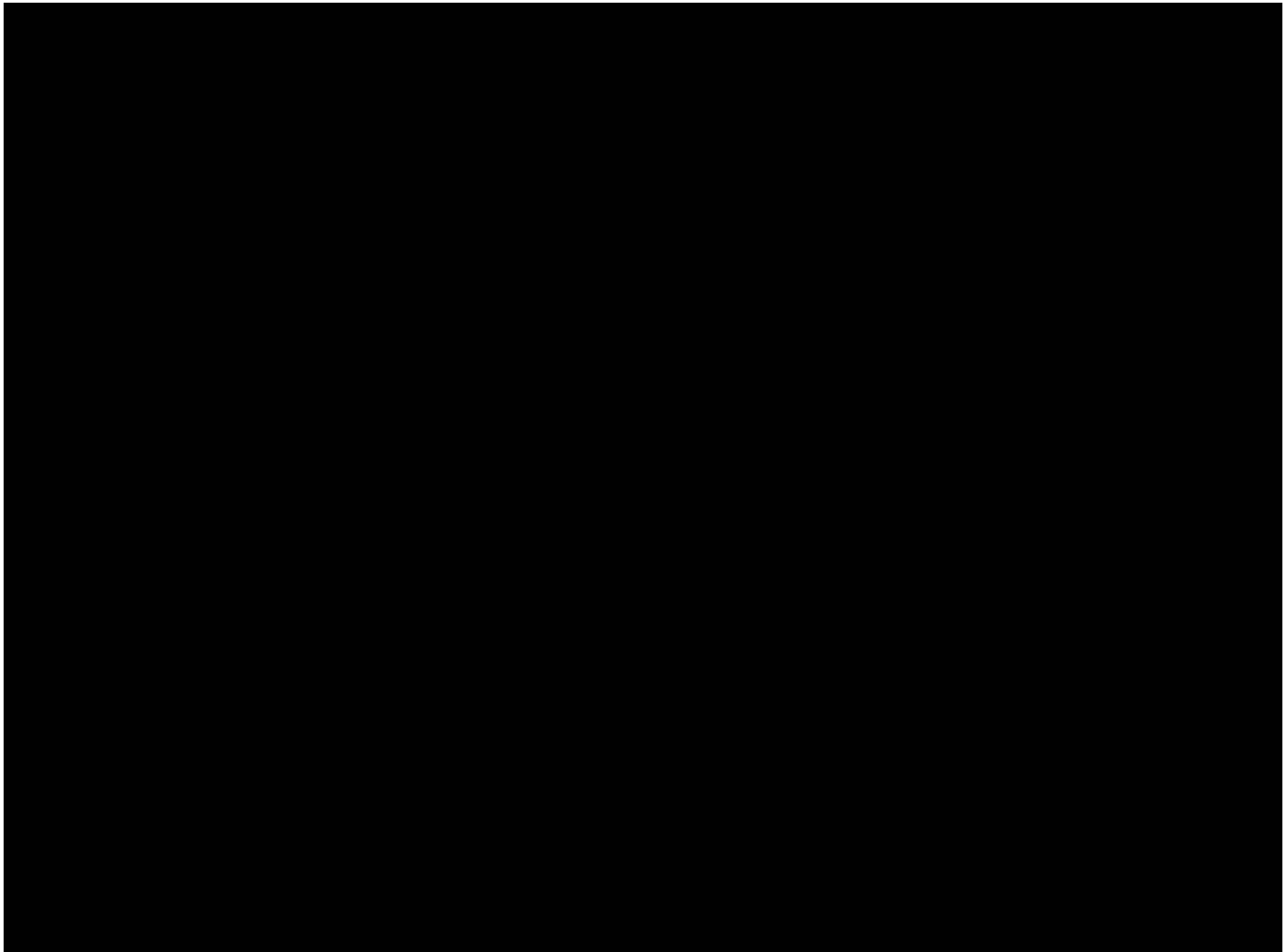
- Actions in a Walking Task: Support/Push/Carry/Swing

Actions in Legged Locomotion (2)

- Actions in a Walking Task: Support/Push/Carry/Swing

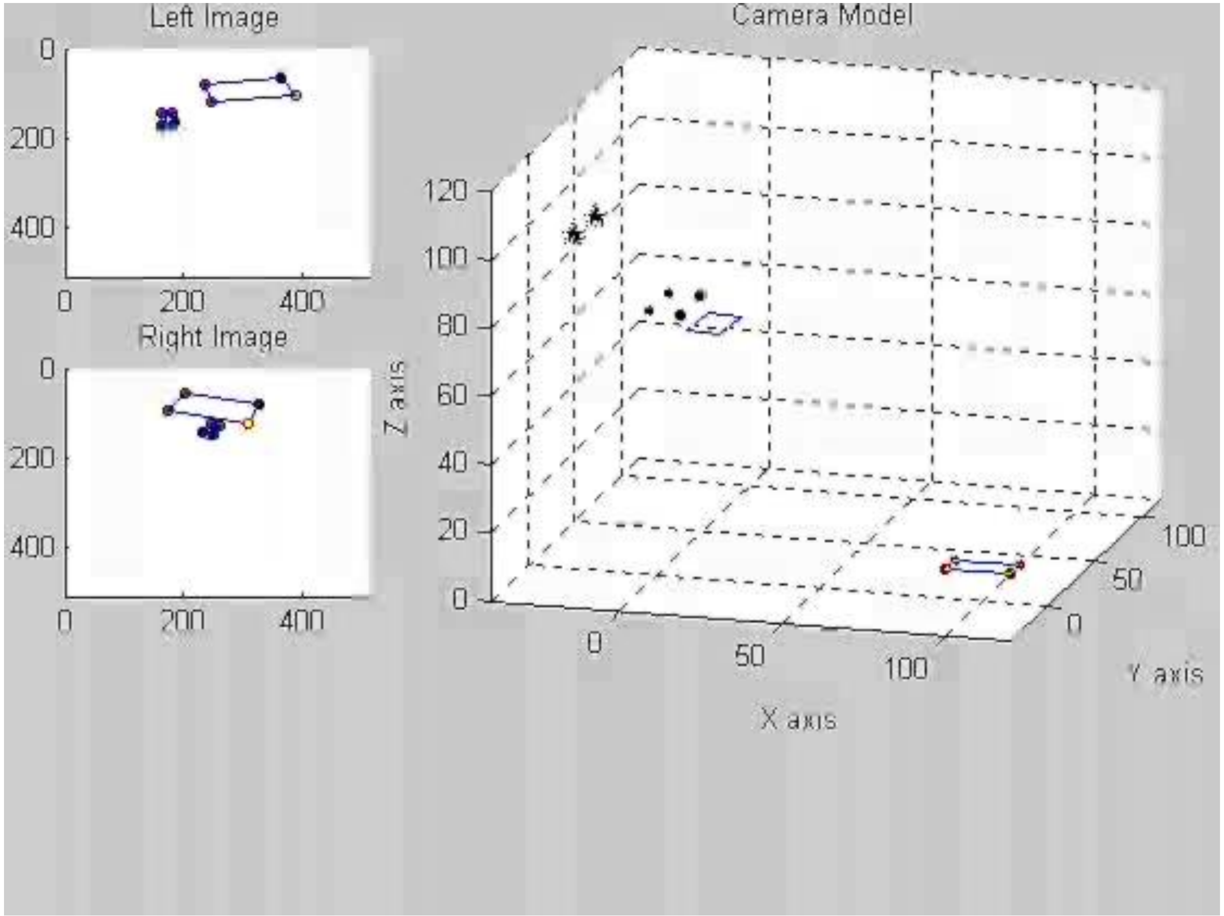


Actions in Legged Locomotion (3)

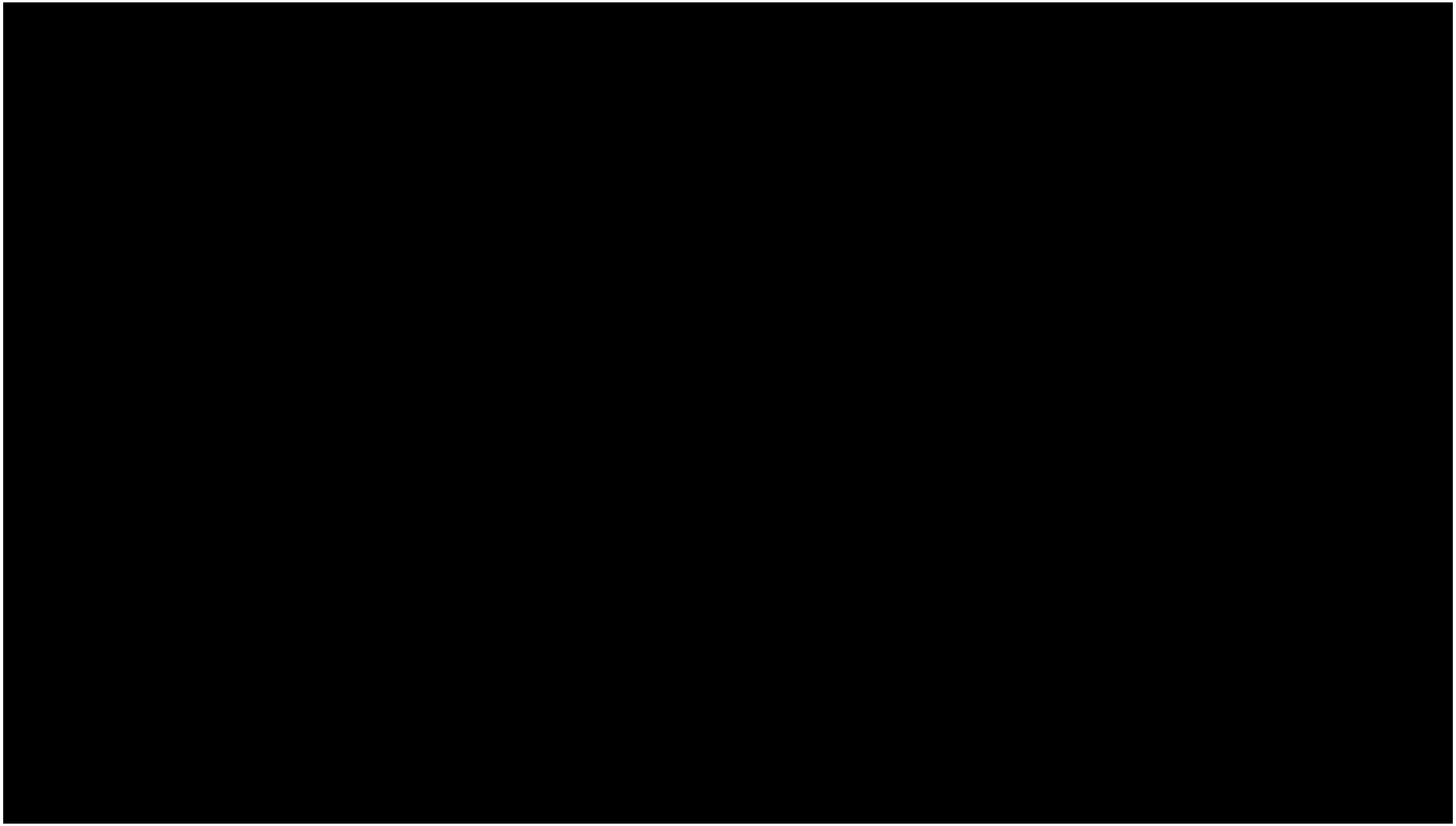


Actions in Flying Locomotion

- Actions in a Flying Task: Take-off/Landing/Turn/Move Forward/Backward/Move Up/Down



More example …



Outline of Lecture 2

- Purpose of Action Planning
- Input to Action Planning
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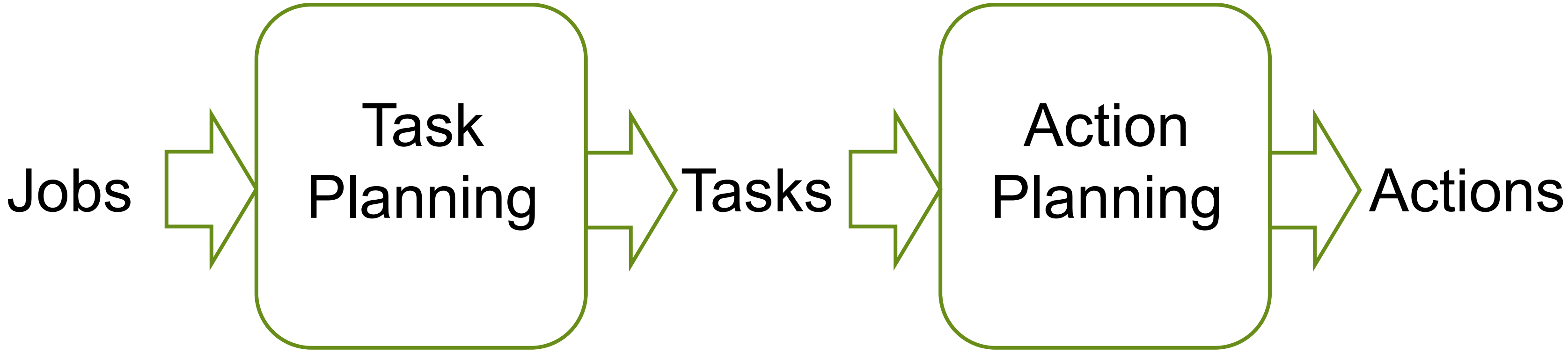
Input to Action Planning

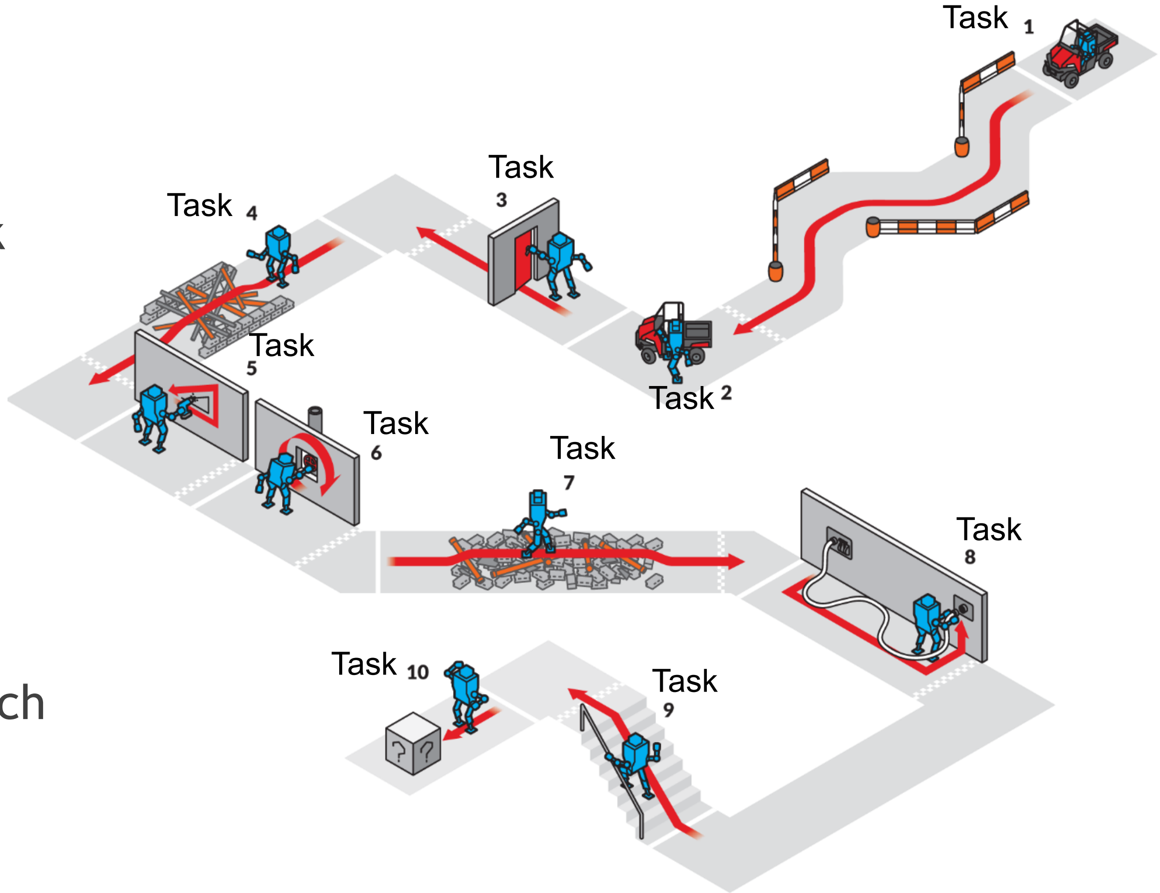
- The input to action planning is the description of a given task in the form of natural languages.



What should be inside the description of a given task?

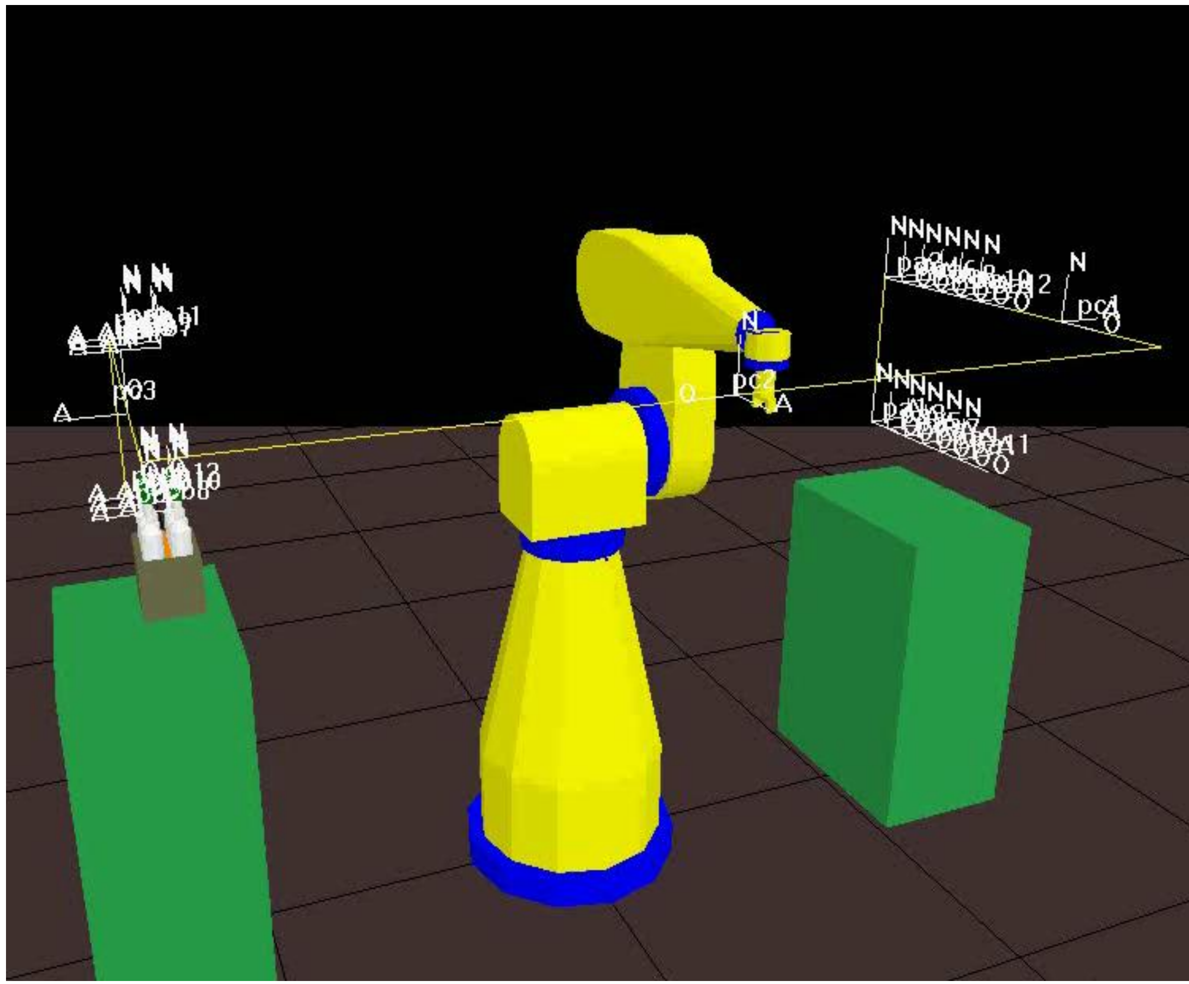
Answer:

- 1. The outcome of task to be achieved
- 2. The space in which the task is to be undertaken
- 3. The time within which the task is to be undertaken



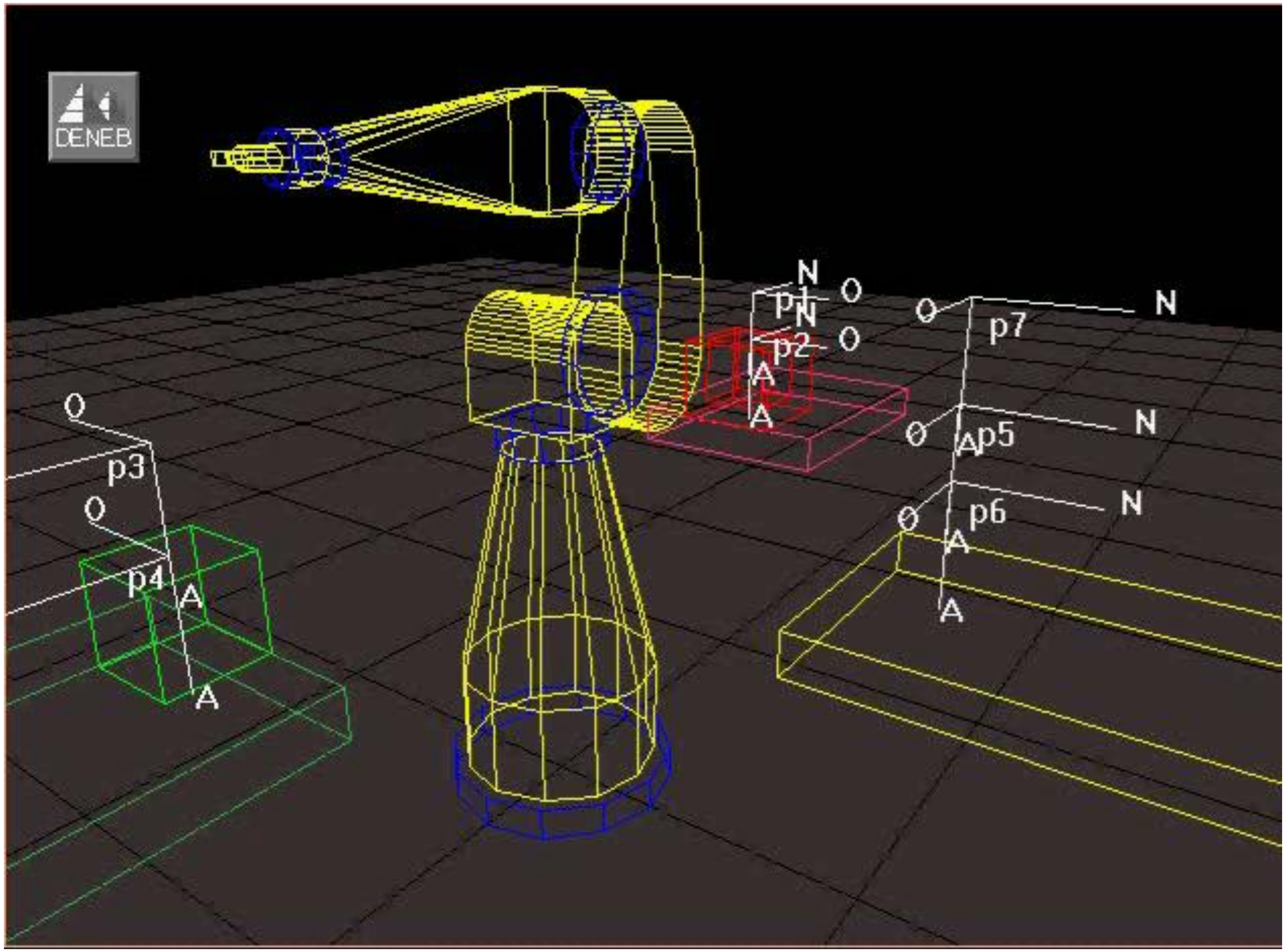
Example of Task Description as Input

- A robot arm picks up a bottle at incoming conveyer, and places it into a specific location inside the box at outgoing conveyor.
- It repeats the operations until the box is full.



Example of Task Description as Input

- An AGV transports part A into the workspace.
- A conveyor transports part B into the workspace.
- A robot arm picks up part A and places it onto the outgoing conveyor.
- The robot arm picks up part B and inserts it into part B.



Outline of Lecture 2

- Purpose of Action Planning
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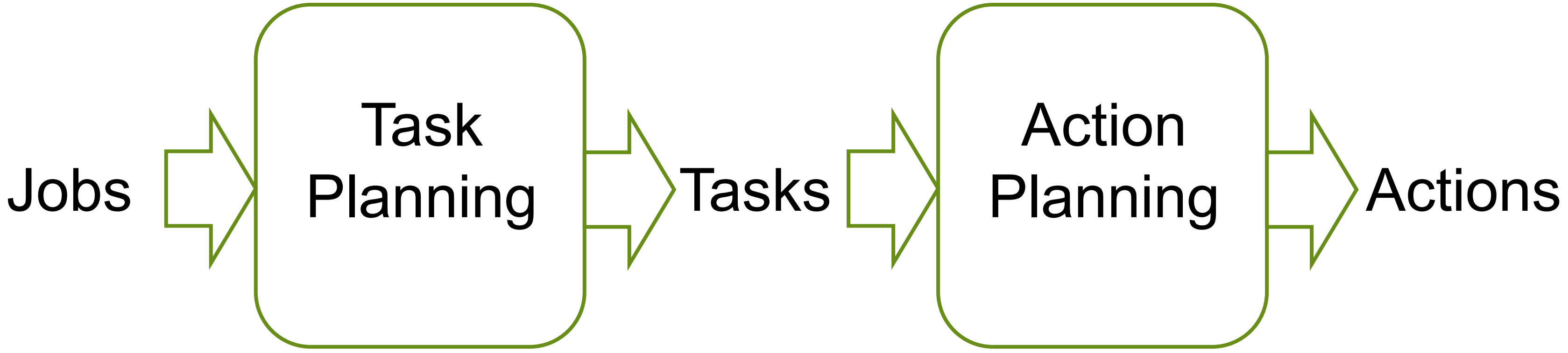
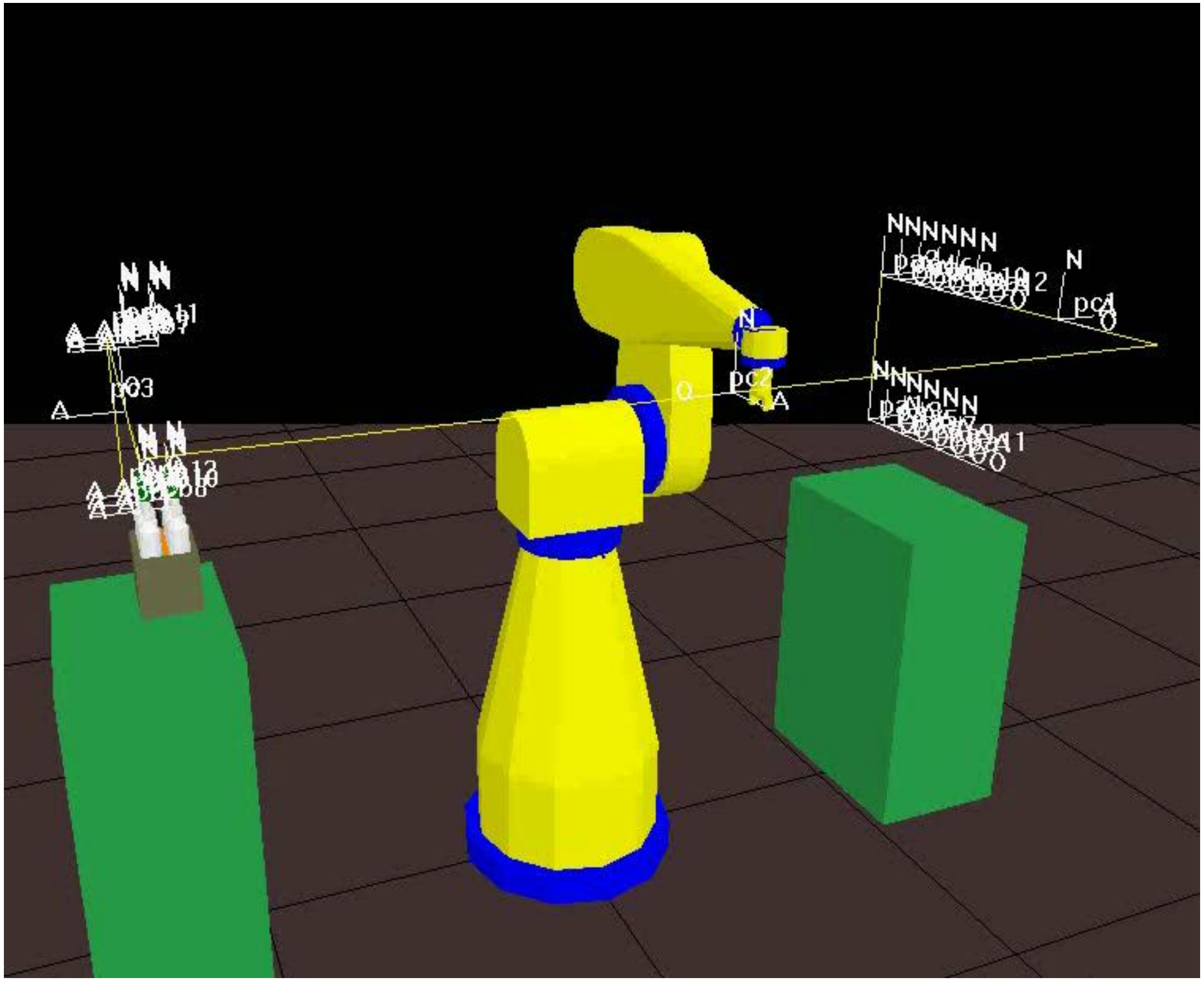
Output from Action Planning

- The output of action planning is a sequence of actions which could lead to the accomplishment of a given task.

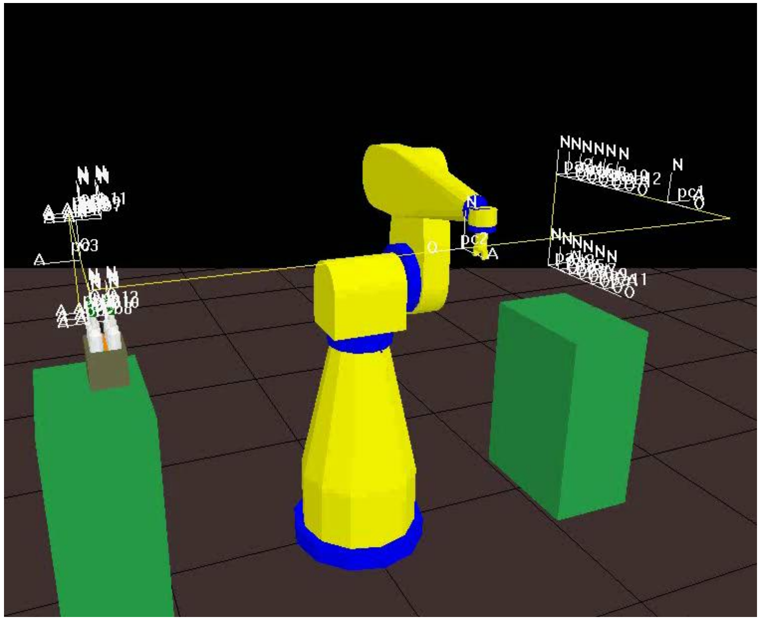


What is the action sequence derived from this material handling task?

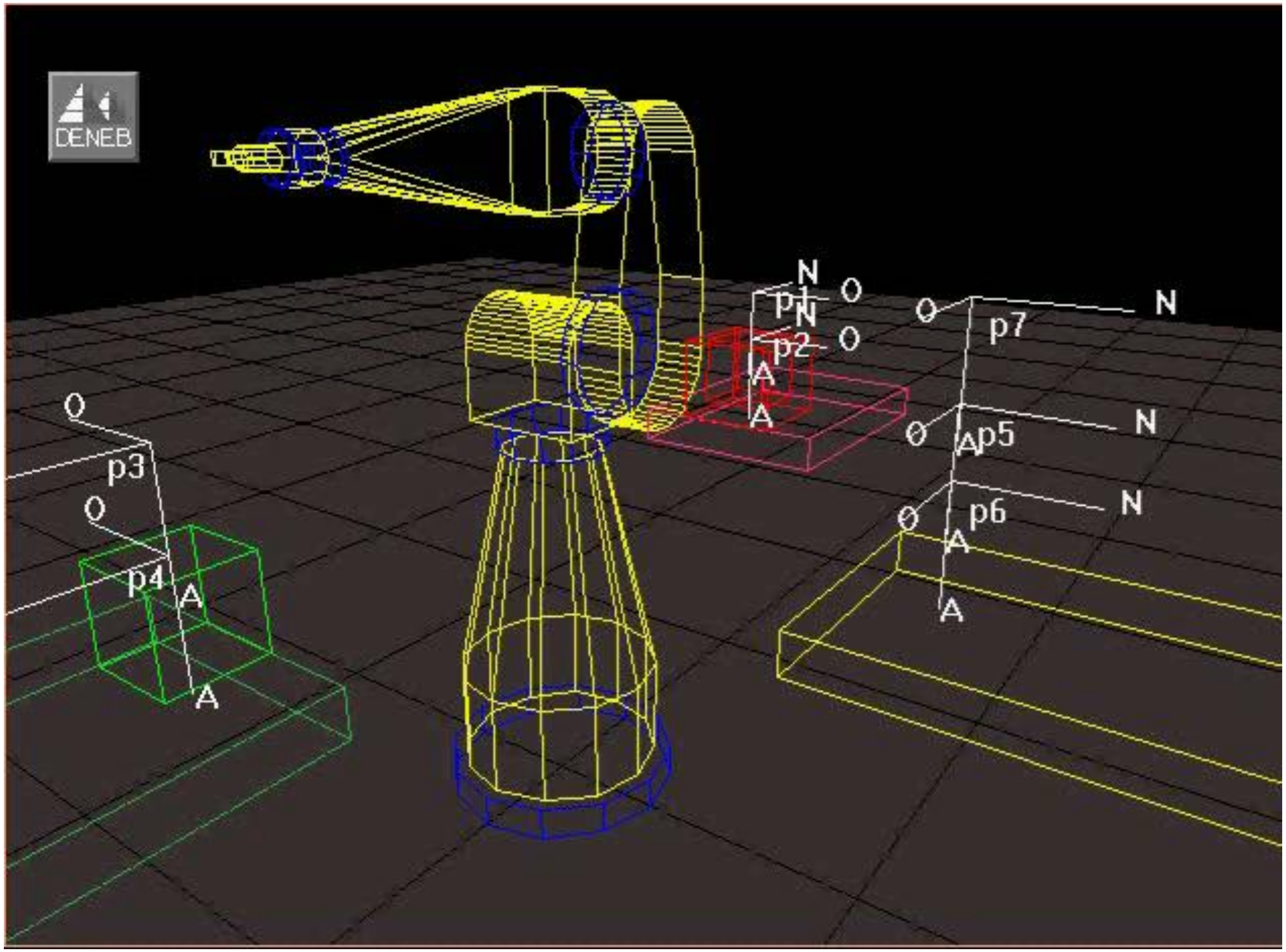


Output of a Sequence of Actions

1. To bring in an empty box to the packaging station.
2. To transport the filled bottles to the packaging station.
3. To pick up a filled bottle and place it into the box.
4. To repeat until the box is full.
5. To shift away the full box and to bring in a new empty box.



What is the action sequence derived from this assembly task?



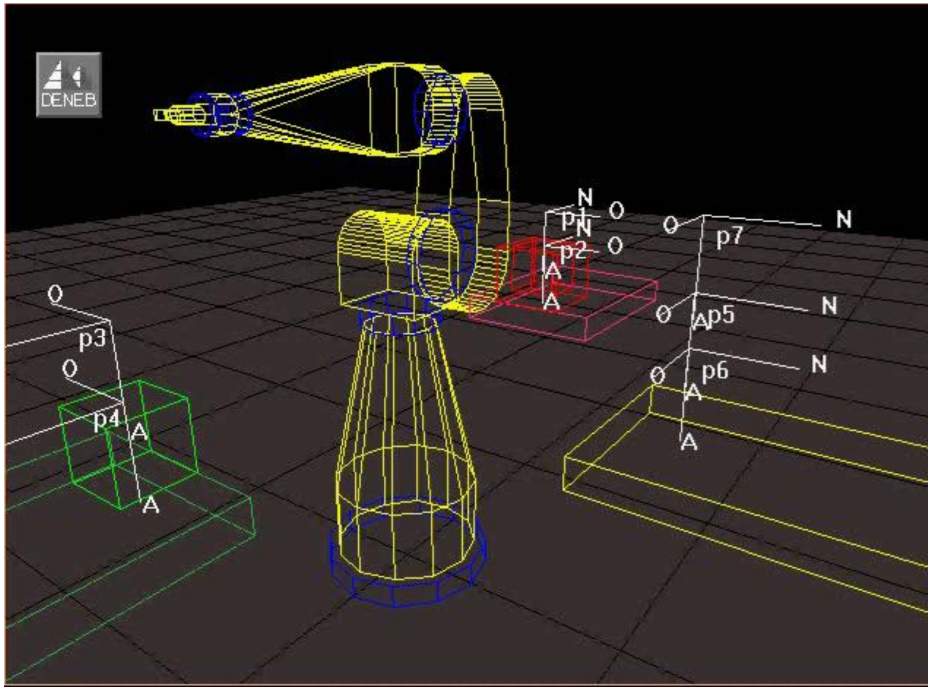
Output of a Sequence of Actions

(1) **To pick up part A from the AGV.**

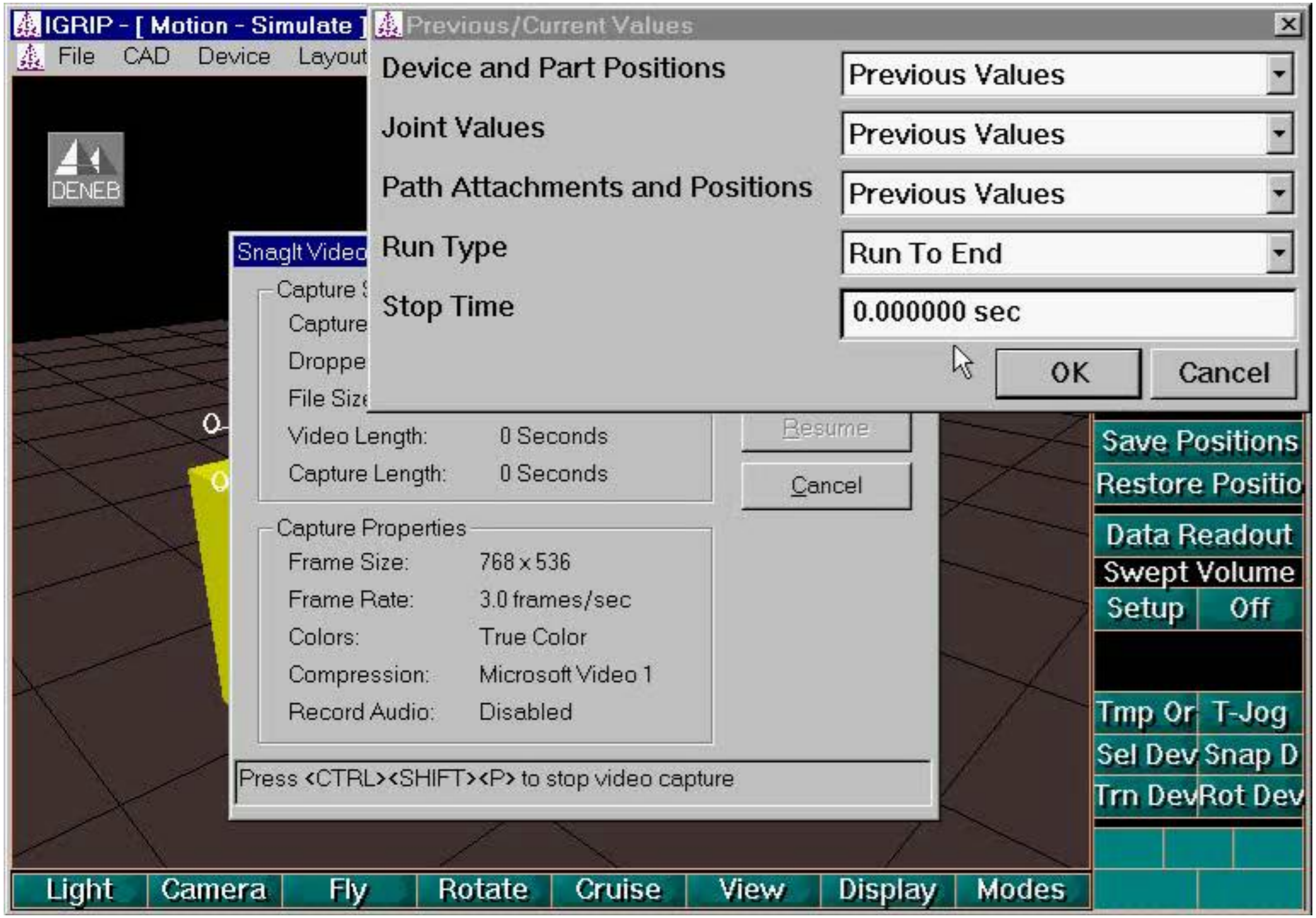
(2) **To place part A onto the outgoing conveyor.**

(3) **To pick up part B from the incoming conveyor.**

(4) **To insert part B into part A.**

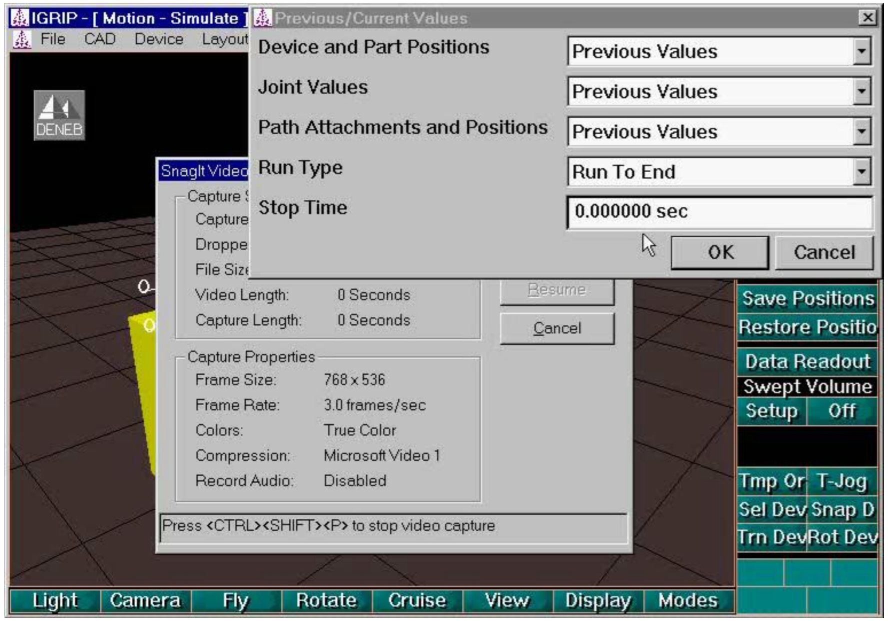


What is the action sequence derived from this sorting task?



Output of a Sequence of Actions

- To pick up a bar of one colour
- To place it onto the table of the same colour
- Repeat for the rest of three bars
- To pick up the bar on a table
- To place it to the middle of the table
- Repeat for the rest of three bars



Outline of Lecture 2

- Purpose of Action Planning
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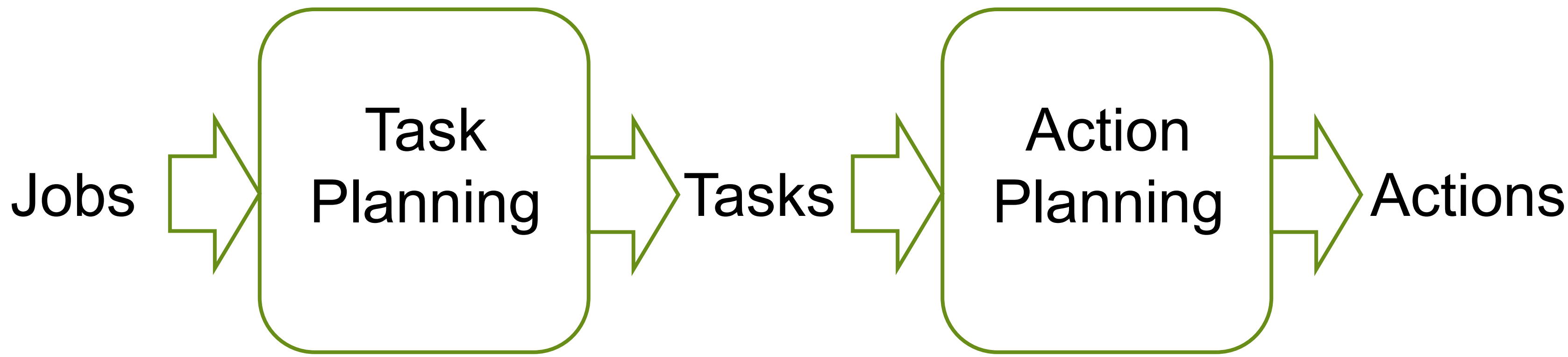
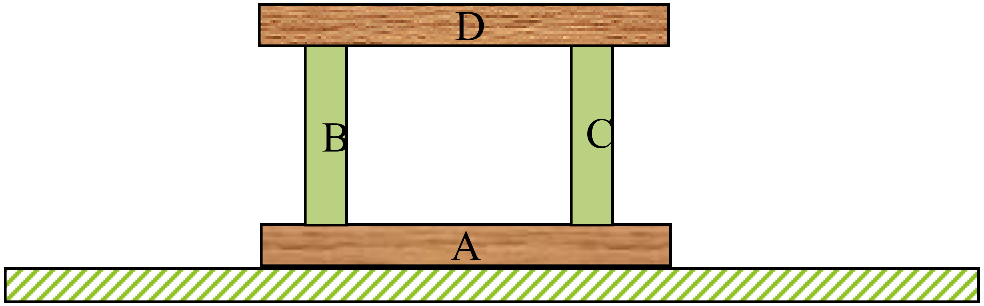
How to do action planning?

- Scenario 1: You are the buyers or users of robots
  - Human-Assisted Action Planning → (Human Assistance)
    - To user natural languages
    - To use programming languages and library functions
- Scenario 2: You are the designers robots
  - Action Planning by Robots → (Robots with Intelligence)
    - Model-Guided Action Planning
    - Sensor-Guided Action Planning



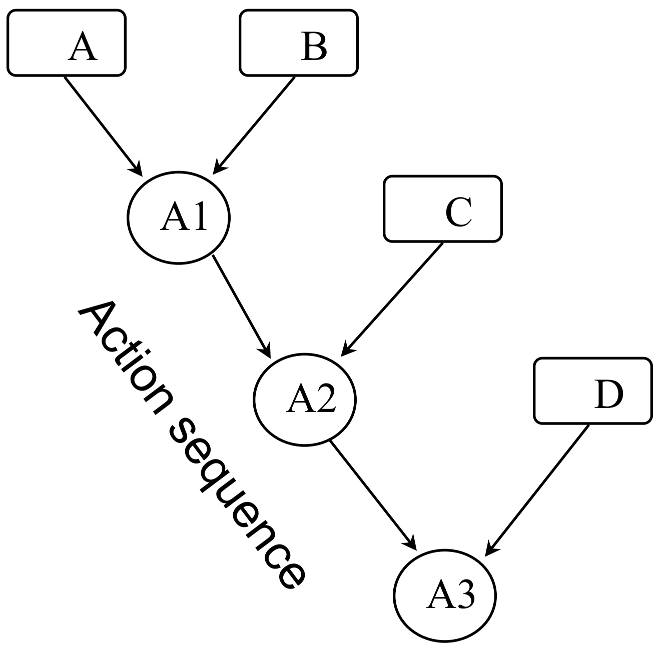
Model-Guided Action Planning

- Use of Assembly Task Model



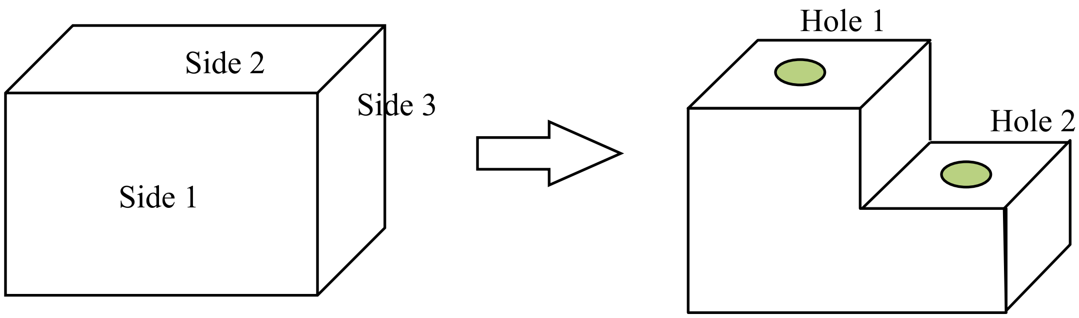
Output of Action Sequence

- Action A1:
  - Put part A and part B together
- Action A2:
  - Add part C to the sub-assembly
- Action A3:
  - Add part D to the sub-assembly



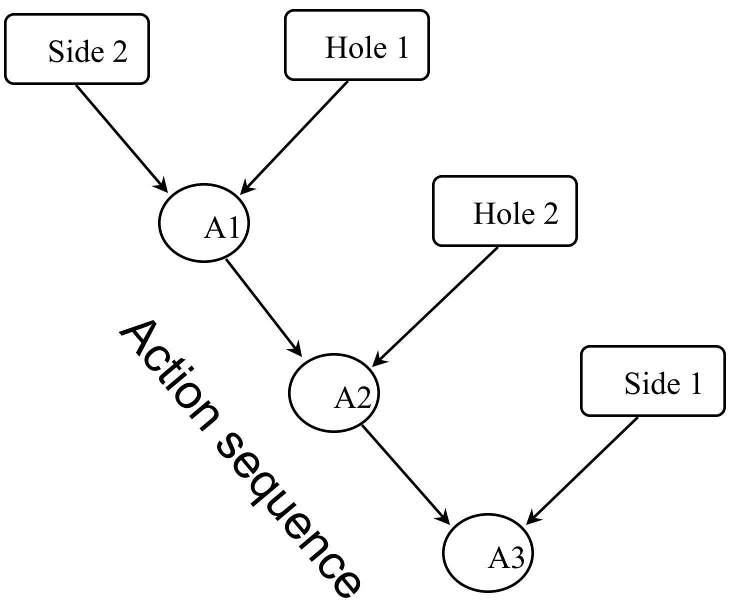
Model-Guided Action Planning

- Use of Machining Task Model

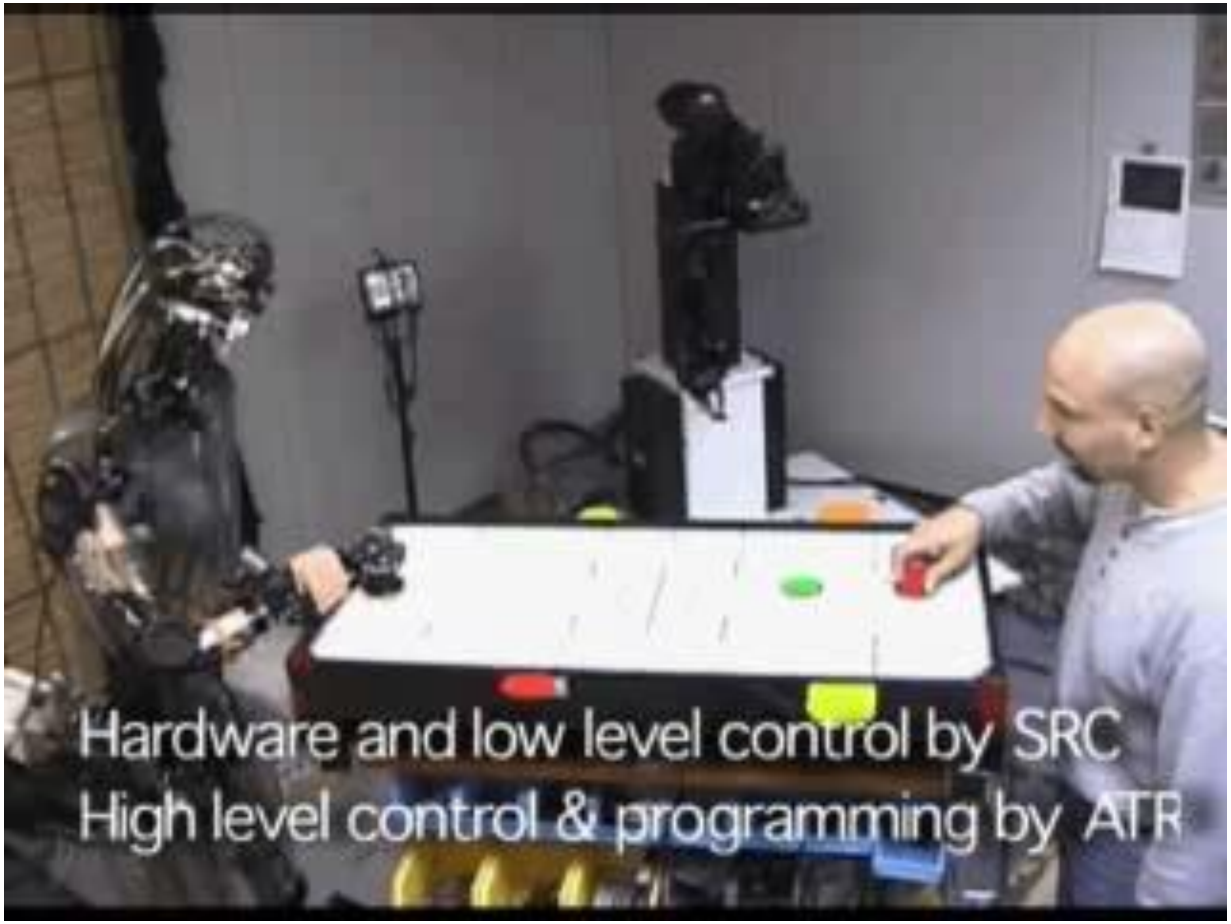


Output of Action Sequence

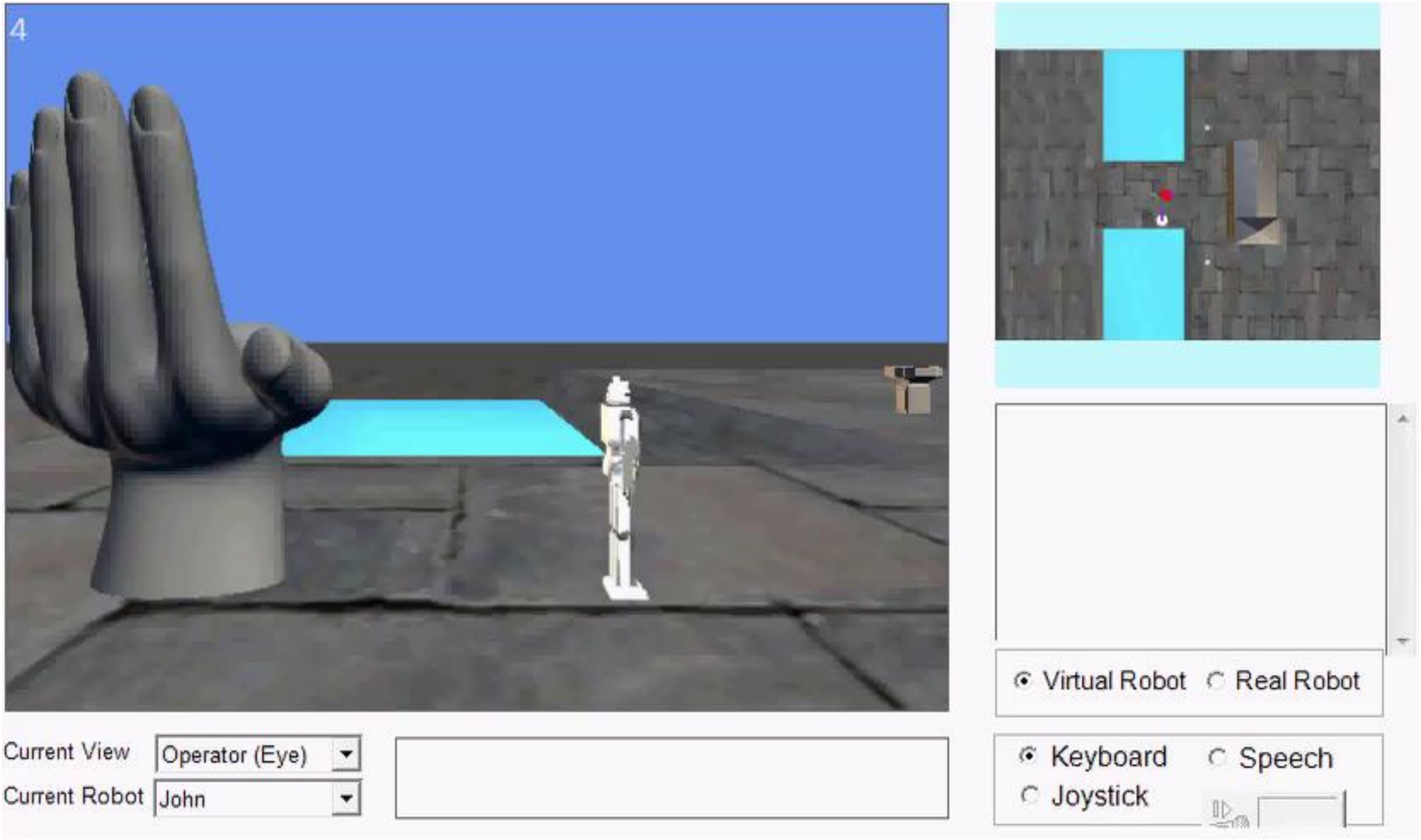
- Action A1:
  - Drill hole 1 at side 2
- Action A2:
  - Drill hole 2 at side 2
- Action A3:
  - Remove a sub-block at side 1



Sensor-Guided Action Planning



Human-Assisted Action Planning



Outline of Lecture 2

- Purpose of Action Planning
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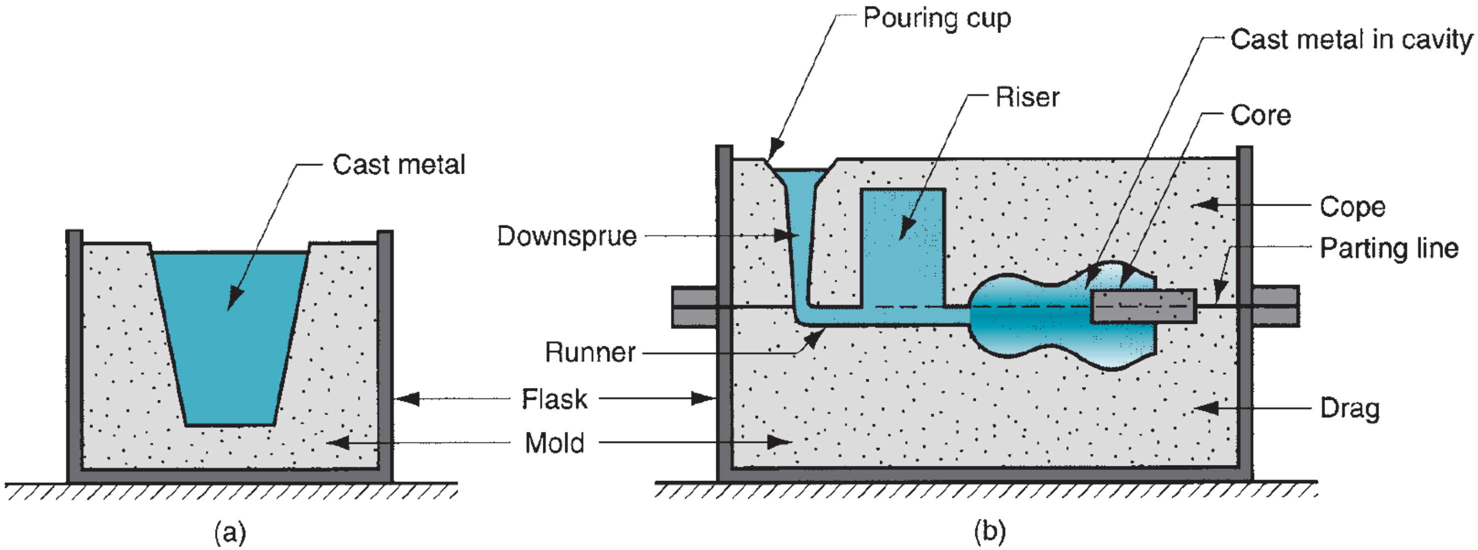
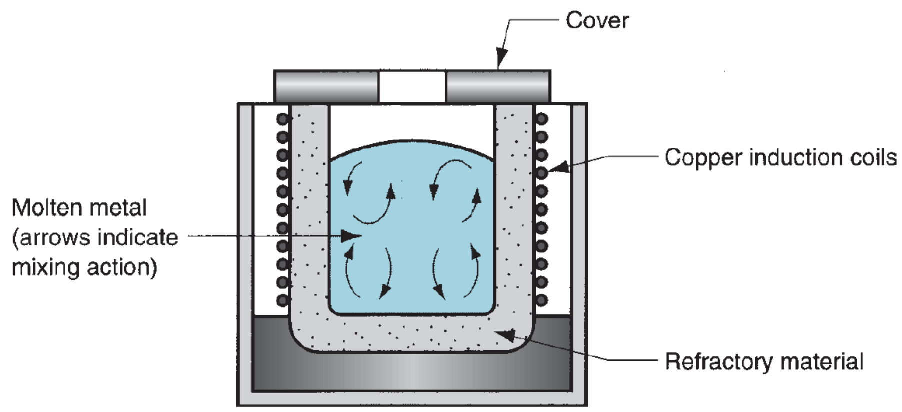
What is the action sequence derived from this task description (Sand Casting)?

- The task is to make use of molten metals and a mould of cavity of a specific shape in order to produce metal parts of the shape of the mould's cavity.



Output of Action Sequence

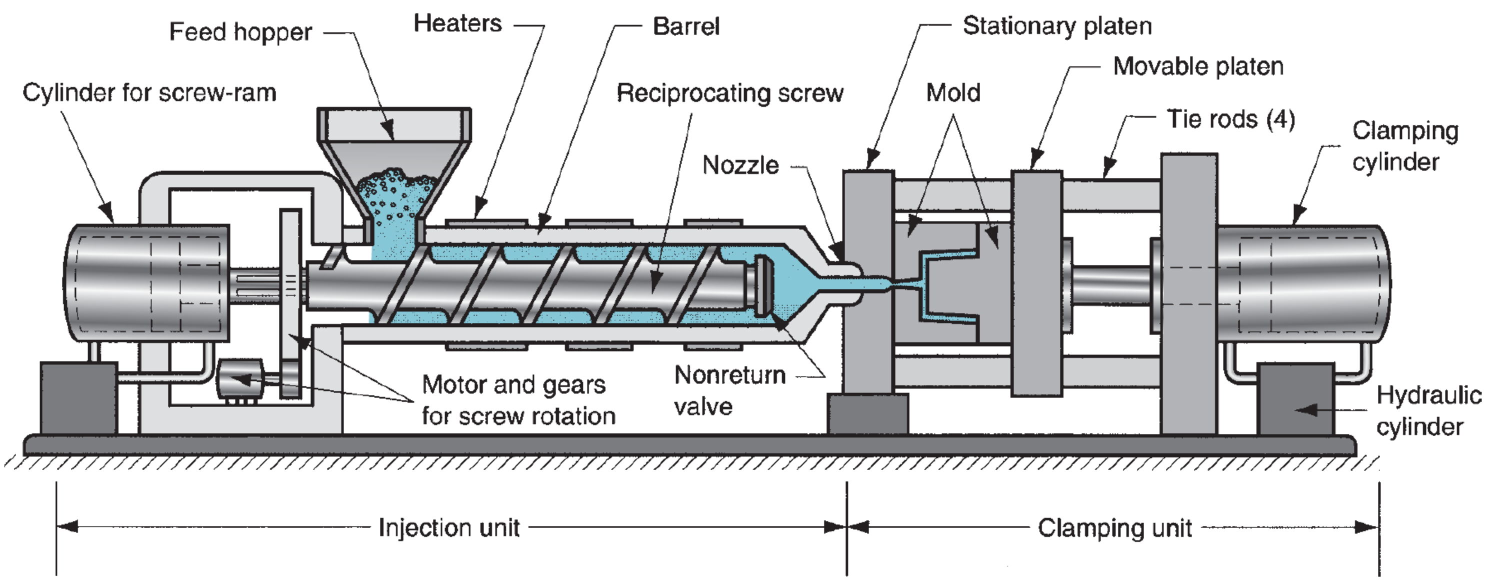
- Melding of Raw Metals
- Pouring of Molten Metals
- Solidification and Cooling
- Removal of Mould
- Cleaning and Inspection of Finished Casting



Induction Furnace

What is the action sequence derived from this task description (Injection Moulding)?

- The task is to make use of molten polymer and a mould of cavity of a specific shape in order to produce plastic parts of the shape of the mould's cavity.

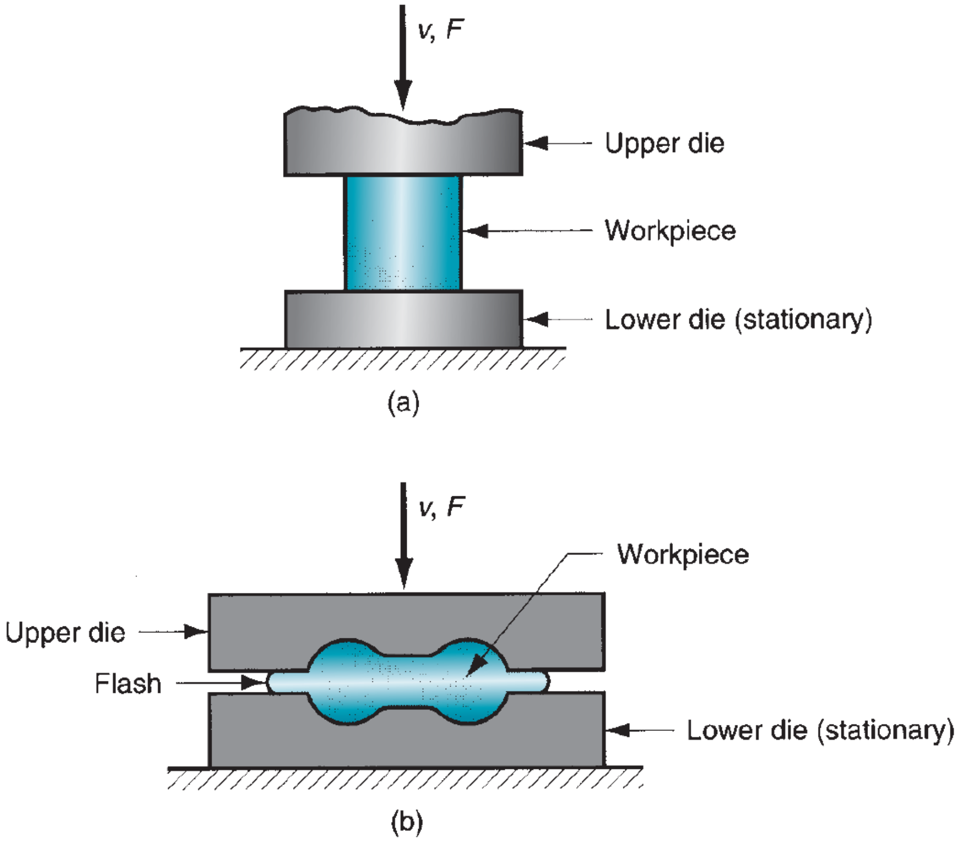


Output of Action Sequence

- To close the mould
- To melt and homogenize the polymer
- To inject the molten polymer into the cavity of mould
- To keep the mould closed during injection
- To open the mould
- To remove the finished parts

What is the action sequence derived from this task description (Forging)?

- The task is to make use of impacts or gradual pressure to compress hot metals into desired shapes.



Output of Action Sequence

Forging by Impact

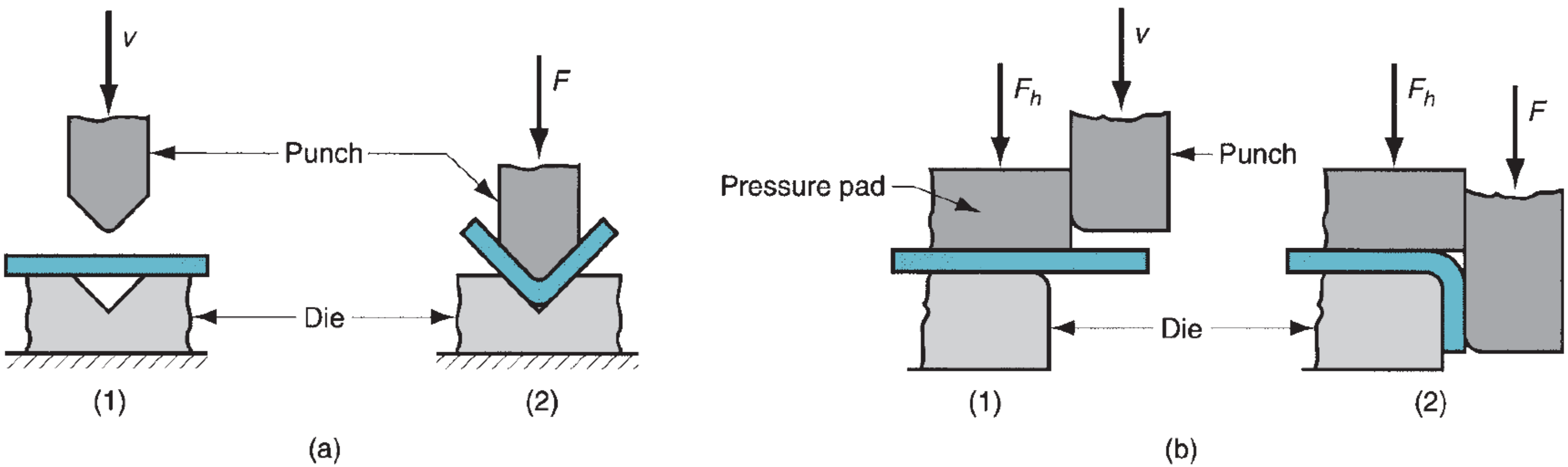
- Workpiece of metal is heated into a temperature.
- Workpiece is placed on lower die.
- Upper die (or punch) exerts an impact onto the workpiece.

Forging by Gradual Pressure

- Workpiece of metal is heated into a temperature.
- Workpiece is placed on lower die.
- Upper die (or punch) exerts gradual pressure onto the workpiece.

What is the action sequence derived from this task description (Bending)?

- The task is to make use of a punch to exert strain along a straight axis on a surface of a metal sheet (which is placed on a die) so as to create a desired bending angle.



V-Bending

Edge-Bending

Output of Action Sequence

V-Bending

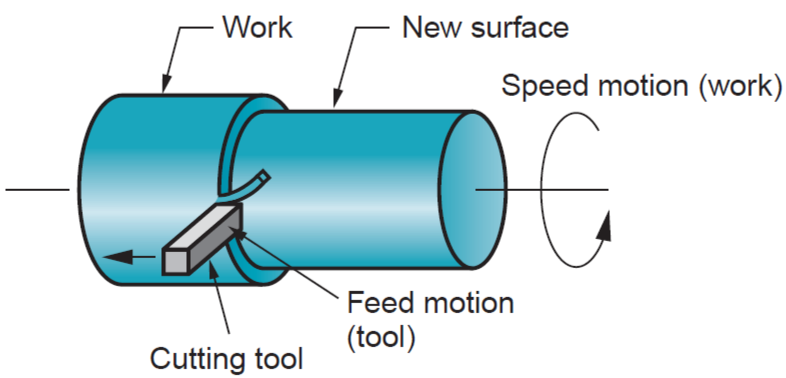
- A metal sheet is placed on a die.
- A punch exerts a force and presses the metal sheet against the die.
- The punch is lifted up,

Edge-Bending

- A metal sheet is placed on a die.
- The metal is held against the die by a pressure pad.
- A punch exerts a force and presses the metal sheet against the die.
- The punch and the pressure pad are lifted up.

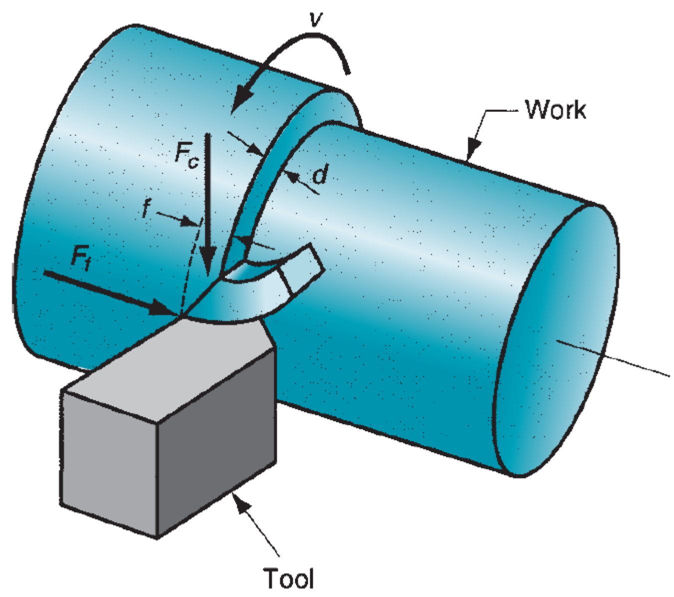
What is the action sequence derived from this task description (Turning)?

- The task is to make use of movable cutting tools which act on a rotating workpiece for the purpose of removing layers of materials (i.e. chips) so that the original surface of the workpiece is transformed into a new surface of a desired shape.



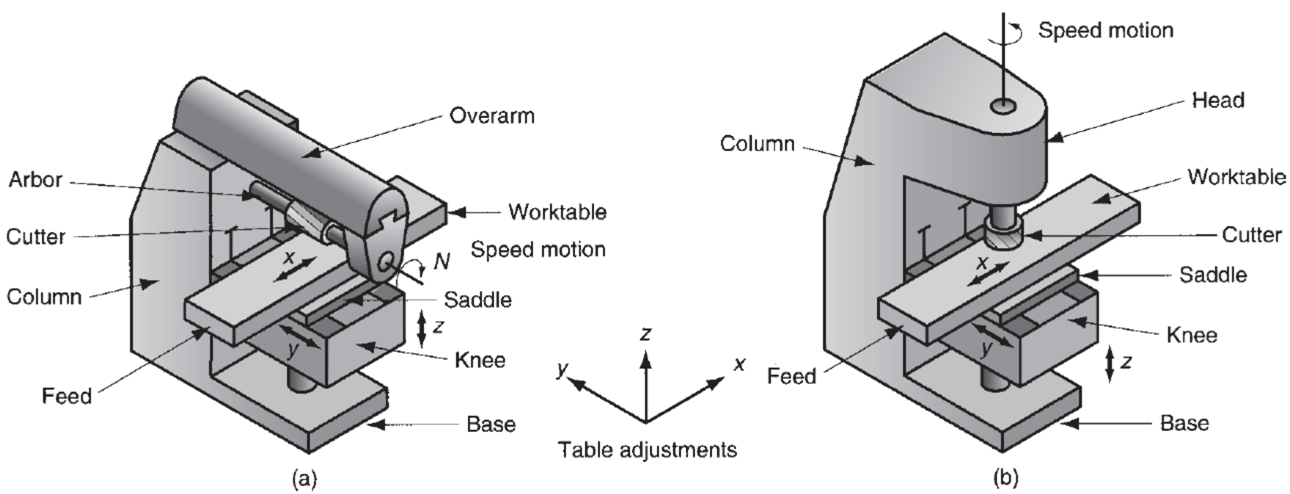
Output of Action Sequence

- To load and fix the workpiece onto a rotating head.
- To rotate the workpiece.
- To advance the cutting tool until the final shape is obtained.



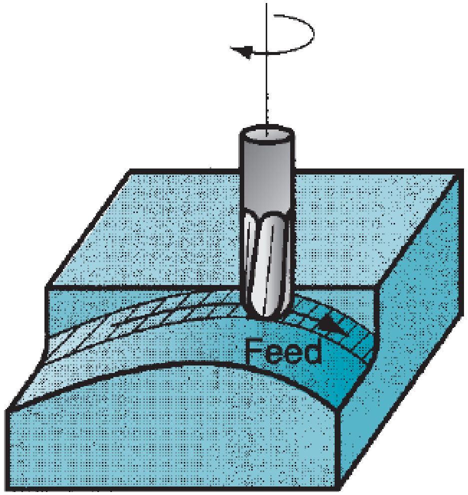
What is the action sequence derived from this task description (Milling)?

- The task is to make use of rotating cylindrical tools with multiple edges in order to remove materials from a movable workpiece so as to produce workpieces with desired surface profiles.



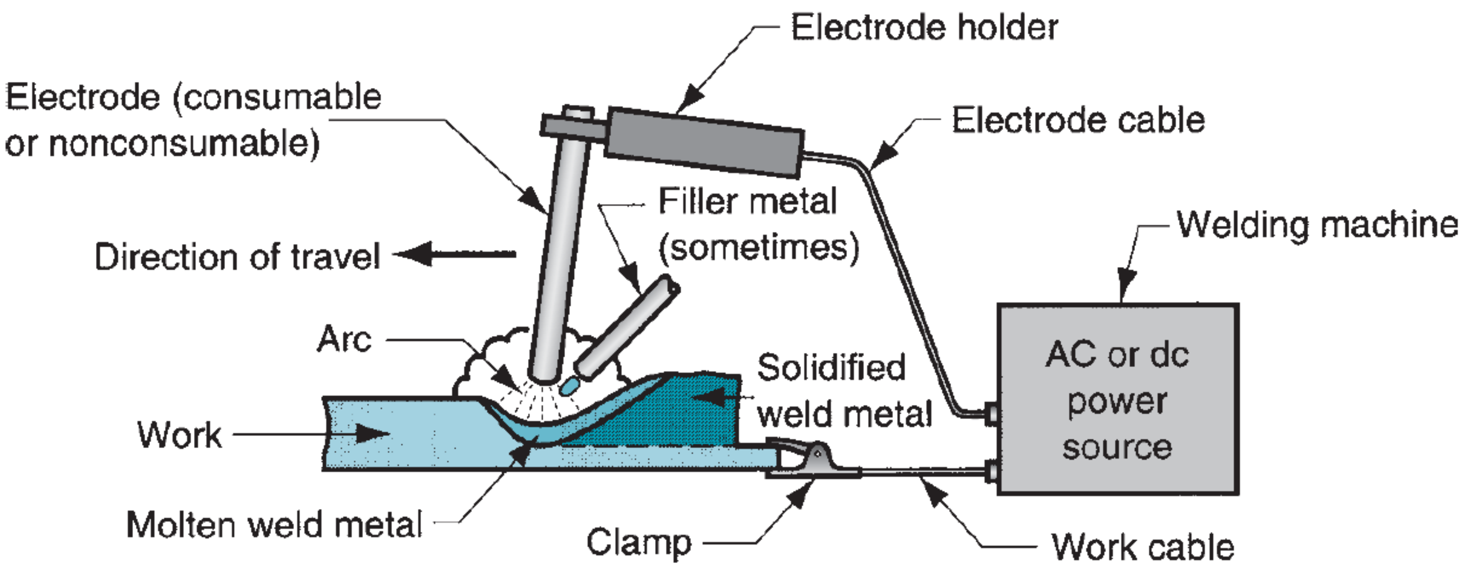
Output of Action Sequence

- To attach and rotate cylindrical tool.
- To fix a workpiece on a movable table.
- To feed the workpiece to the rotating tool until the desired surface profile is obtained.



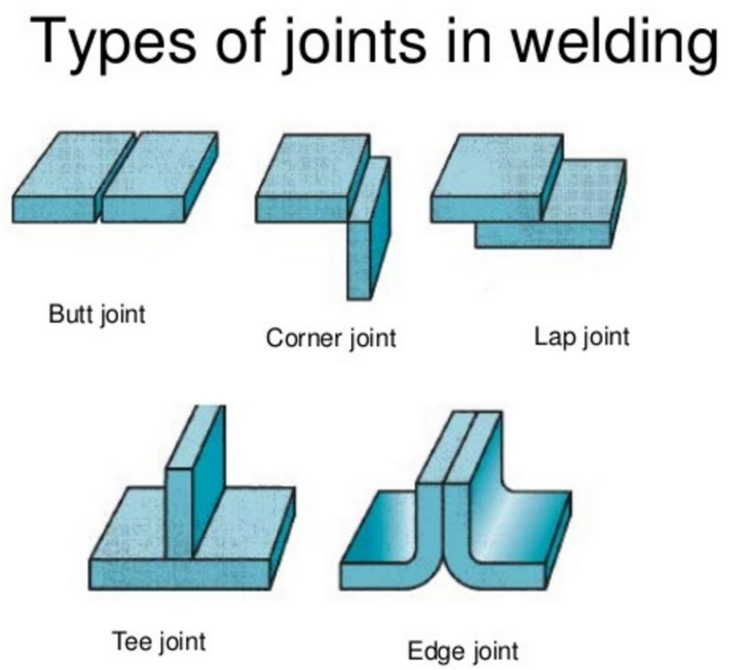
What is the action sequence derived from this task description (Welding)?

- The task is to make use of heat and/or pressure in order to join two workpieces together with their contacting surfaces or faying surfaces.



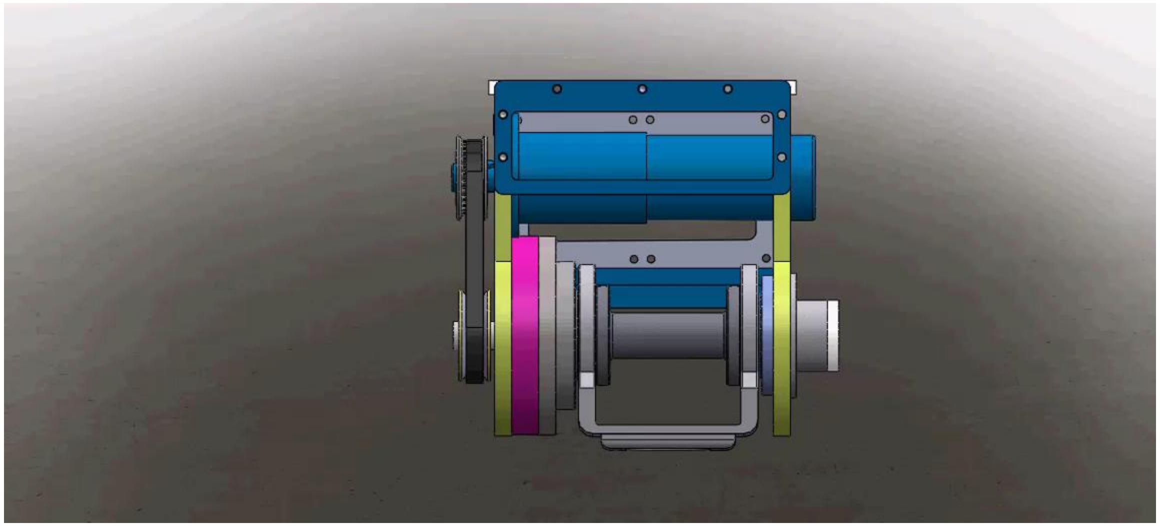
Output of Action Sequence

- To place two workpieces so as to form the desired joint.
- To generate and apply heat and/or pressure along the contacting surface while supplying filler metals if needed.
- To inspect the welded joints.



What is the action sequence derived from this task description (Assembly)?

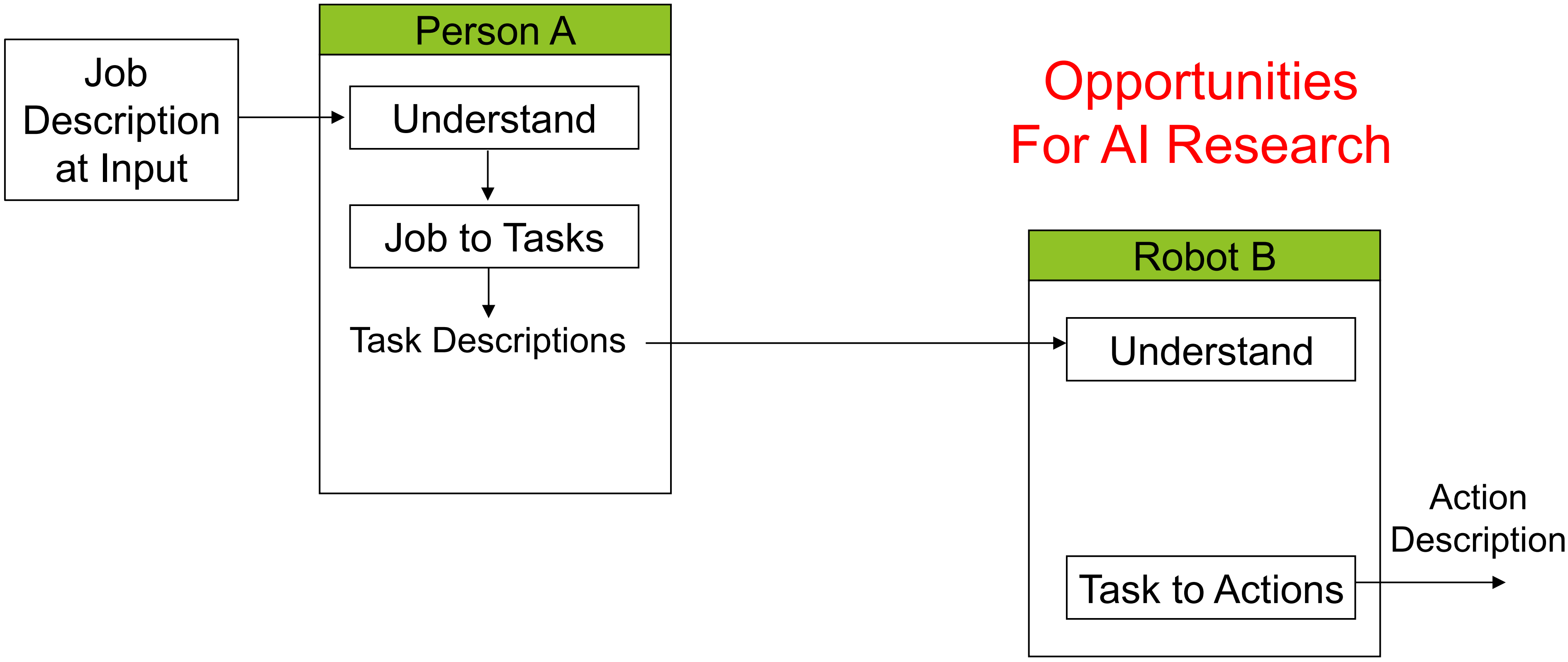
- The task is to make use of fasteners which is to mechanically attach two or more parts together in order to create a final product of a desired structure or mechanism.



Output of Action Sequence

- To place two parts together with correct alignment.
- To insert fasteners
- To tighten fasteners

Future Goal of Making Robots to Understand Task Descriptions or Models



Summary of Lecture 2

- Purpose of Action Planning
- Input to Action Planning
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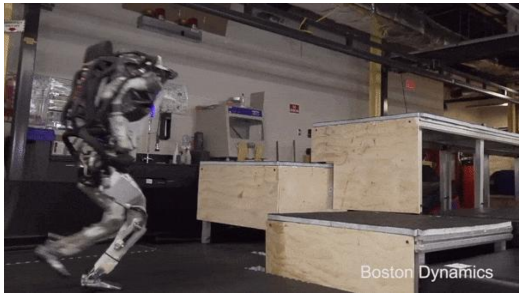
Outline of Module 3

Conceptual World

- Task Planning
- Action Planning
- Motion Planning



Physical World

- Path Planning
- Trajectory Planning

Module 3

MA4825 Robotics

Lecture 3

Motion Planning

Xie Ming, PhD (France)
http://personal.ntu.edu.sg/mmxie

Outline of Lecture 3

- Purpose of Motion Planning
- Input to Motion Planning
- Output from Motion Planning
- Process of Motion Planning

Outline of Lecture 3

- Purpose of Motion Planning
- Input to Motion Planning
- Output from Motion Planning
- Process of Motion Planning

Definition of Motion Planning

- Motion planning is a process which takes the description of a given action as input and produces a sequence of motions as output, which are to be performed by robots.

What are motions that robots can perform?

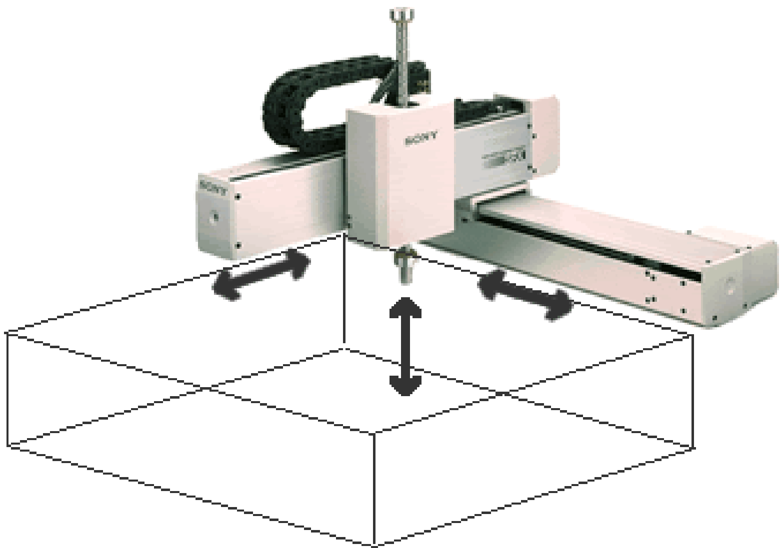
Linear Motion

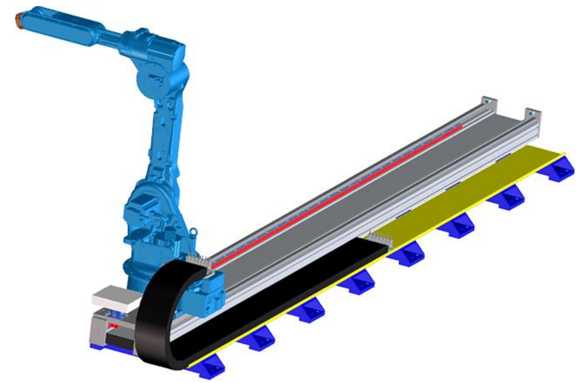
- Move To Linear Position
- Move With Linear Displacement
- Set Linear Speed To
- Set Linear Acceleration To

Angular Motion

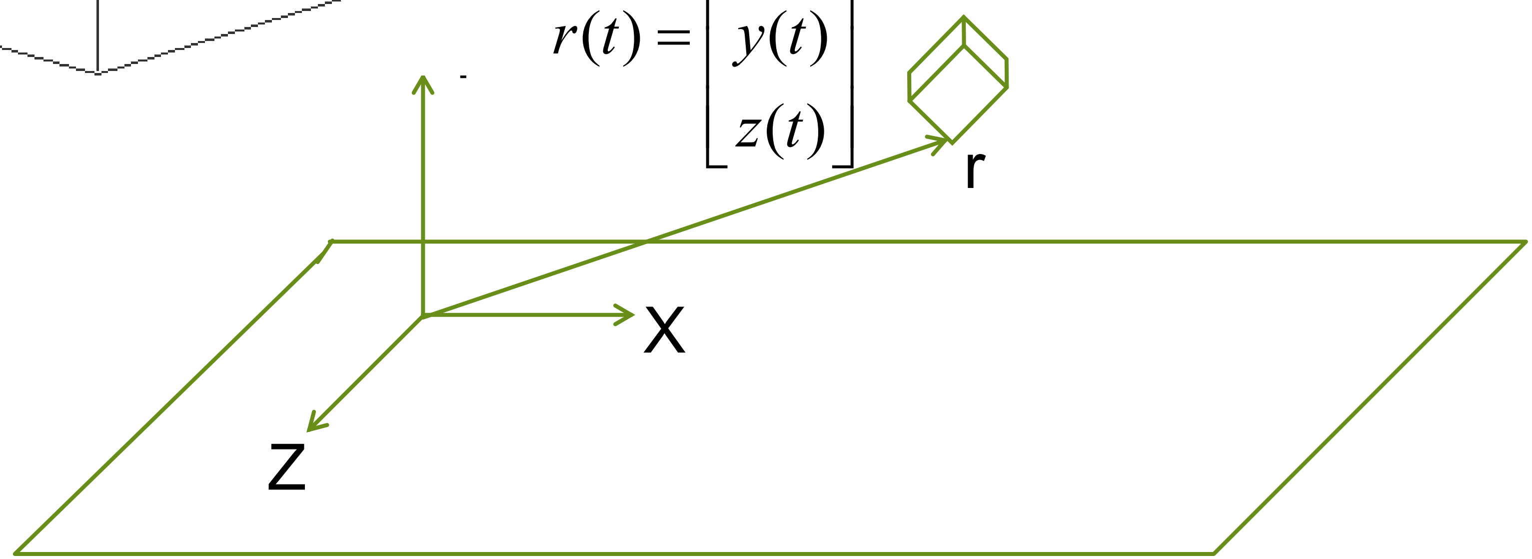
- Rotate To Angular Position
- Rotate With Angular Displacement
- Set Angular Speed To
- Set Angular Acceleration To

<Move To> Linear Position



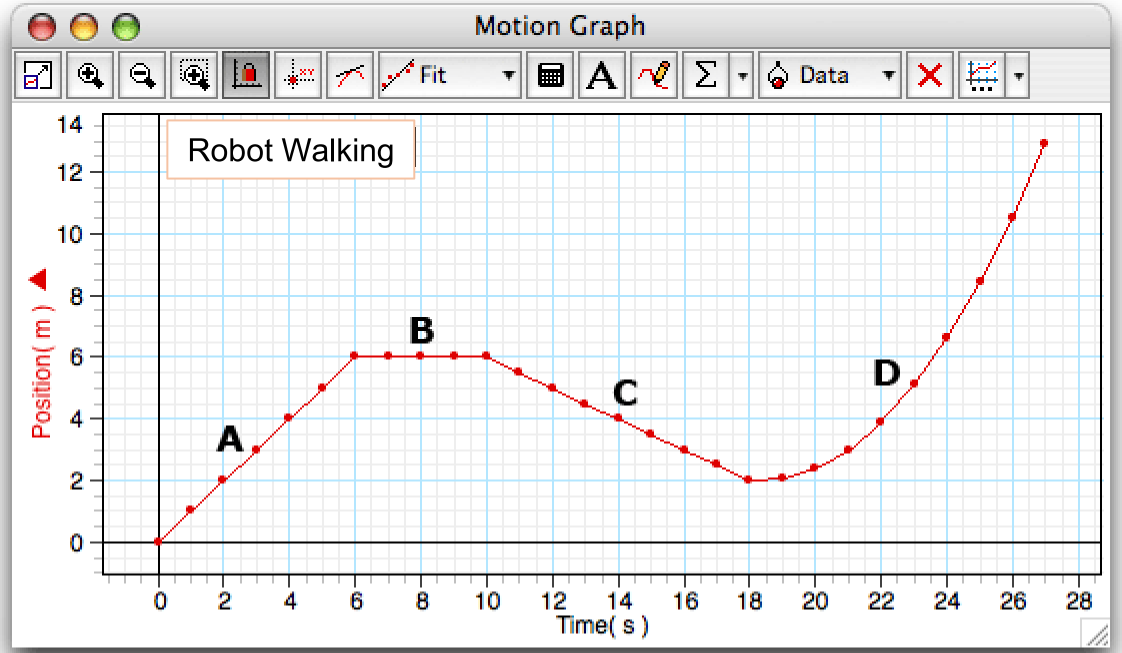


$$r(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}$$



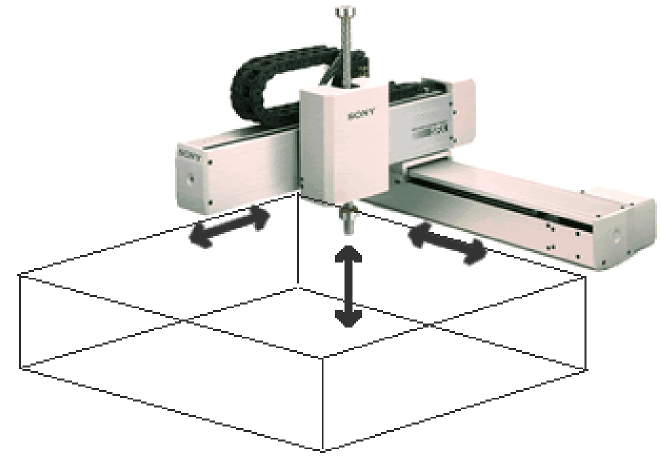
Example

- One ASIMO robot walks along a X axis. The record of the robot's positions is shown in the figure below. What is the position of the robot at the time instance of 18 seconds?



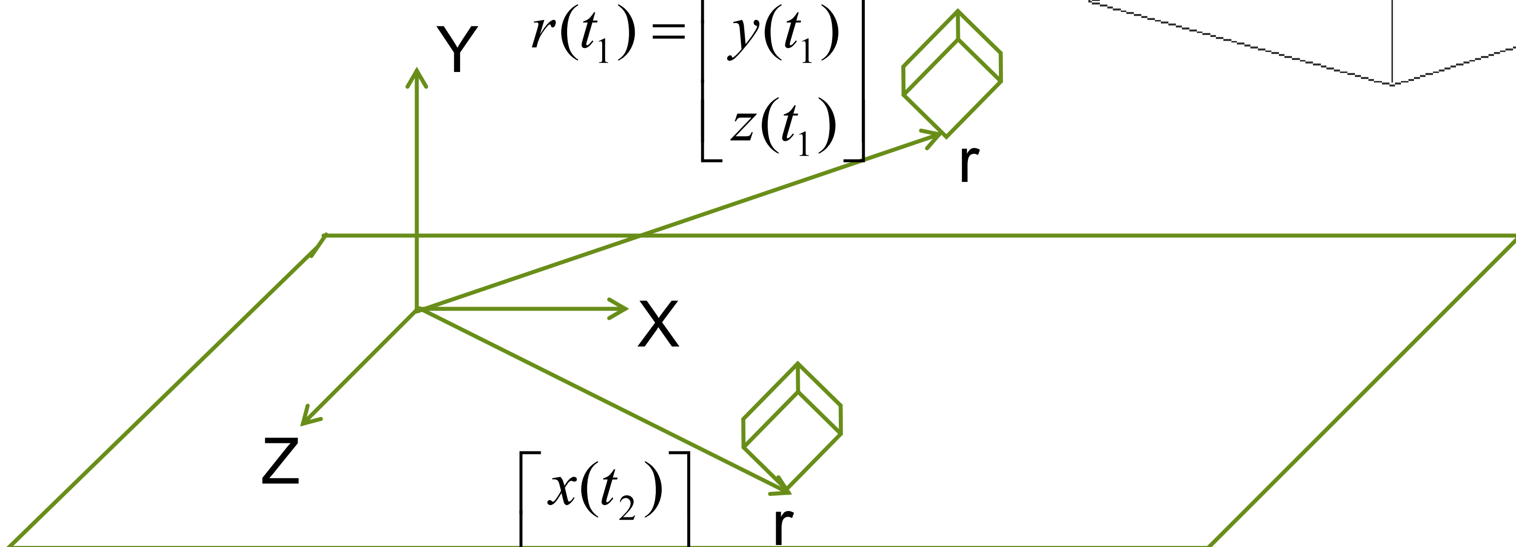
- Answer: the position when t = 18 seconds is: x = 2.0 meters.

<Move With> Linear Displacement



$$\Delta r = r(t_2) - r(t_1)$$

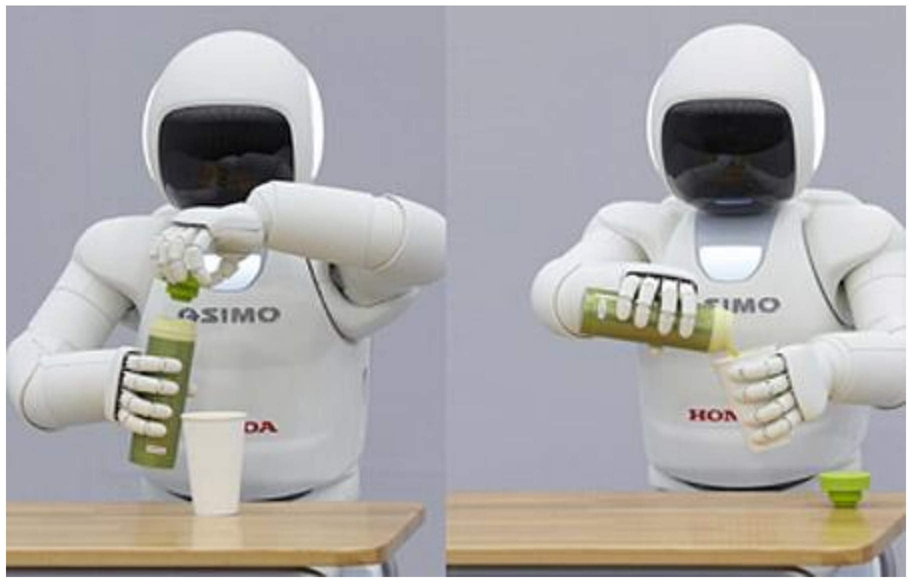
$$r(t_1) = \begin{bmatrix} x(t_1) \\ y(t_1) \\ z(t_1) \end{bmatrix}$$



$$r(t_2) = \begin{bmatrix} x(t_2) \\ y(t_2) \\ z(t_2) \end{bmatrix}$$

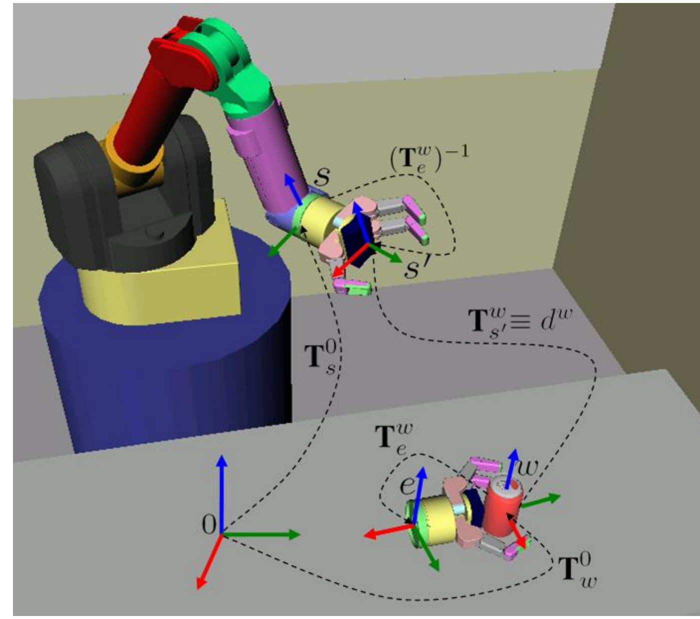
Example

- One ASIMO robot opens the cap of a cup at location (10.0, 20.0, 30.0) (cm) and places it at location (30.0, -10.0, 10.0) (cm). What is the displacement of the cap?



- Answer:
  - It is: (30.0, -10.0, 10.0) – (10.0, 20.0, 30.0) = (20.0, -30.0, -20.0) (cm)

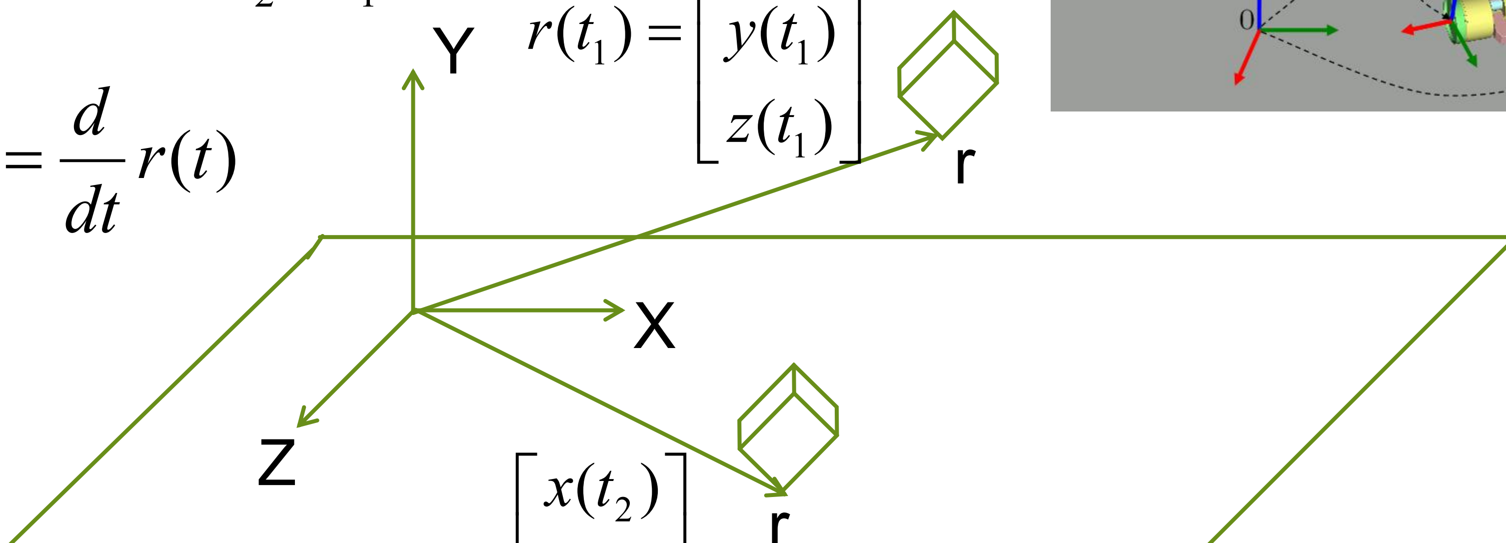
Set Linear Speed To



$$v(t_1, t_2) = \frac{r(t_2) - r(t_1)}{t_2 - t_1}$$

$$v(t) = \frac{d}{dt} r(t)$$

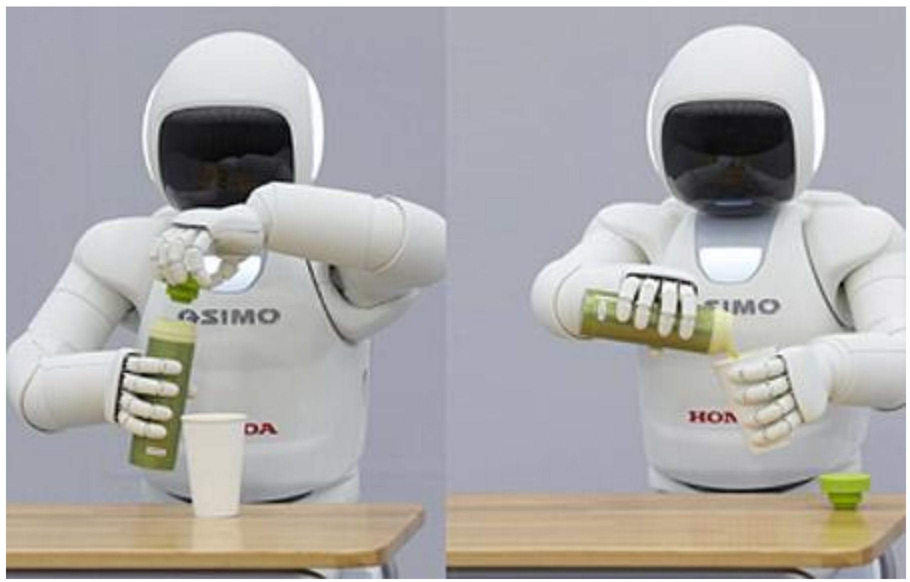
$$r(t_1) = \begin{bmatrix} x(t_1) \\ y(t_1) \\ z(t_1) \end{bmatrix}$$



$$r(t_2) = \begin{bmatrix} x(t_2) \\ y(t_2) \\ z(t_2) \end{bmatrix}$$

Example

- One ASIMO robot opens the cap of a cup at location (10.0, 20.0, 30.0) (cm) and places it at location (30.0, -10.0, 10.0) (cm). It completes the motion within 2.0 seconds. What is the average velocity of the cap's motion?

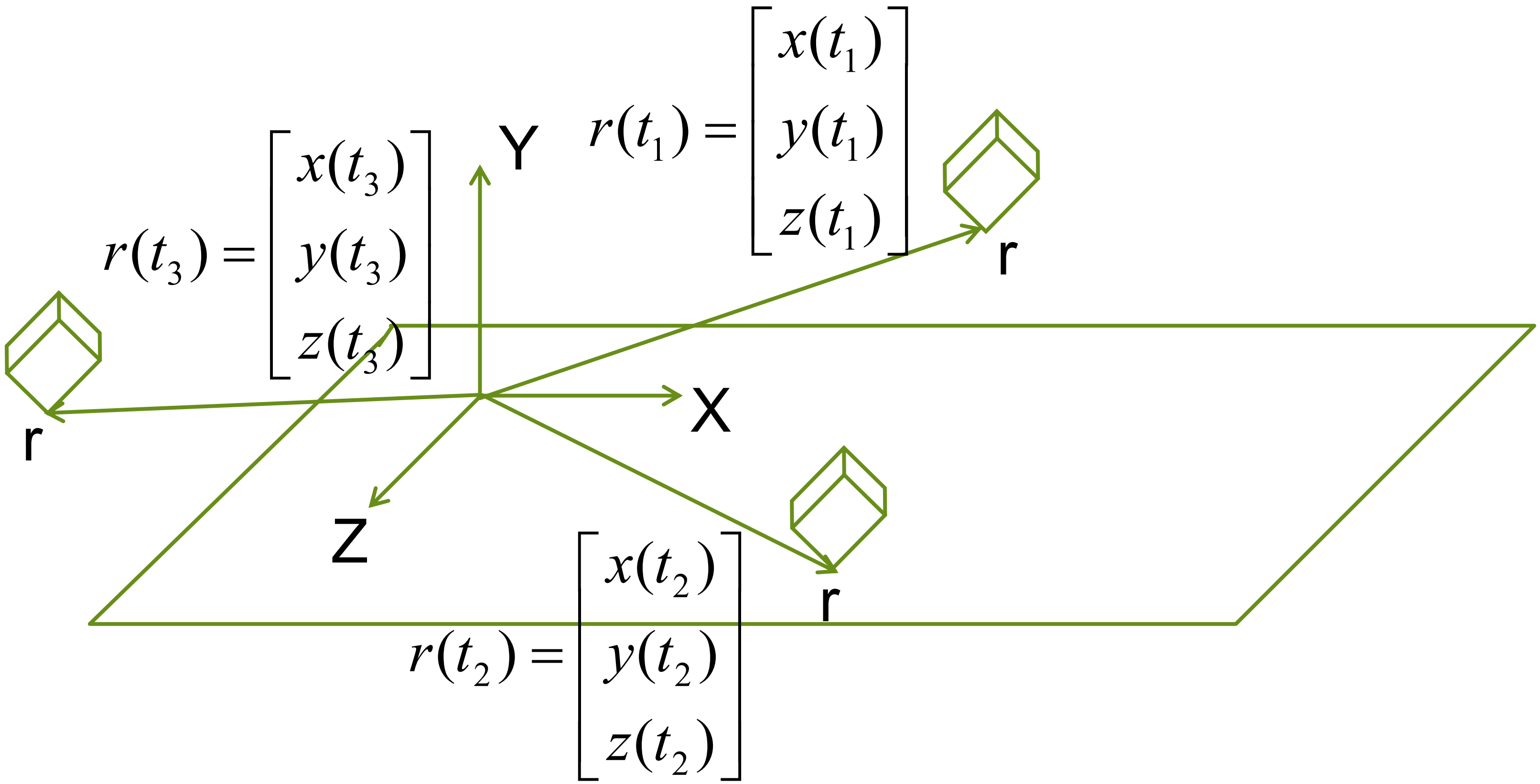


- Answer:
  - It is: (20.0, -30.0, -20.0) (cm) / 2.0 (s) = (10.0, -15.0, -10.0) (cm/s)

Set Linear Acceleration To

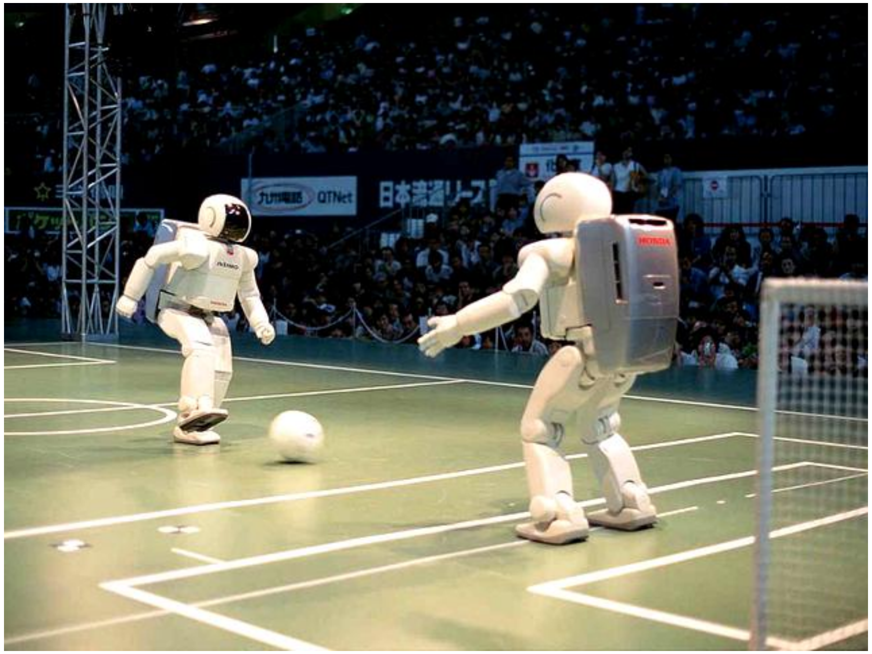
$$a(t_1, t_3) = \frac{v(t_2, t_3) - v(t_1, t_2)}{t_3 - t_1}$$

$$a(t) = \frac{d}{dt} v(t)$$



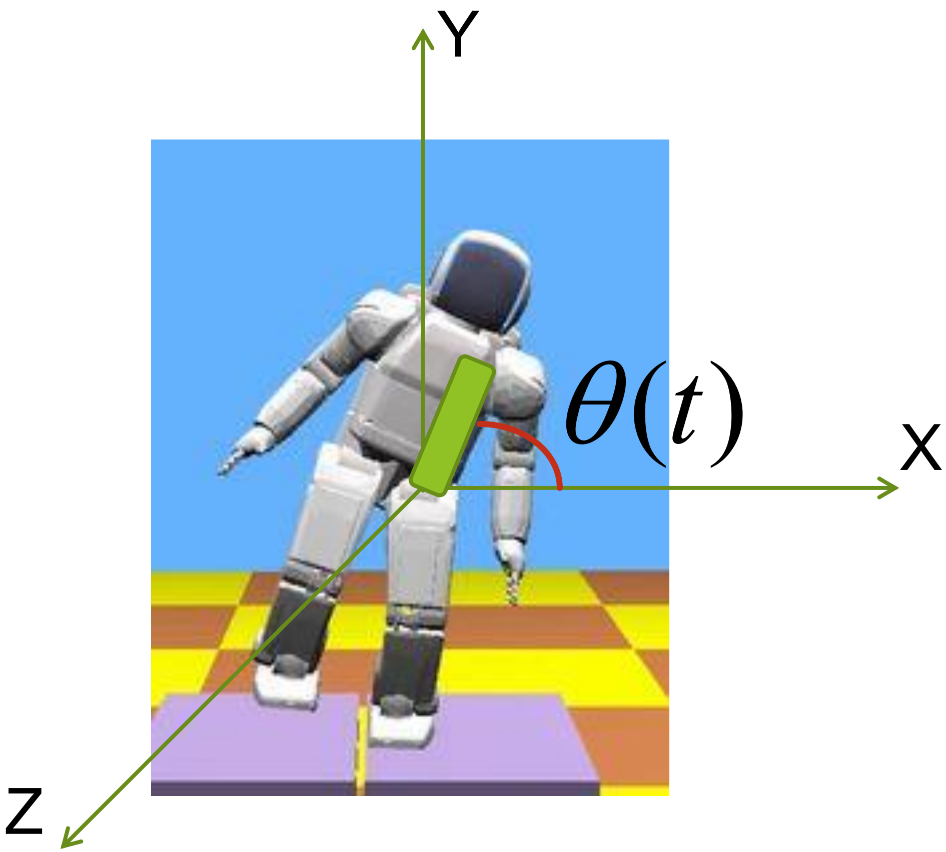
Example

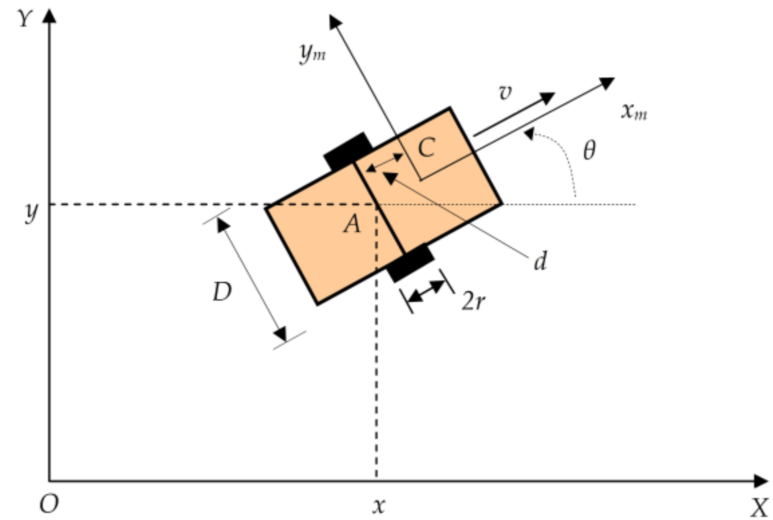
- A ball on a floor is at rest. Then, one ASIMO robot kits the ball which reaches the velocity of (1.5, 0.0, 0.2) (m/s) within a time period of 0.5 seconds. What is the average acceleration of the ball during the kick-off?

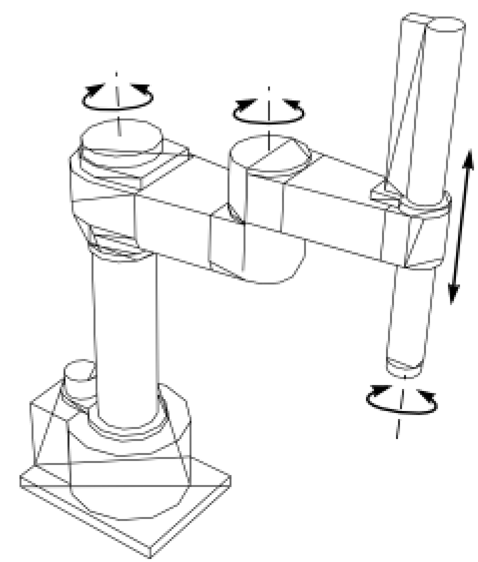


- Answer:
  - It is: (1.5, 0.0, 0.2) (m) / 0.5 (s) = (3.0, 0.0, 0.4) (m/s2)

<Rotate To> Angular Position

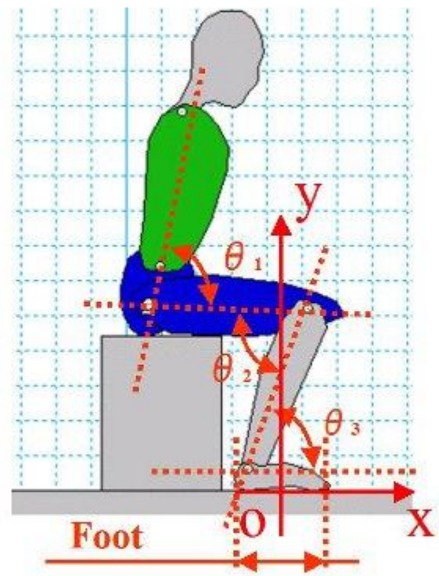






Example

- A person is sitting on a bloc as shown in the figure below. What is the angular position of the leg's lower limb (i.e. tibia link)? And, what is the angular position of the leg's upper limb (i.e. femur link)?

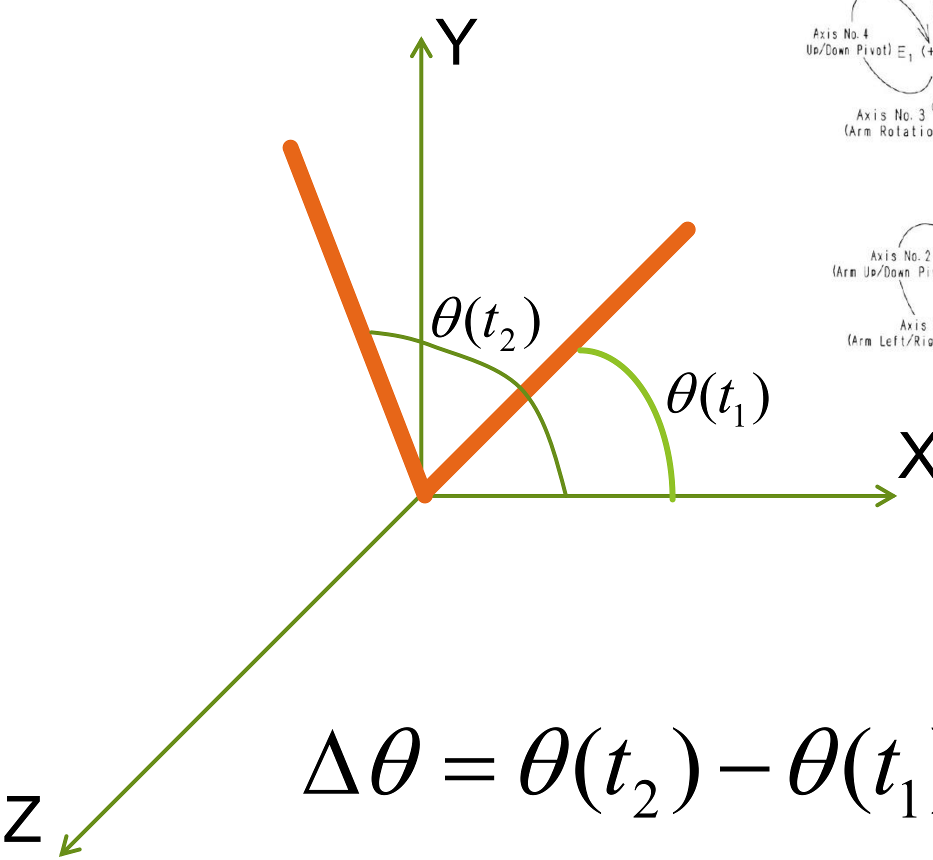


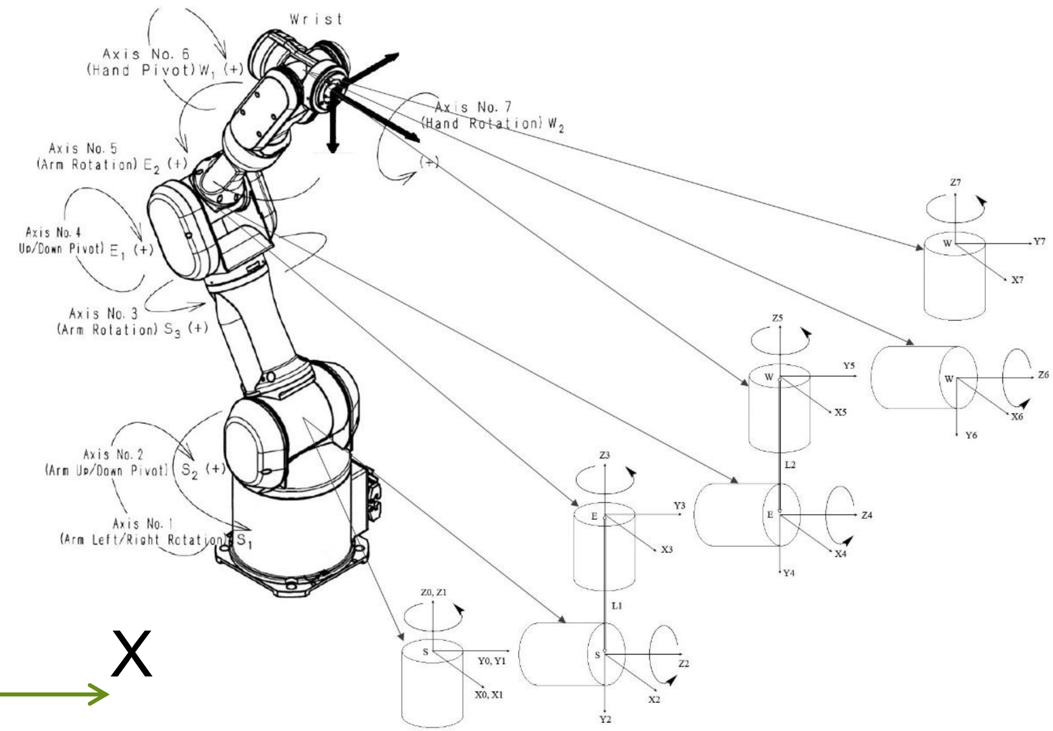
- Answer:

$$\text{The angular position of leg's lower limb is : } \theta_3$$

$$\text{The angular position of leg's upper limb is : } \theta_2$$

<Rotate With> Angular Displacement

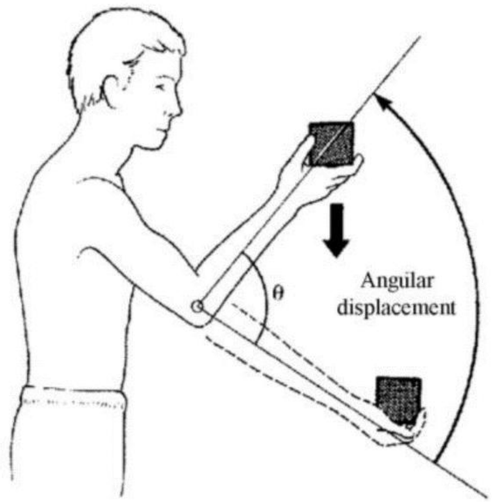




$$\Delta\theta = \theta(t_2) - \theta(t_1)$$

Example

- A person is lifting up an object by the angular motion of one elbow joint as shown in the figure below. What is the angular displacement depicted in the figure?

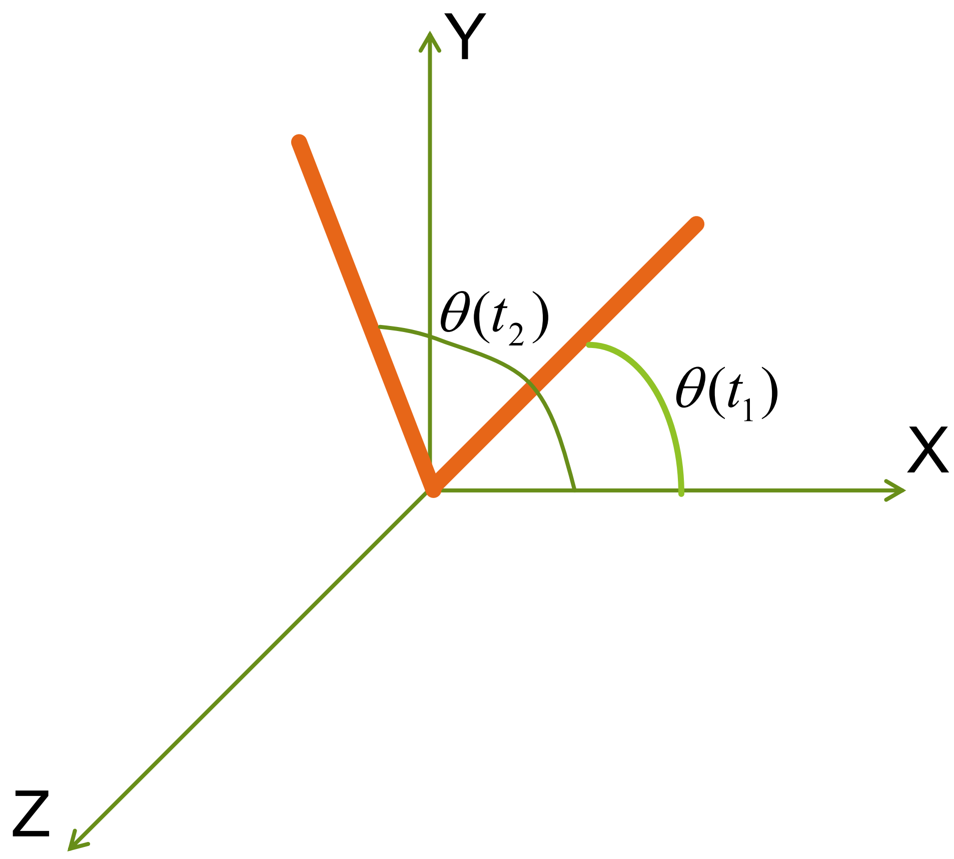


- Answer: the angular displacement of elbow joint is 90.0 degrees.

Set Angular Speed To

$$\omega(t_1, t_2) = \frac{\theta(t_2) - \theta(t_1)}{t_2 - t_1}$$

$$\omega(t) = \frac{d}{dt}\theta(t)$$



Example

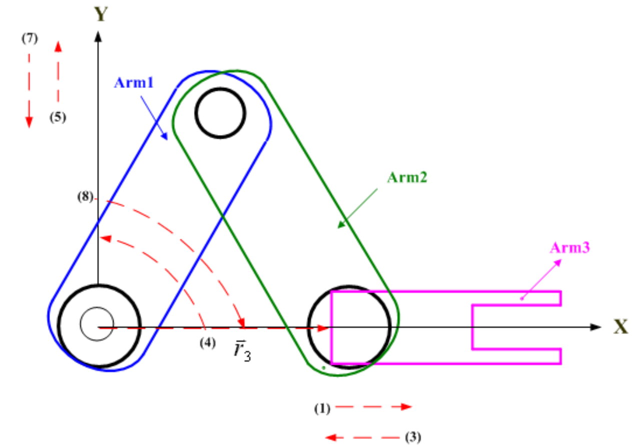
- A robot consists of three arms (or links) as shown in the figure. The records of its three joint angles are shown in three plots respectively.

- What is the angular velocity of the second joint within the time internal of [3, 6] seconds?

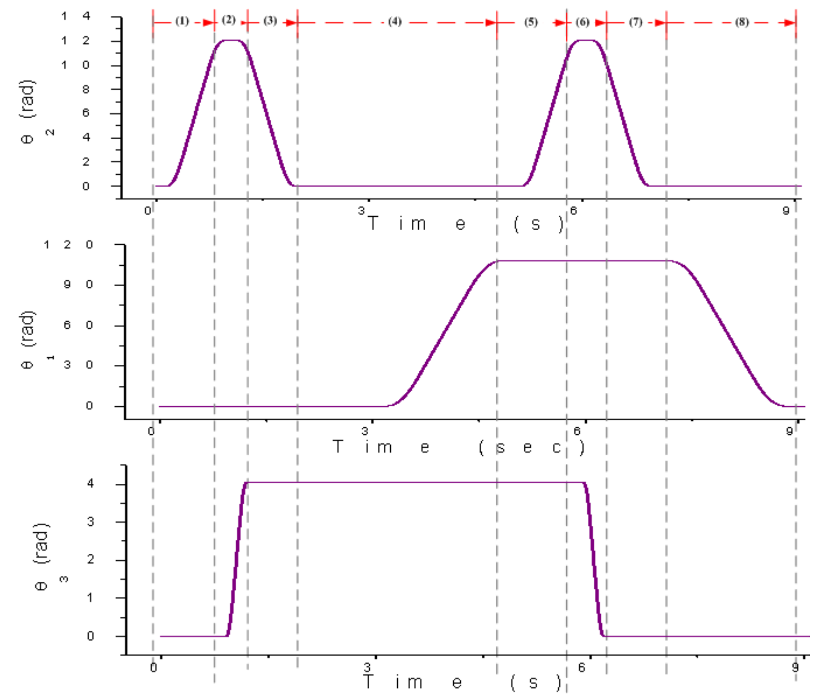
- Answer:

$$\omega_2(3.0{,}6.0) = \frac{11.5 - 0.0}{6.0 - 3.0} \times \frac{\pi}{180}$$

$$= 0.067 \text{ rad/s}$$



(a)

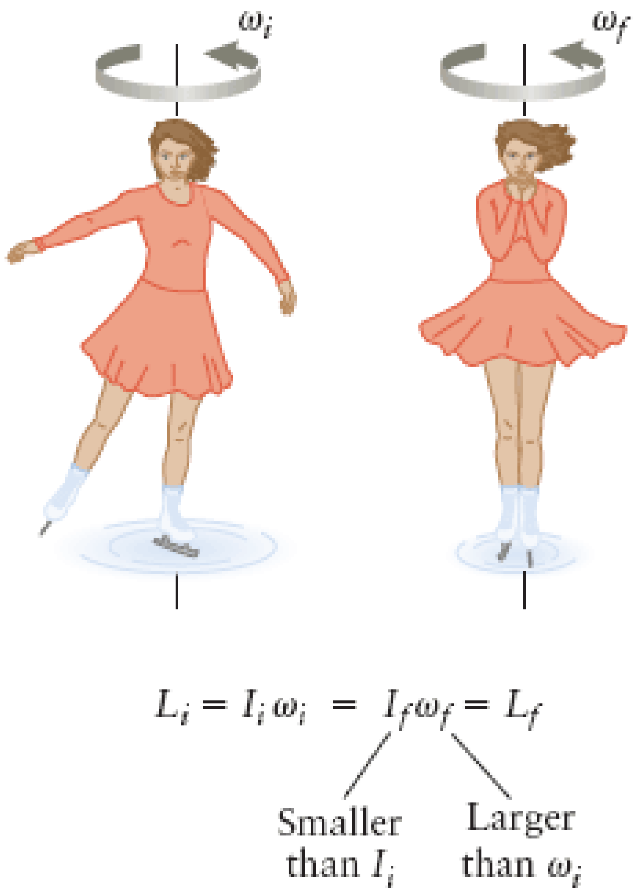


(b)

Set Angular Acceleration To

$$\alpha(t_1, t_2) = \frac{\omega(t_2) - \omega(t_1)}{t_2 - t_1}$$

$$\alpha(t) = \frac{d}{dt}\omega(t)$$



Example

- One QRIO robot throws a ball as shown in the figure. The robot throws out the ball within a time duration of 0.5 seconds. And, the angular velocity of the ball is 0.8 rad/s when it exists the hand of the robot. What is the angular acceleration of the ball during the throwing process?

- Answer:

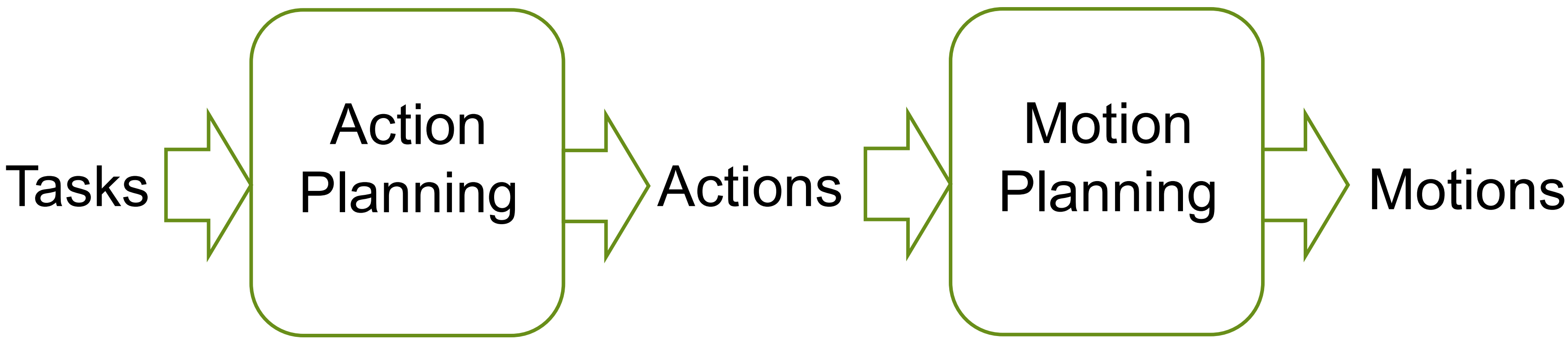
$$\alpha(0.0,0.5) = \frac{0.8-0.0}{0.5-0.0} = 1.6\ \text{rad/s}^2$$

Outline of Lecture 3

- Purpose of Motion Planning
- Input to Motion Planning
- Output from Motion Planning
- Process of Motion Planning

Input to Motion Planning

- The input to motion planning is the description of a given action in the form of natural languages.



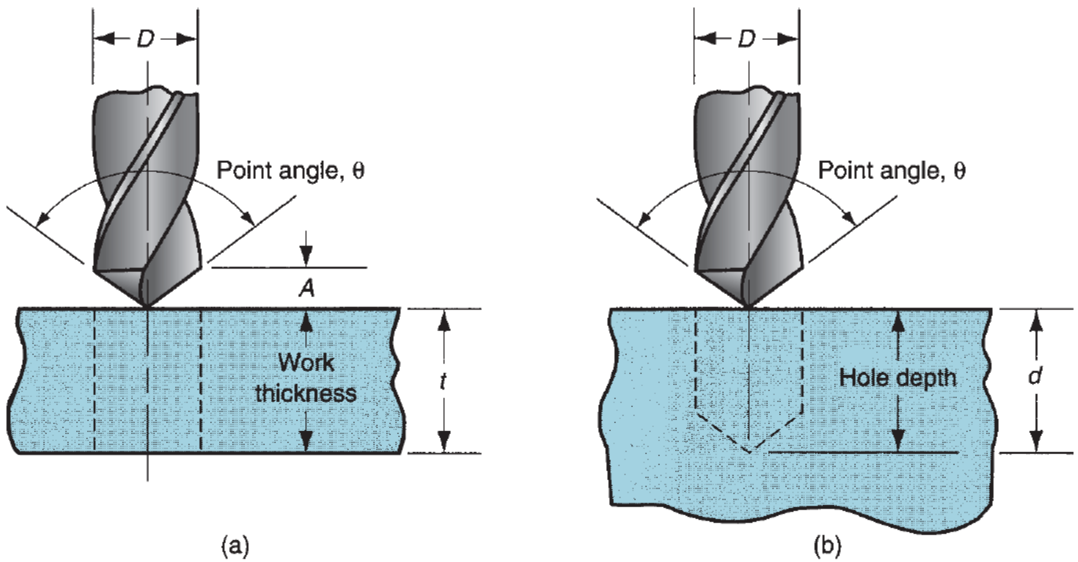
What should be inside the description of a given action?

Answer:

- 1. The outcome of action to be achieved
- 2. The space in which the action is to be undertaken
- 3. The time within which the action is to be undertaken

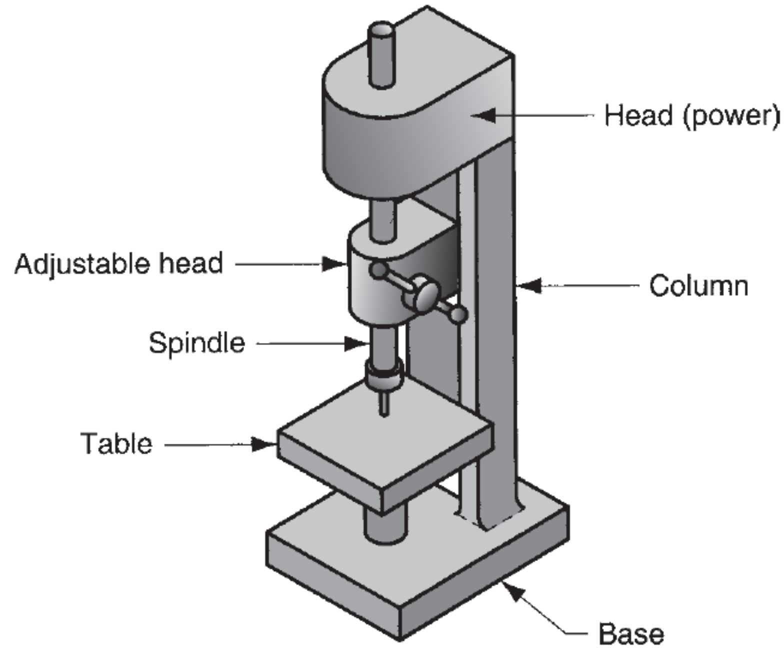
What is the action sequence derived from this task description (Drilling)?

- The task is to make use of rotating drill or drill bit in order to create holes of desired dimensions on stationary workpieces.



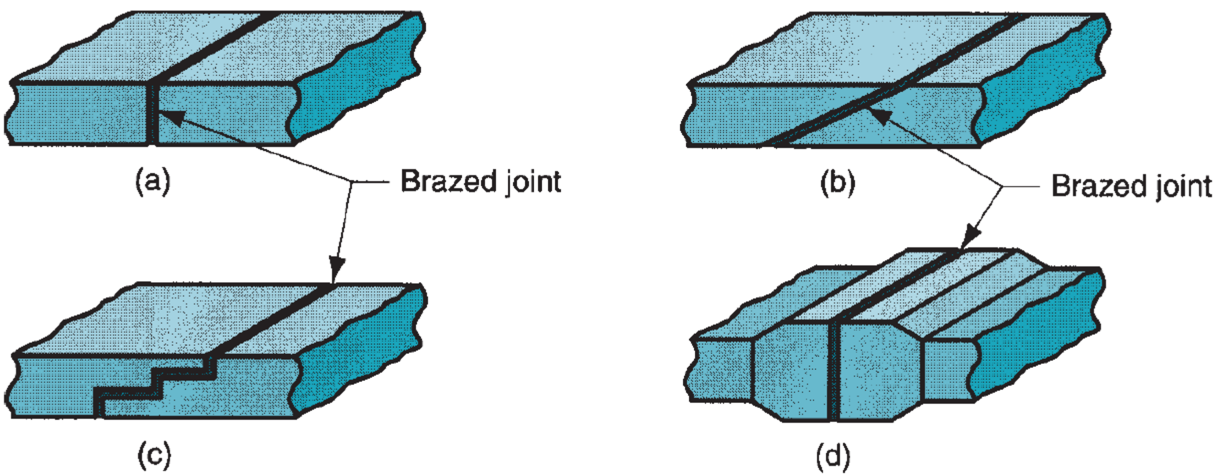
Output of Action Sequence (Drilling)

- To hold drill bit onto spindle
- To rotate the drill bit
- To press the rotating drill bit against a workpiece
- To move the rotating drill bit down into the workpiece



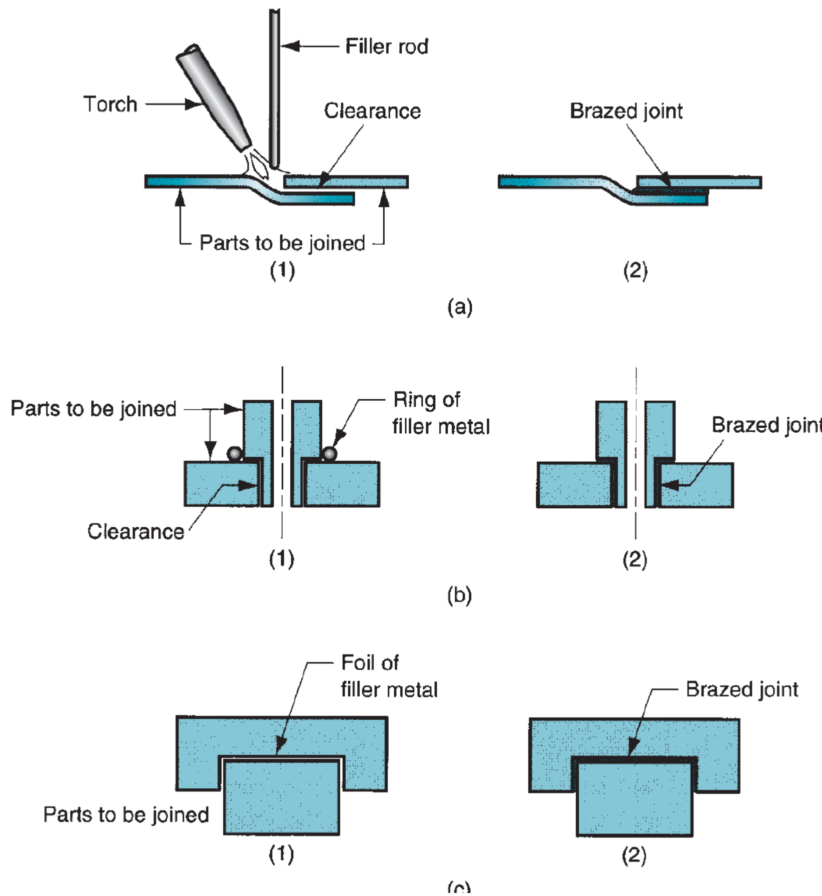
What is the action sequence derived from this task description (Soldering)?

- The task is to make use of filler metal which is melted and distributed into the faying surfaces of two metal parts in order to join or bond them together so as to form a permanent joint.



Output of Action Sequence

- Two metal parts are placed according to the desired joint.
- To move the torch and filler rod together along the faying surface
- Or, to place the filler metal and to apply heat so as to melt the filler metal.

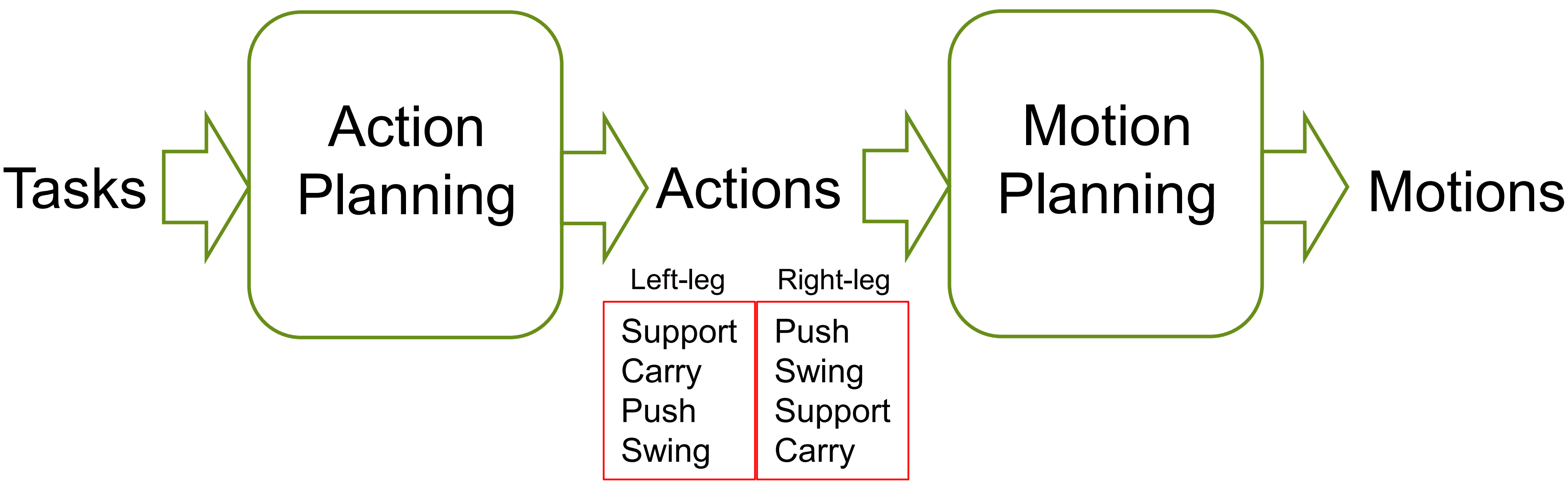


Outline of Lecture 3

- Purpose of Motion Planning
- Input to Motion Planning
- Output from Motion Planning
- Process of Motion Planning

Output from Motion Planning

- The output of motion planning is a sequence of motion descriptions in the form of script languages (commands) or library functions, which could lead to the execution of a given action.



How to describe motions?

Use of Script Languages

- Move to
- Move with
- Rotate to
- Rotate with

Software Design

Use of Library Functions

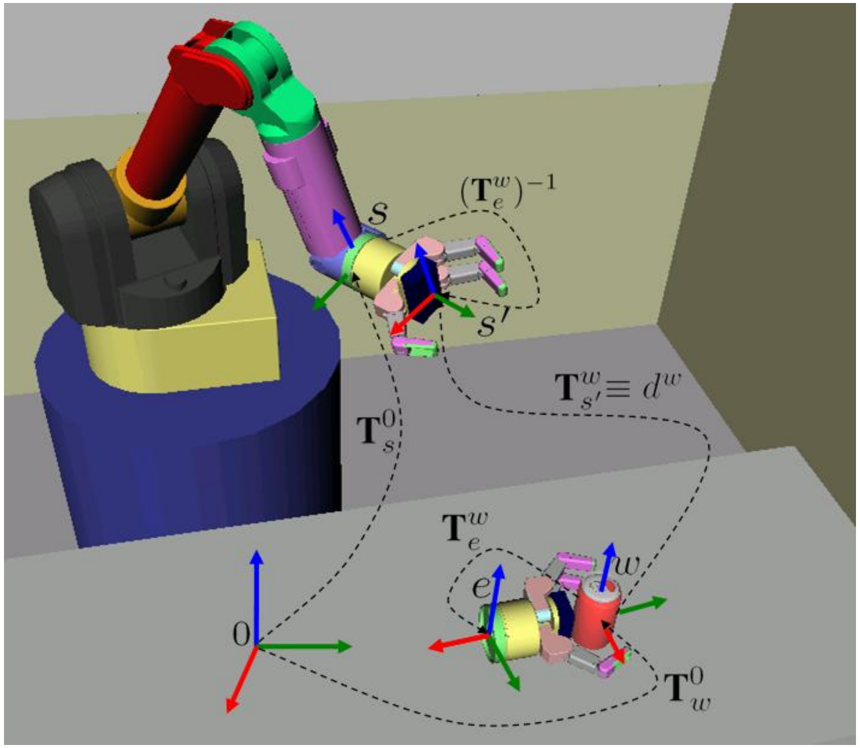
- MoveTo()
- MoveWith()
- RotateTo()
- RotateWith()

Software Design

What should be inside the description (commands or library functions) of motions?

Answer:

- 1. Initial Linear and/or Angular Positions
- 2. Space and Time Constraints.
- 3. Final Linear and/or Angular Positions.



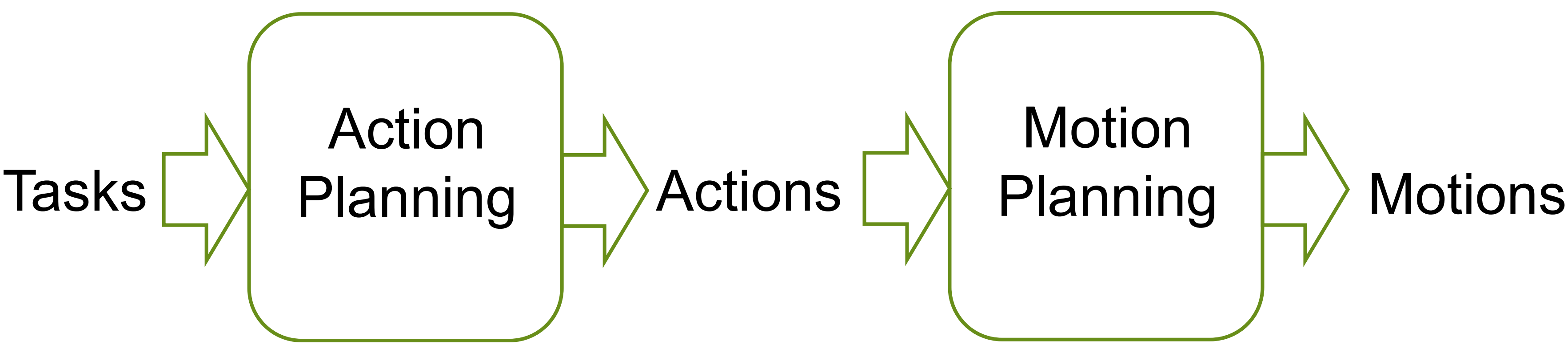
Outline of Lecture 3

- Purpose of Motion Planning
- Input to Motion Planning
- Output from Motion Planning
- Process of Motion Planning

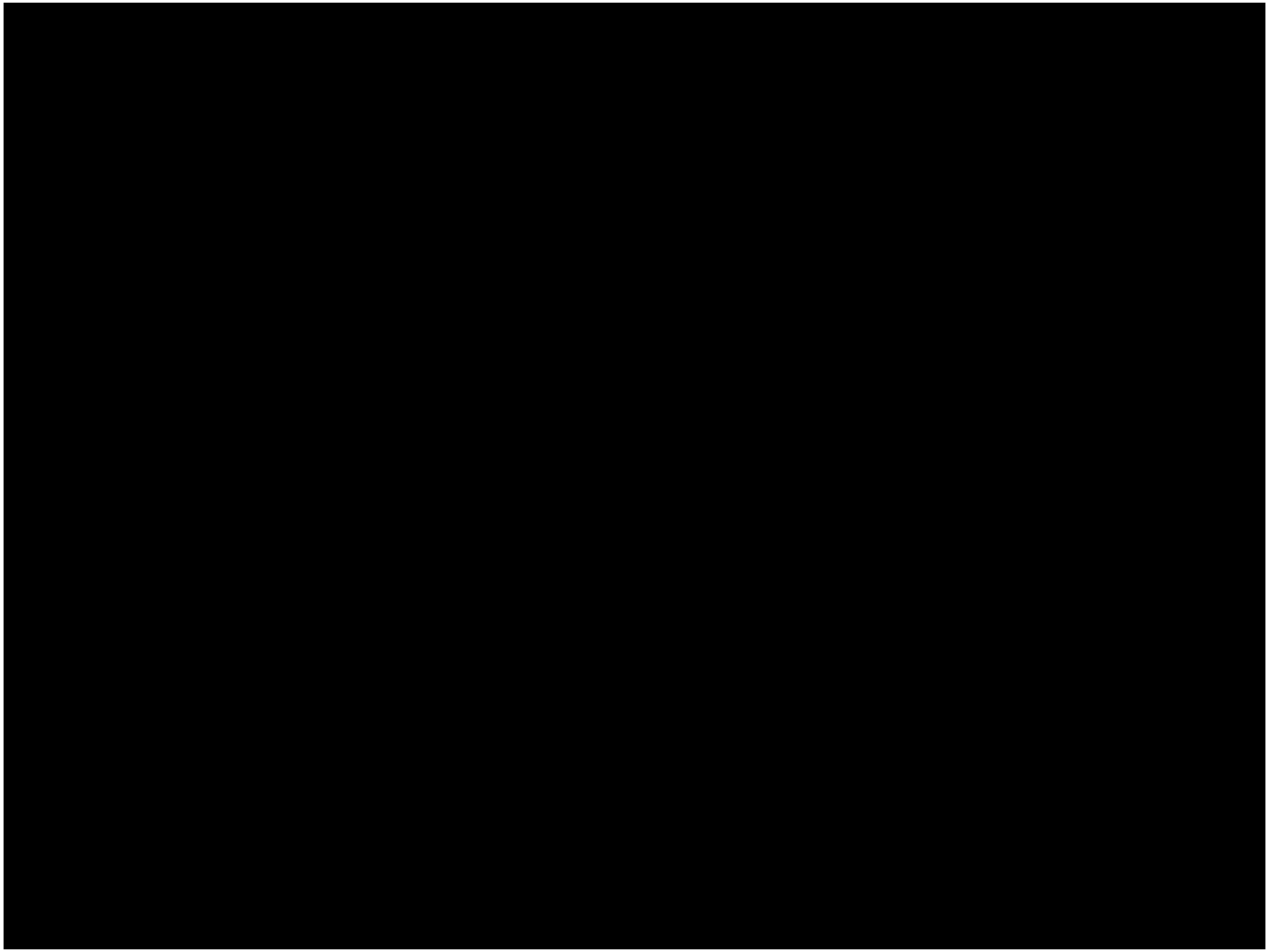
How to do motion planning?

- Scenario 1: You are the buyers or users
  - Human-Assisted Motion Planning
    - Use of programming languages and library functions

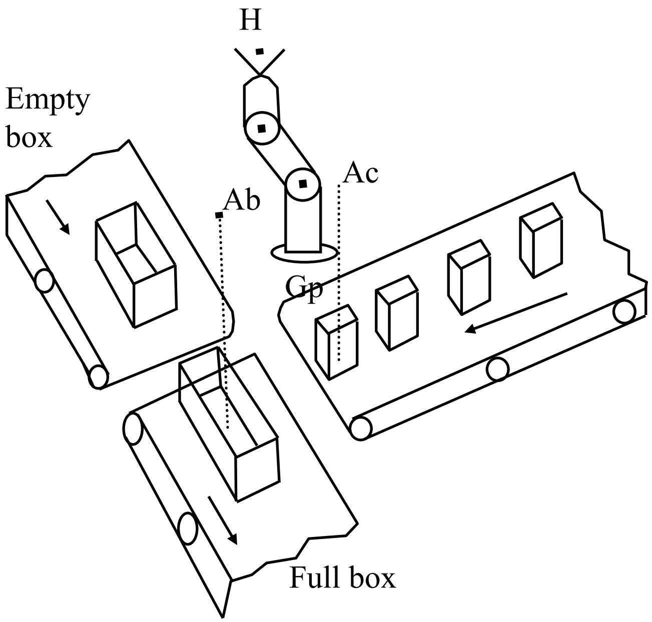
- Scenario 2: You are the designers of robots
  - Autonomous Motion Planning by Robots



Example 1 of Human-Assisted Motion Planning



Example 2 of Human-Assisted Motion Planning



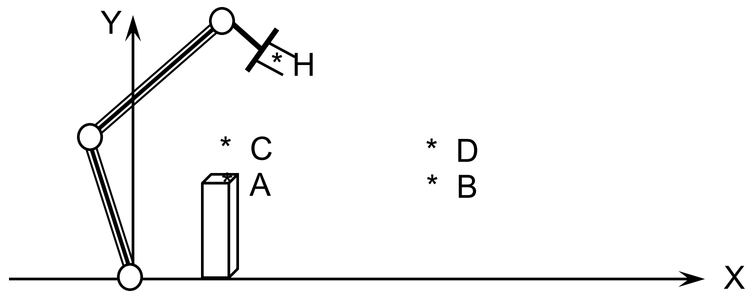
Action:

To pick up an object.

Motion Sequence:

(1) Move to point Ac.
(2) Open the gripper.
(3) Move to point Gp.
(4) Close the gripper.
(5) Move back to point Ac.

Example 3 of Human-Assisted Motion Planning



Motion Sequence:

Define Point H
Define Point A
Define Point B
Define Point C
Define Point D
Open Gripper
Move To A
Close Gripper
Move To C
Move To D
Move To B
Open Gripper
Move To D
....

Motion Sequence:

```
setp H = (x1, y1)
setp A = (x2, y2)
setp B = (x3, y3)
setp C = (x4, y4)
setp D = (x5, y5)
open
moveto(A)
close
moveto(C)
moveto(D)
moveto(B)
open
moveto(D)
....
```

Example 4 of Human-Assisted Motion Planning

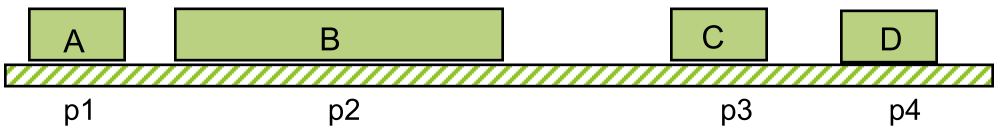
Initial State before Motion:

AT_POSITION(A, p1)

AT_POSITION(B, p2)

AT_POSITION(C, p3)

AT_POSITION(D, p4)



Final State after Motion:

ON_TOP(B, A)

ON_TOP(C, B) and AT_LEFT(C, B)

ON_TOP(D, B) and AT_RIGHT(D, B)

OpenHand();
Set p = p2 – offset1;
MoveTo(p);
CloseHand();
Set p = p1 – offset2;
MoveTp(p);
OpenHand();

Set p = p3;
MoveTo(p);
CloseHand();
Set p = p1 – offset3;
MoveTo(p);
OpenHand();

Set p = p4 – offset4;
MoveTo(p);
CloseHand();
Set p = p1 – offset5;
MoveTo(p);
OpenHand();

Motion Sequence

Example 5 of Human-Assisted Motion Planning

Initial State before Motion:

IN_ROOM(Robot, r1)

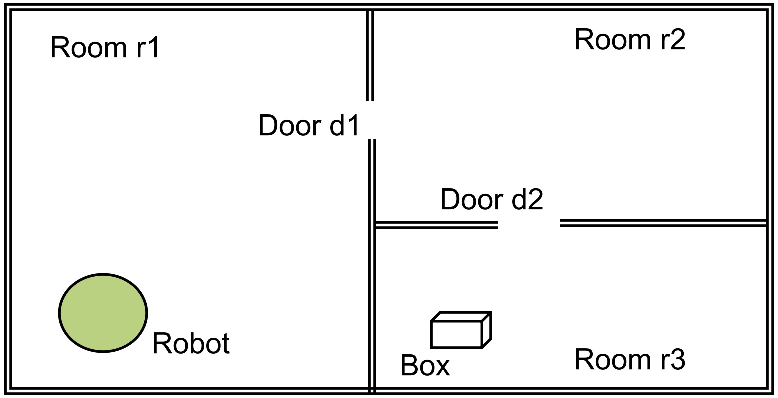
IN_ROOM(Box, r3)

CONNECT(d1, r1, r2)

CONNECT(d2, r2, r3)

Final State after Motion:

IN_ROOM(Box, r1)



Motion Sequence:

MOVE_TO(Robot, r2, d1)

MOVE_TO(Robot, r3, d2)

PUSH_TO(Box, r2, d2)

PUSH_TO(Box, r1, d1)

How to implement script languages or library functions?

Manually by Designers

- Move To
- Move With
- Speed To
- Accelerate To
- Open/Close Hand

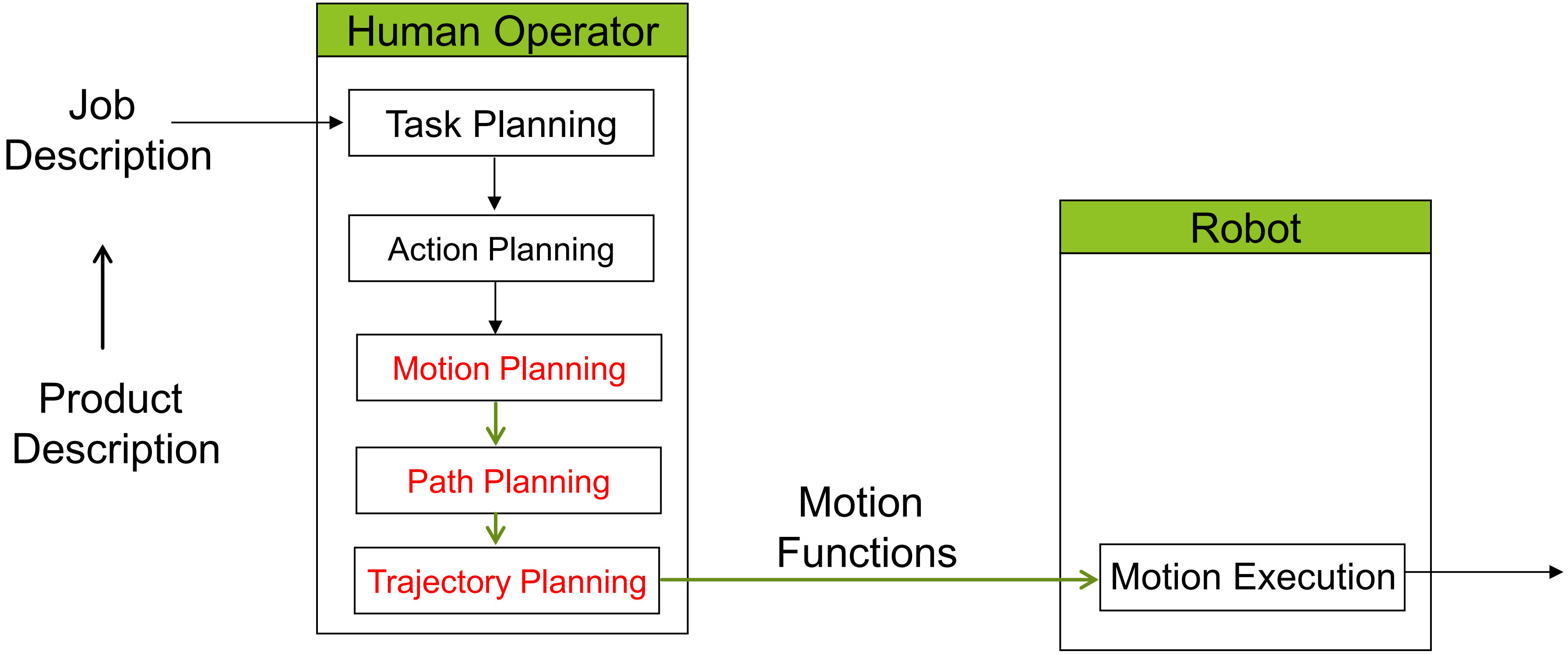
- MoveTo(arguments)
- MoveWith(arguments)
- SpeedTo(arguments)
- AccelerateTo(arguments)
- HandOpenClose(arguments)

Autonomously by Robots

- Move To
- Move With
- Speed To
- Accelerate To
- Open/Close Hand

- MoveTo(arguments)
- MoveWith(arguments)
- SpeedTo(arguments)
- AccelerateTo(arguments)
- HandOpenClose(arguments)

Discussion: Level 0 of Robot Intelligence or Intelligence Readiness



(TL: Technological Level of Readiness for Autonomous Vehicles)

Execution of Grasping Motions

Execution of Motion of Push, Pick and Place

Execution of Continuous Motion

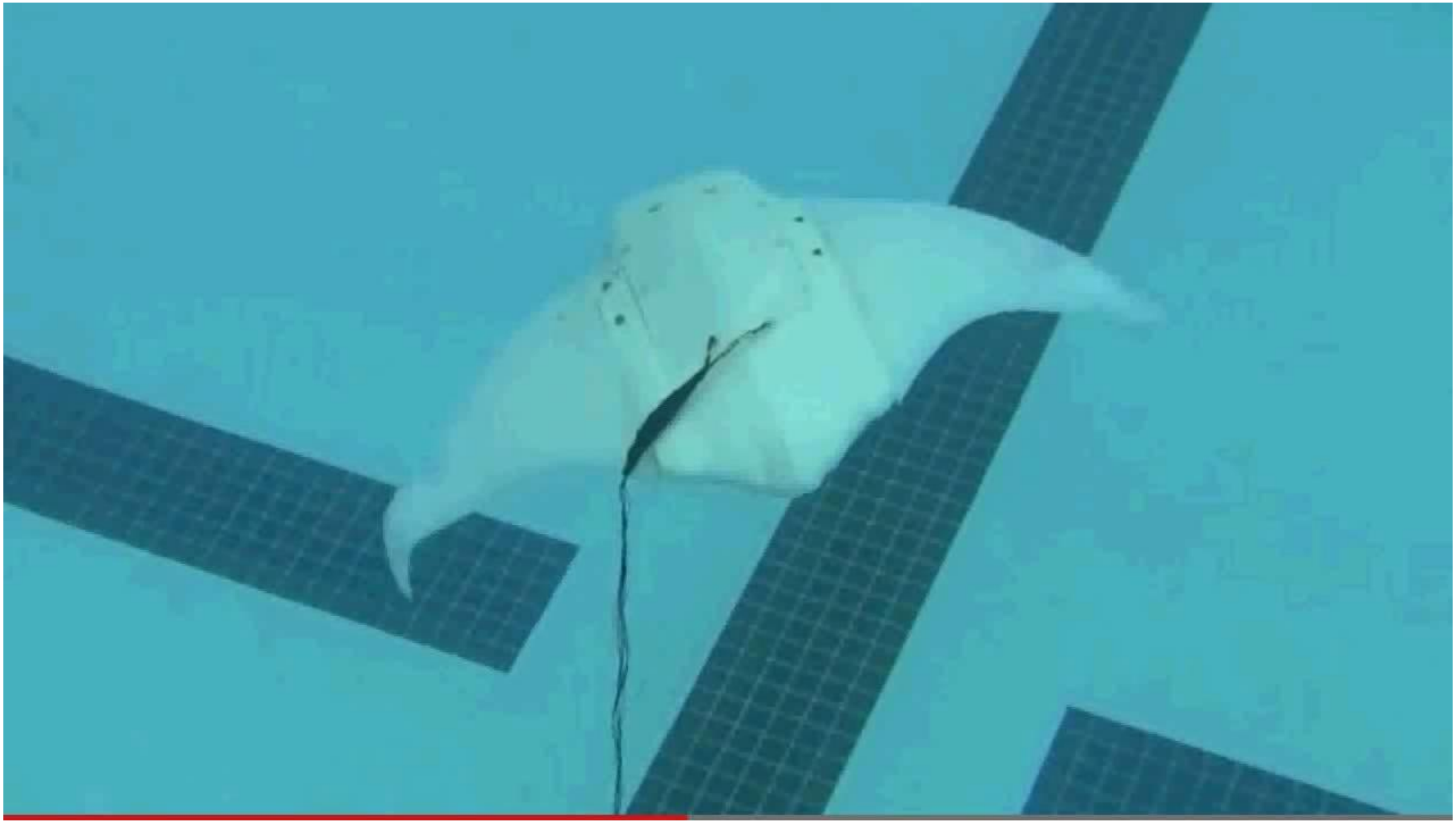
Execution of Motion on Land

Execution of Biped Walking on Land

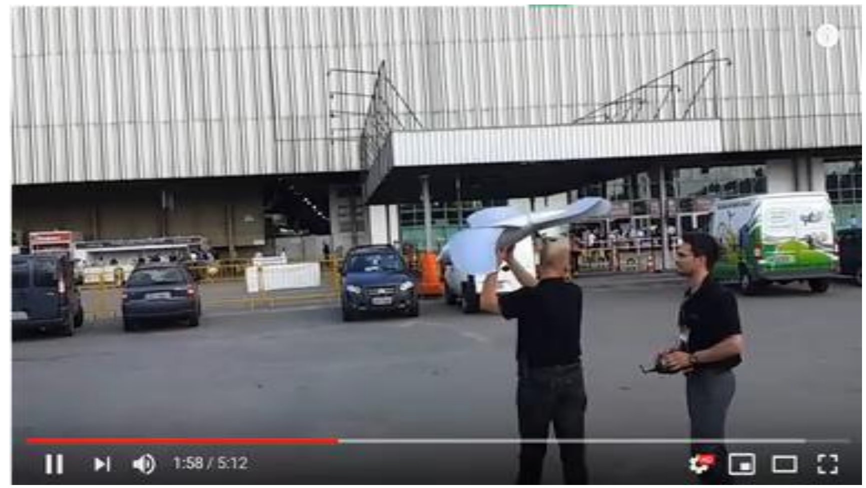
Execution of Snake-like Motion

Execution of Motion on Water

Execution of Motion in Water

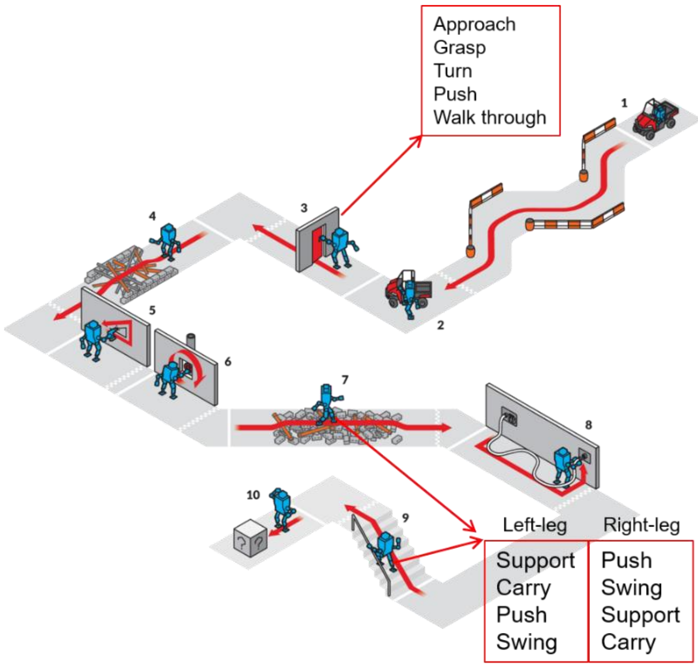


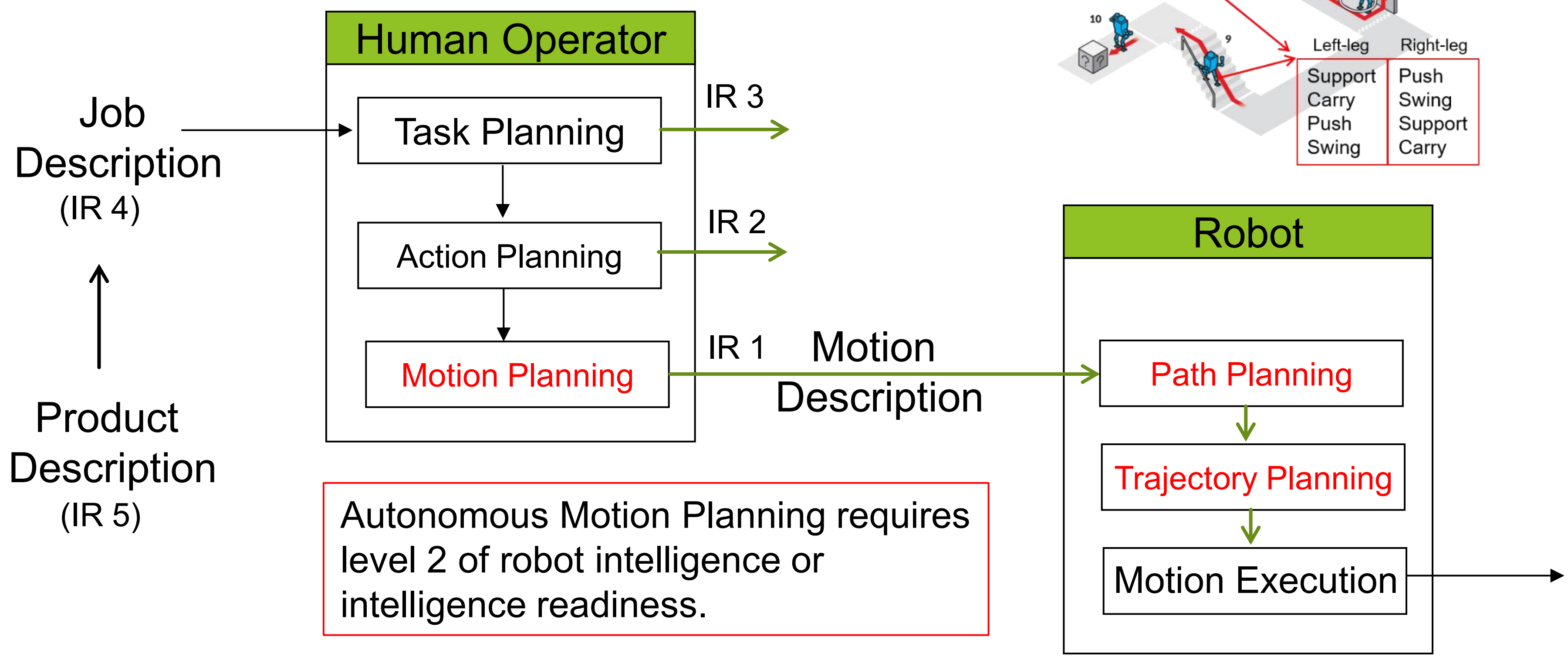
Execution of Motion in Air



Discussion: Level 1 of Robot Intelligence or Intelligence Readiness

(IR: Intelligence Readiness)



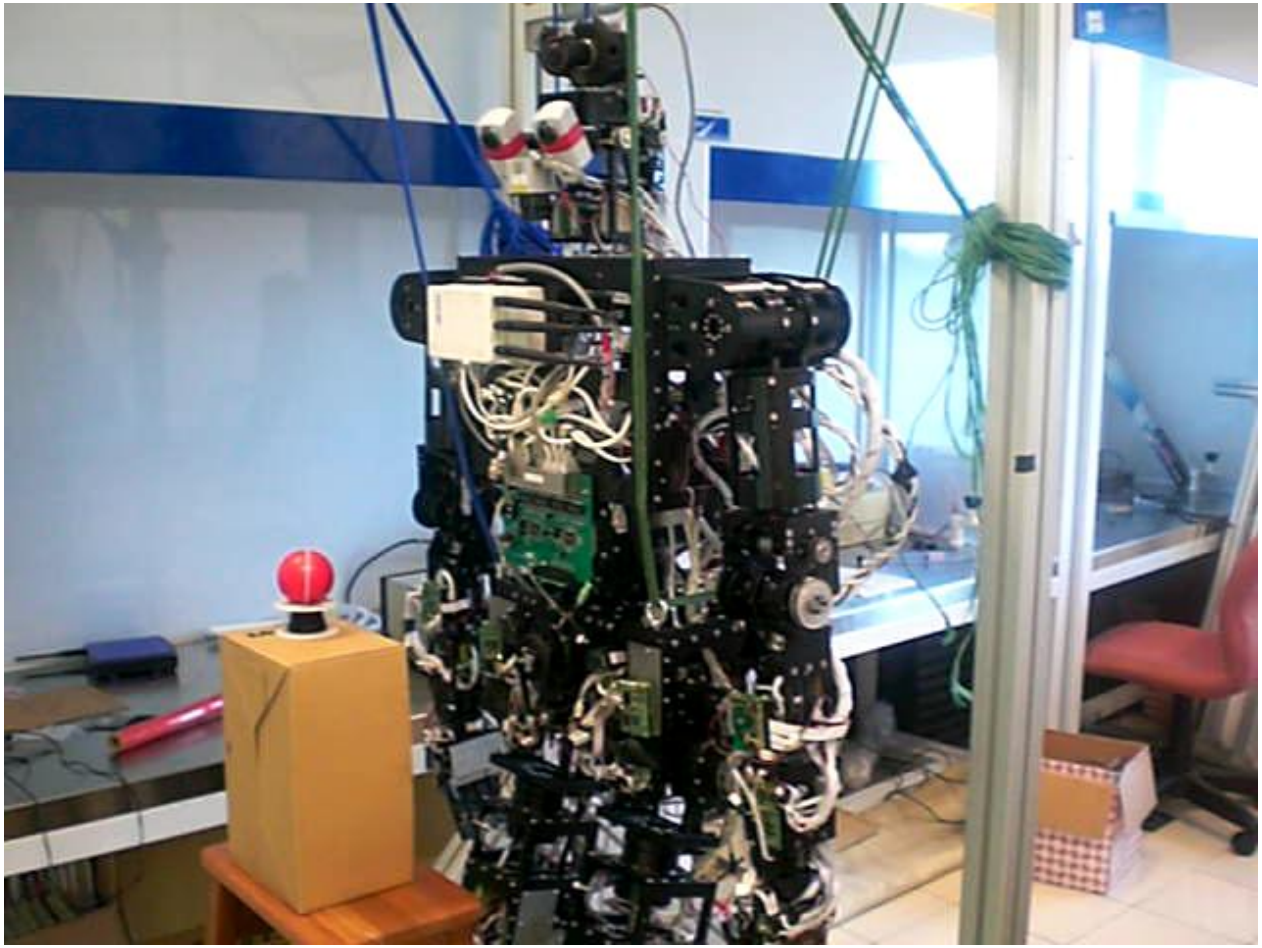


Example of Mobility with Level 1 of Robot Intelligence or Intelligence Readiness

Year 2000

Example of Grasping with Level 1 of Robot Intelligence or Intelligence Readiness

Year 2008



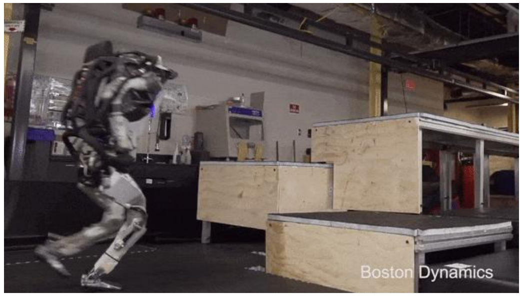
Summary of Lecture 3

- Purpose of Motion Planning
- Input to Motion Planning
- Output from Motion Planning
- Process of Motion Planning

Outline of Module 3

Conceptual World

- Task Planning
- Action Planning
- Motion Planning



Physical World

- Path Planning
- Trajectory Planning

School of Mechanical & Aerospace Engineering

Design, Machine, Control, Intelligence

Module 3

MA4825 Robotics

Lecture 4

Path Planning

Xie Ming, PhD (France)

http://personal.ntu.edu.sg/mmxie

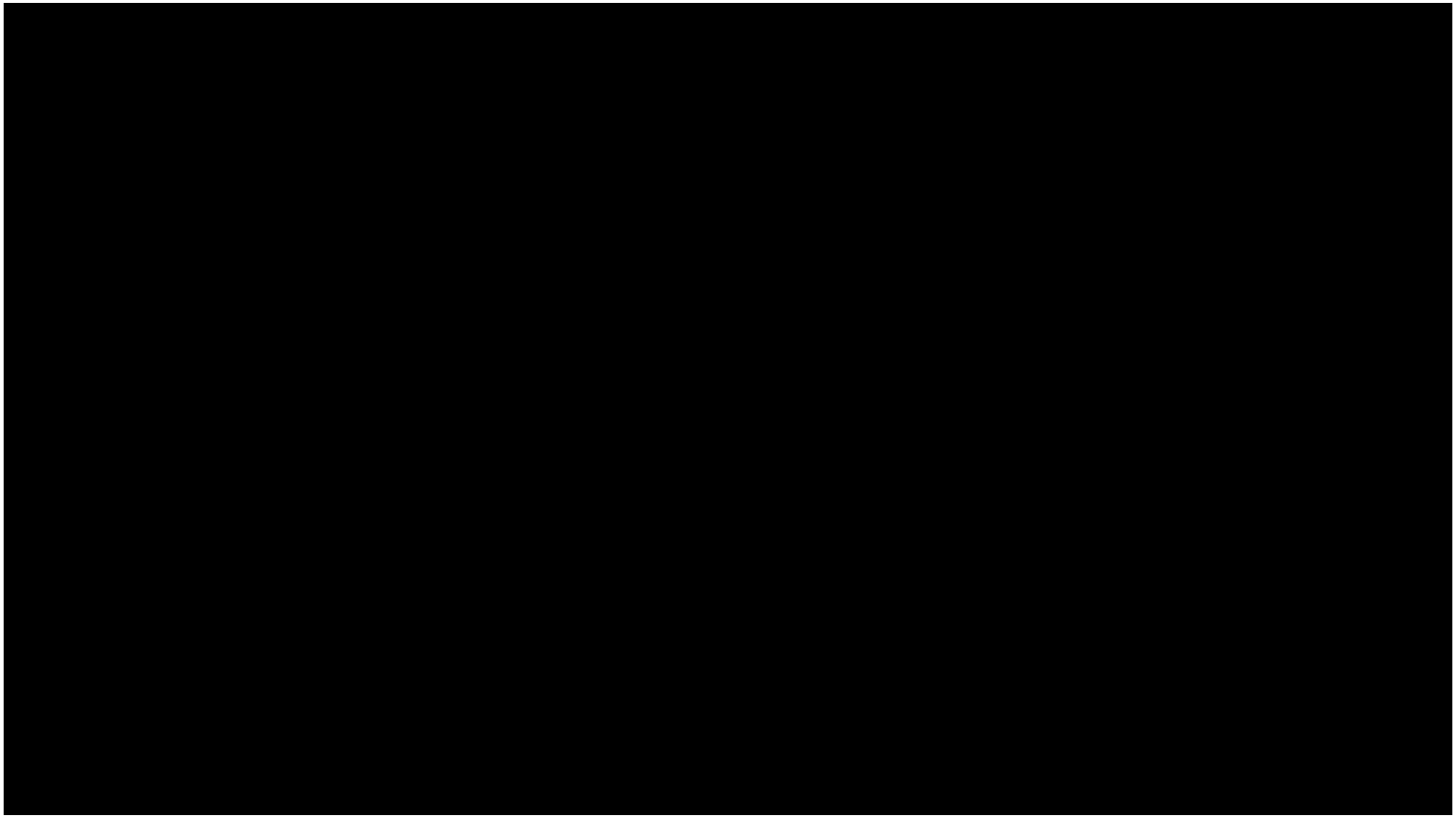
Outline of Lecture 4

- Background Knowledge
- Purpose of Path Planning
- Input to Path Planning
- Output from Path Planning
- Process of Autonomous Path Planning
  - Case of Arm Manipulators
  - Case of Car-like Mobile Bases

Outline of Lecture 4

- Background Knowledge
- Purpose of Path Planning
- Input to Path Planning
- Output from Path Planning
- Process of Autonomous Path Planning
  - Case of Arm Manipulators
  - Case of Car-like Mobile Bases

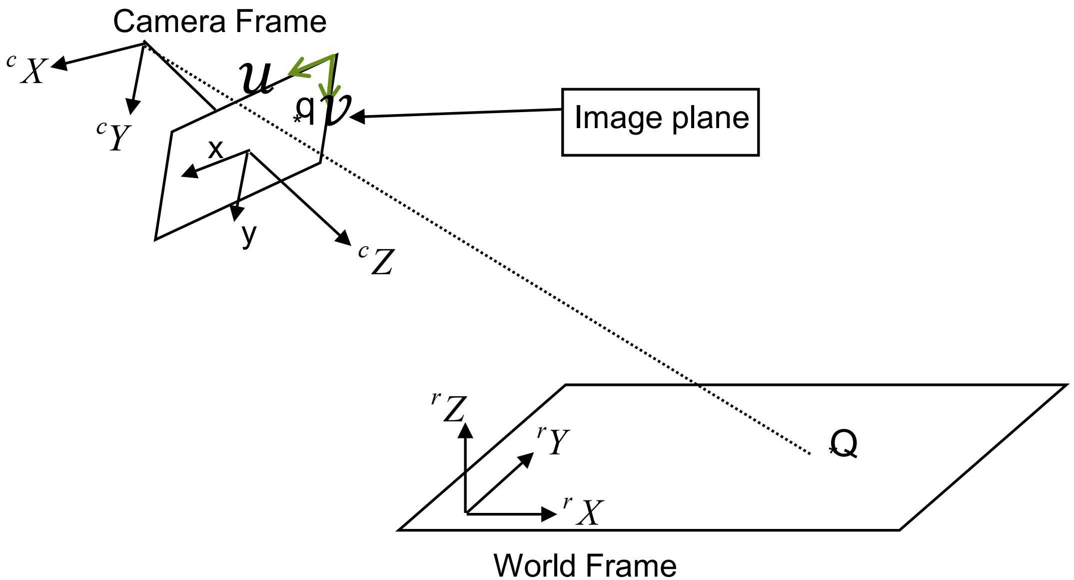
What is the scenario of coordinate transformations?



Basics of Homogeneous Transformation in Vision

$$H_{camera} = \begin{bmatrix} R_{camera} & T_{camera} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix}$$

$$H_{world} = \begin{bmatrix} R_{camera}^{-1} & -R_{camera}^{-1} \times T_{camera} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix}$$

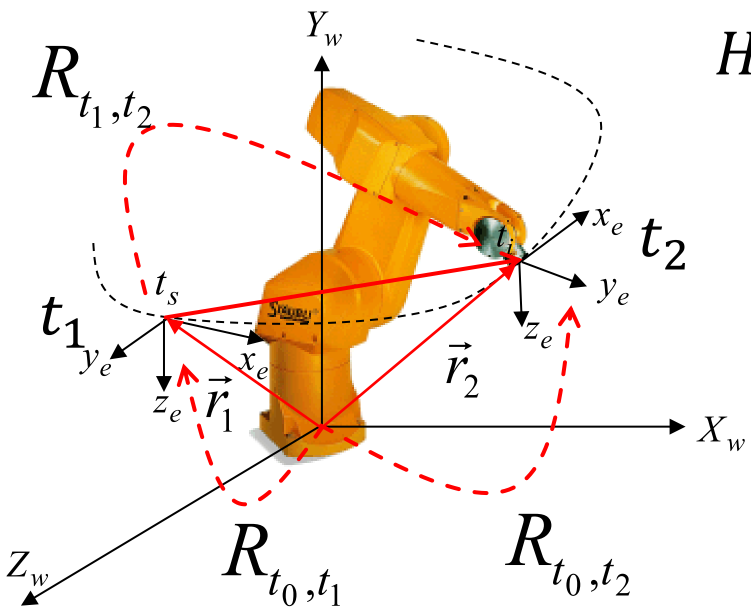


$$H_{camera} \times H_{world} = I_{4\times4}$$

$$H_{camera} = H_{world}^{-1}$$

Basics of Homogeneous Transformation in Robotics

$$H_{tooltip} = \begin{bmatrix} R_{tooltip} & T_{tooltip} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix}$$



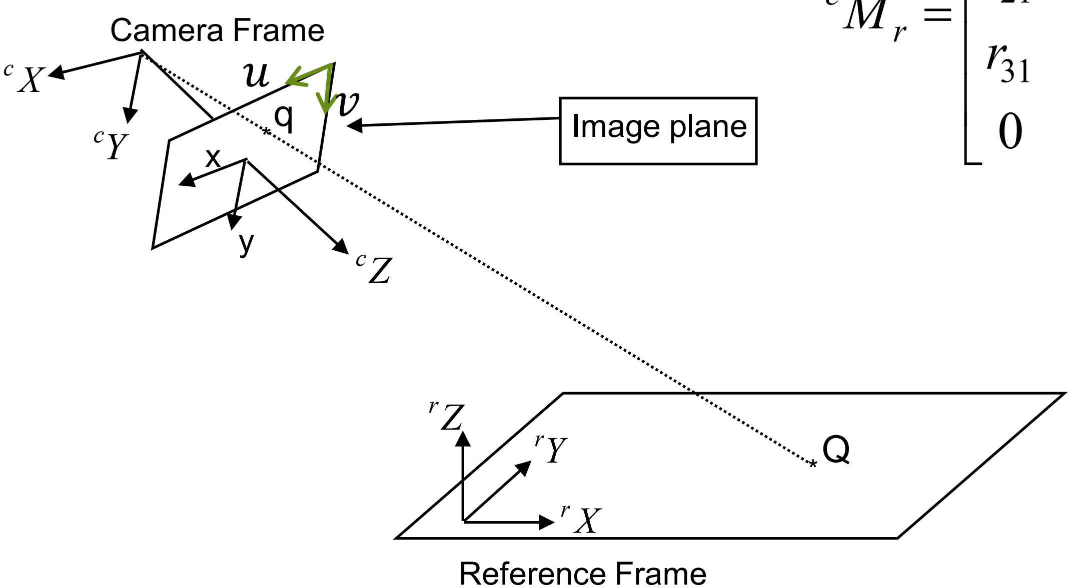
$$H_{world} = \begin{bmatrix} R_{tooltip}^{-1} & -R_{tooltip}^{-1} \times T_{tooltip} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix}$$

$$H_{tooltip} \times H_{world} = I_{4\times4}$$

$$H_{tooltip} = H_{world}^{-1}$$

Transformation of Coordinates in 3D Space

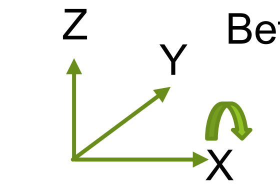
- The coordinates in the reference frame can be transformed into the coordinates in the camera frame.



$$
{}^{c}M_{r} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

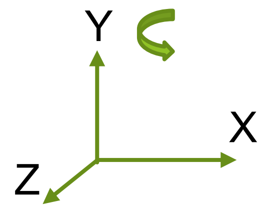
$$
\begin{bmatrix} {}^{c}X \\ {}^{c}Y \\ {}^{c}Z \\ 1 \end{bmatrix} = {}^{c}M_{r} \bullet \begin{bmatrix} {}^{r}X \\ {}^{r}Y \\ {}^{r}Z \\ 1 \end{bmatrix}
$$

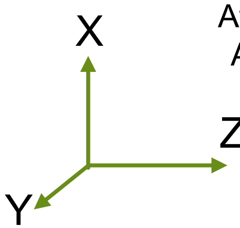
Practices with Rotational Transformation



Before Rotation

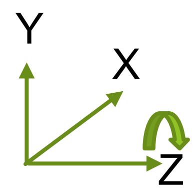
After Rotation About X Axis



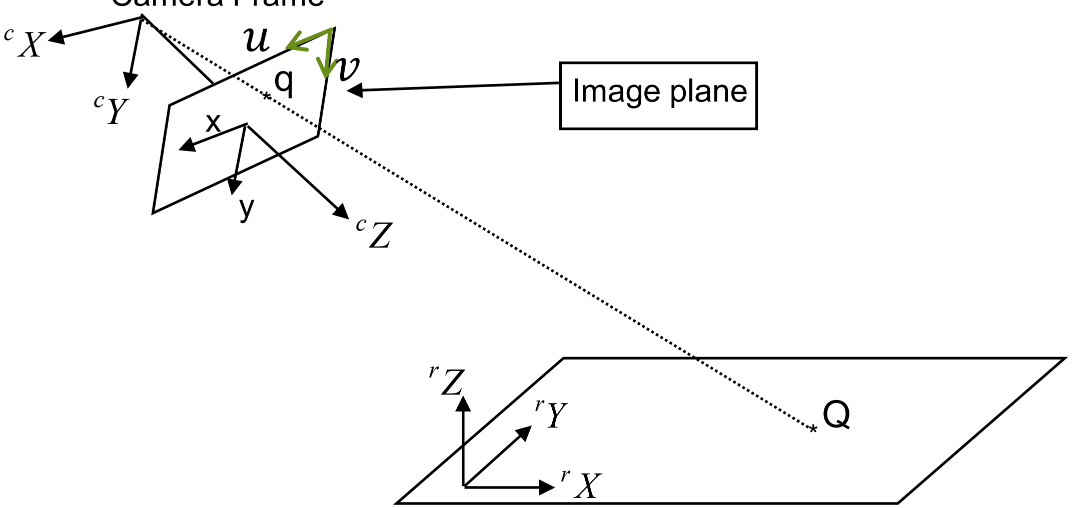


After Rotation About Z Axis

After Rotation About Y Axis



Camera Frame



Reference Frame

```
ax = 90*pi/180;
rx = [1    0          0 ;
      0  cos(ax)   -sin(ax);
      0  sin(ax)    cos(ax)];

ay = 90*pi/180;
ry = [cos(ay)    0     sin(ay);
        0        1     0 ;
     -sin(ay)    0     cos(ay)];

az = 90*pi/180;
rz = [cos(az)   -sin(az)   0;
      sin(az)    cos(az)   0;
      0          0         1];

r = rx*ry*rz
```

Command Window

New to MATLAB? See resources for Getting Started.

```
r =

    0.0000   -0.0000    1.0000
    0.0000   -1.0000   -0.0000
    1.0000    0.0000    0.0000
```

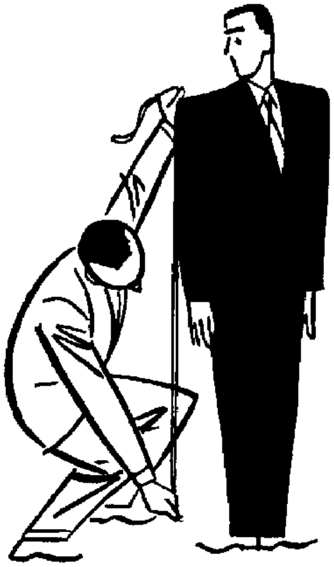
Outline of Lecture 4

- Background Knowledge
- Purpose of Path Planning
- Input to Path Planning
- Output from Path Planning
- Process of Autonomous Path Planning
  - Case of Arm Manipulators
  - Case of Car-like Mobile Bases

Definition of Path

- Path refers to a series of linear positions (i.e. positions) and angular positions (i.e. orientations) without time constraints.
- "Position + Orientation" is called Pose.

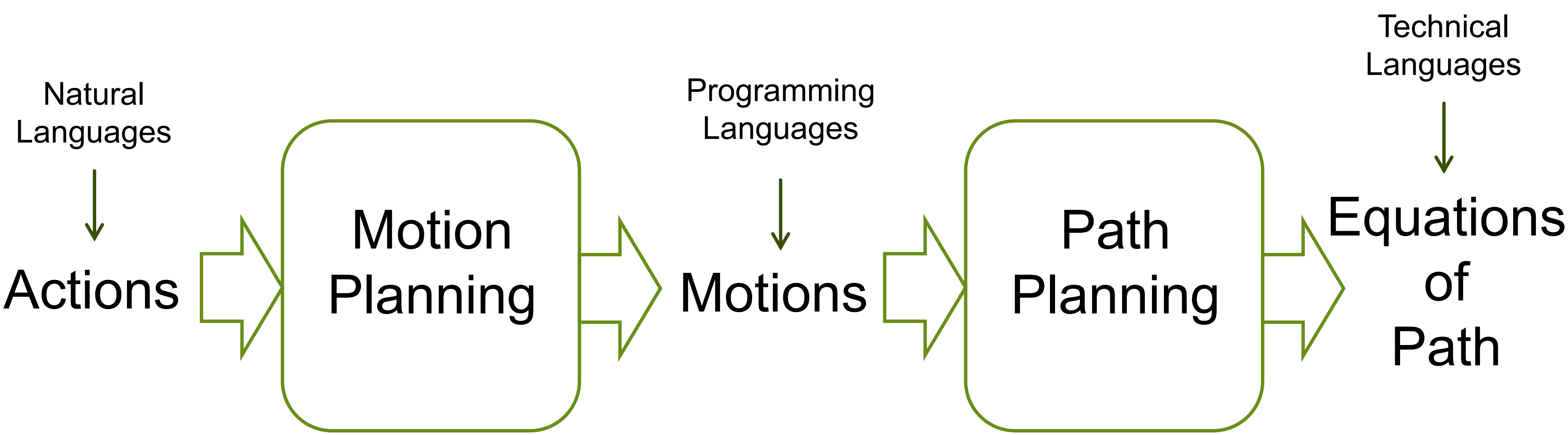
Examples of Paths and Discussions



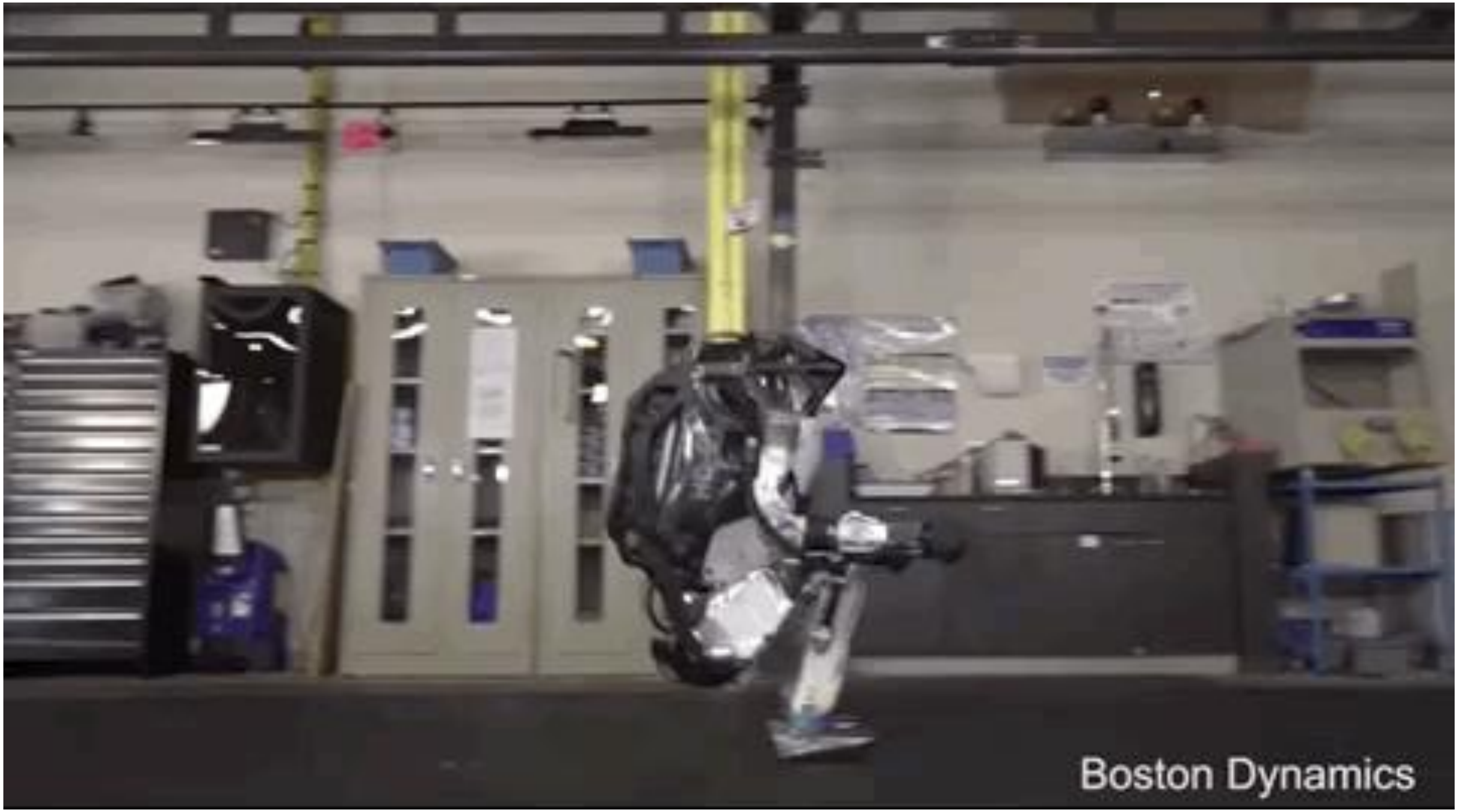
- Discussions:
  - Are the motions performed by human beings fuzzy or crisp?
  - How do we perform crisp motions?
  - Could robots outperform human beings in terms of motion execution?

Definition of Path Planning

- Path planning is a process which takes the description (commands or library functions) of a given motion as input and produces a set of equations of path as output.

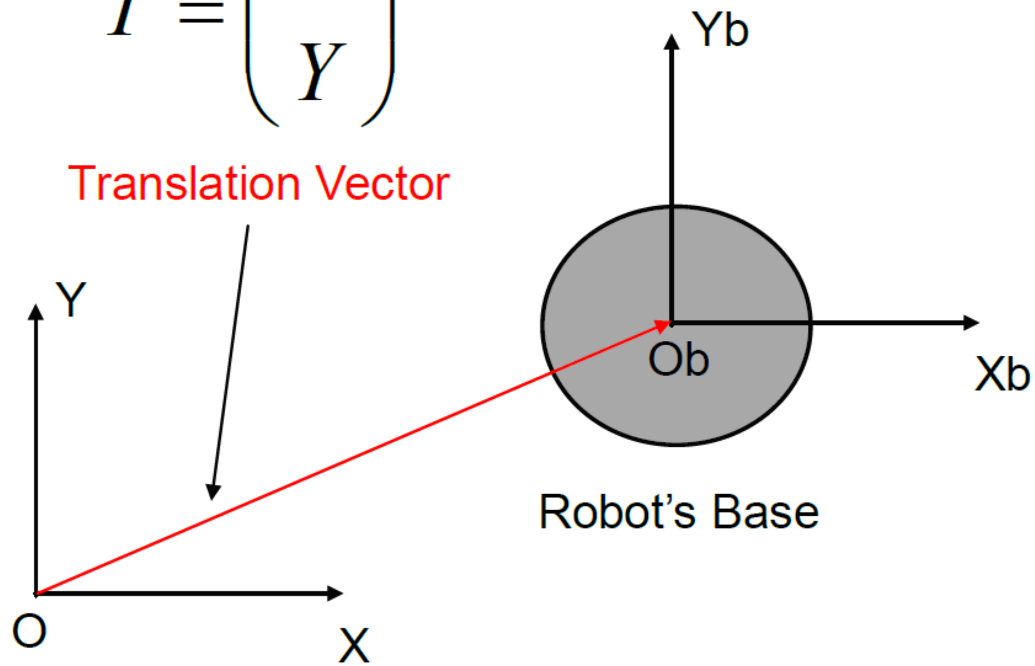


Example of Motion Performed by Robot



How to represent Linear Positions?

$$T = \begin{pmatrix} X \\ Y \end{pmatrix}$$



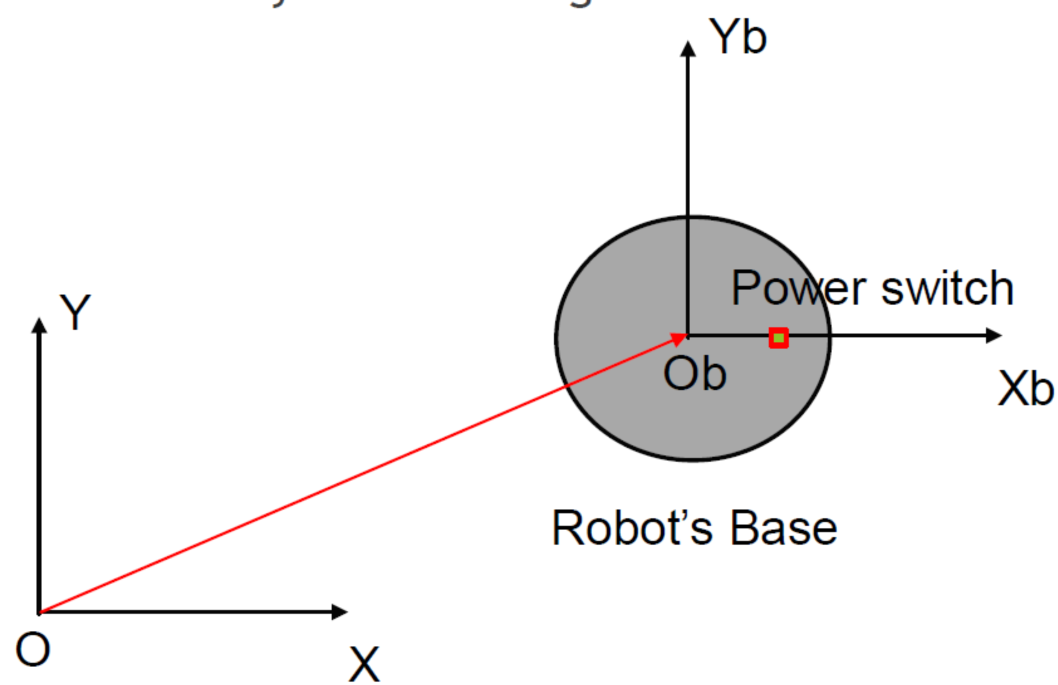
Example

- The origin of a mobile robot's base is at the coordinates (6.0, 4.0) (m) with respect to the reference coordinate system on the ground. A power switch of the mobile robot is at the coordinates (10.0, 0.0) (cm) with respect of the robot base's coordinate system. What are the coordinates of the power switch with respect to the reference coordinate system on the ground?

- Answer:

$$ {}^{w}P = {}^{b}P + T $$

$$ {}^{w}P = \begin{pmatrix} 0.1 \\ 0.0 \end{pmatrix} + \begin{pmatrix} 6.0 \\ 4.0 \end{pmatrix} = \begin{pmatrix} 6.1 \\ 4.0 \end{pmatrix} \text{(m)} $$

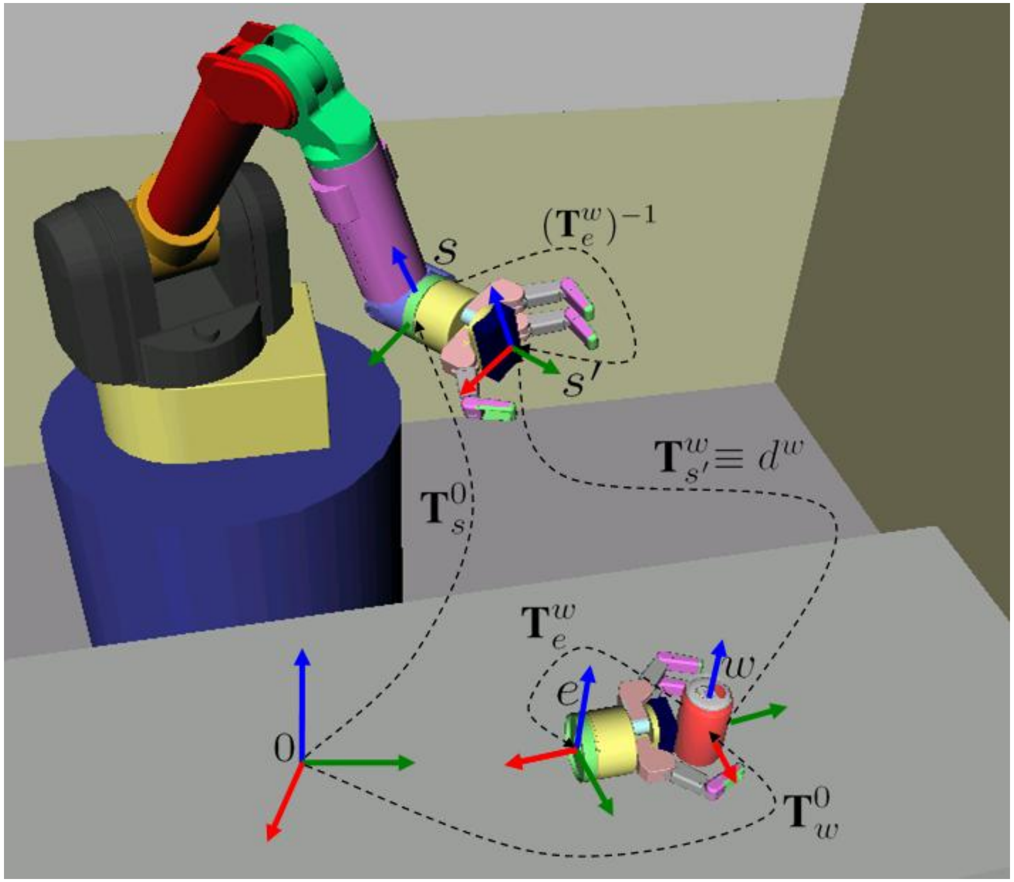


How to represent angular positions?

Answer:

- Use of unit vectors of coordinate system assigned to a rigid body or a robot's body.

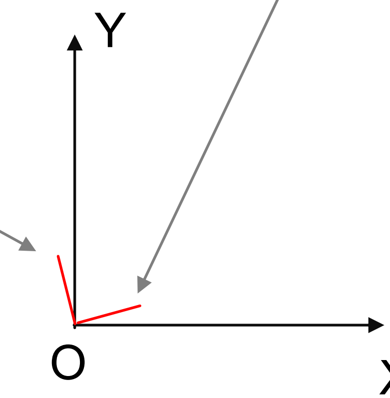
- Use of rotated angles about a reference axis.

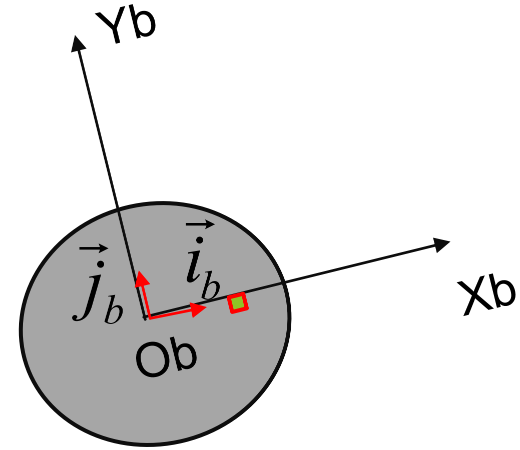


Representation of Angular Positions in 2D Space

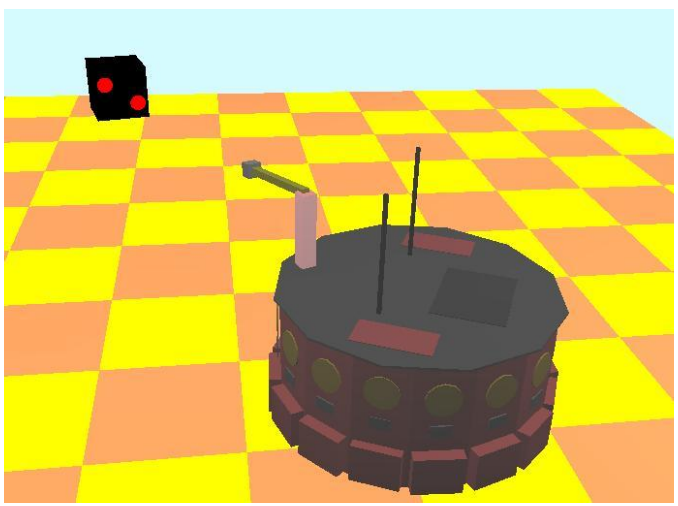
$$\vec{j}_b = \begin{pmatrix} -\sin(\theta) \\ \cos(\theta) \end{pmatrix}$$

$$\vec{i}_b = \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix}$$





Robot's Base



$$R_{t_0,t} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$$

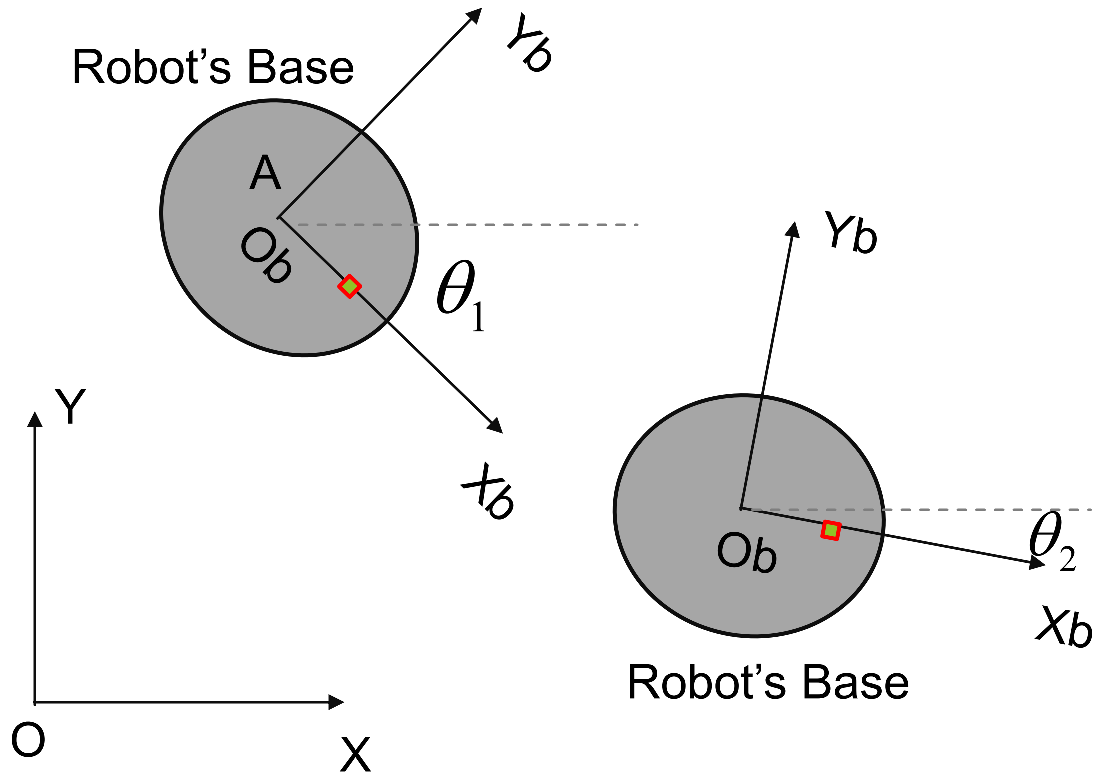
This is the orientation at time t, as the
result of the change of orientation from time t0 to time t.

Exercise

- What are the orientations of the mobile base at times t1 and t2?
- Answer:

$$\theta(t_1) = \theta_1$$

$$\theta(t_2) = \theta_2$$



Exercise

- If we put the unit vectors of axes together to form a matrix. What is the physical meanings of such a matrix?

$$\vec{i}_b = \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix}$$

$$\vec{j}_b = \begin{pmatrix} -\sin(\theta) \\ \cos(\theta) \end{pmatrix}$$

$$R = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$$

Answer: Rotation Matrix.

Exercise

- A robot’s base has rotated 70.0 degrees about Z axis. What is the corresponding rotation matrix?

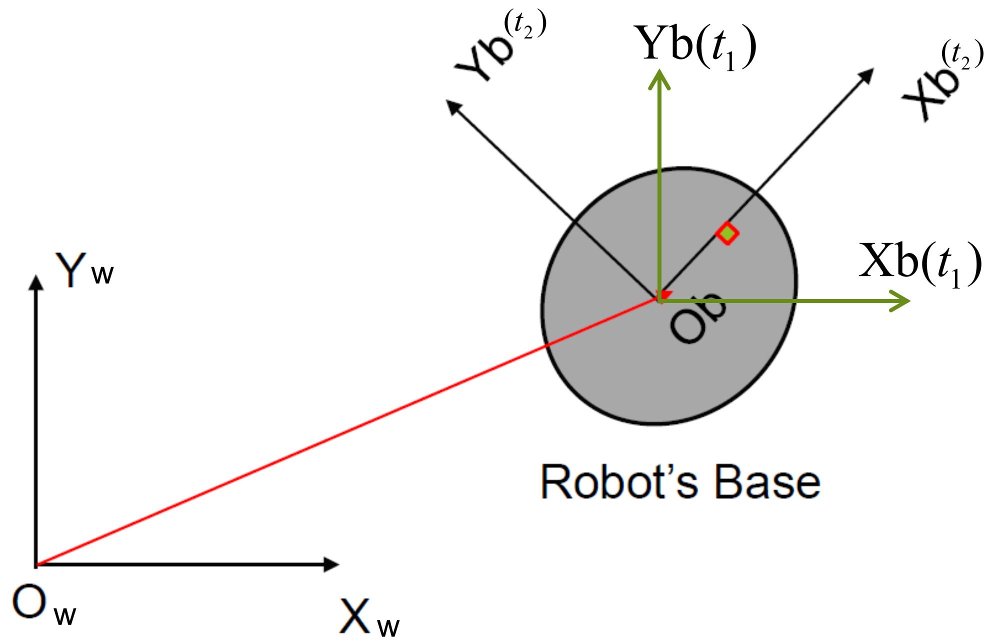
- Answer:

$$R = \begin{bmatrix} \cos(70^0) & -\sin(70^0) \\ \sin(70^0) & \cos(70^0) \end{bmatrix}$$

⇩

$$R_{t_1,t_2}$$

This is the orientation at time t2, as the
result of the change of orientation from time t1 to time t2.



$$P_{t_1} = R_{t_1,t_2} \bullet P_{t_2}$$

Exercise

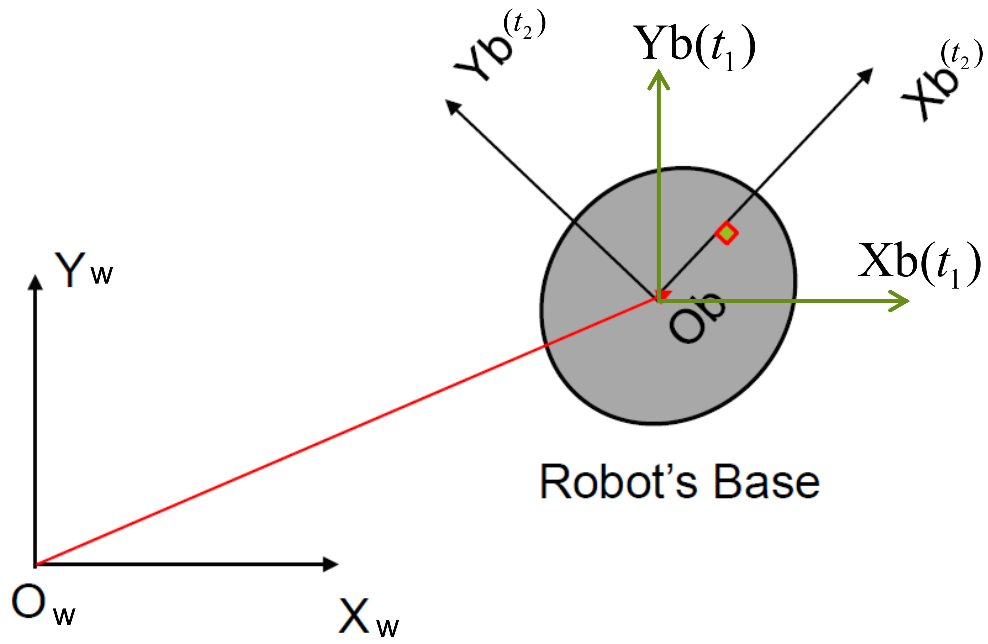
- A power switch is located at the position of (10.0, 0.0) (cm) with respect to the coordinate system of a mobile robot. If the mobile robot makes a rotation of 70.0 degrees about the Z axis, what will be the coordinates of the power switch with respect to the robot's initial coordinate system (that is the one before motion)?

- Answer:

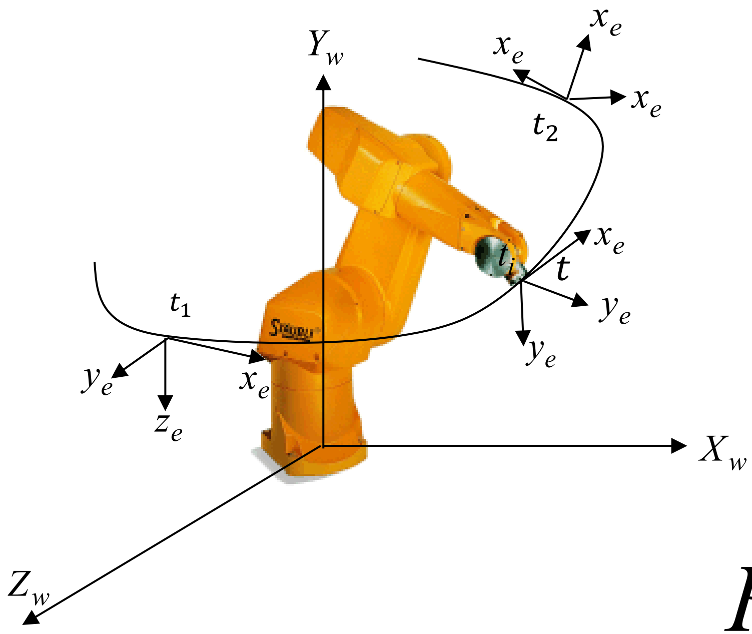
$$P_{t1} = R_{t_1,t_2} \times P_{t_2}$$

$$P_{t1} = \begin{bmatrix} \cos(70^0) & -\sin(70^0) \\ \sin(70^0) & \cos(70^0) \end{bmatrix} \times \begin{bmatrix} 10.0 \\ 0.0 \end{bmatrix}$$

$$P_{t_1} = \begin{bmatrix} 10.0\cos(70^0) \\ 10.0\sin(70^0) \end{bmatrix}$$



Representation of Angular Positions in 3D Space



X Axis's Unit Vector | Y Axis's Unit Vector | Z Axis's Unit Vector

$$R_{t_0,t} = \begin{bmatrix} x_i(t) & x_j(t) & x_k(t) \\ y_i(t) & y_j(t) & y_k(t) \\ z_i(t) & z_j(t) & z_k(t) \end{bmatrix}$$

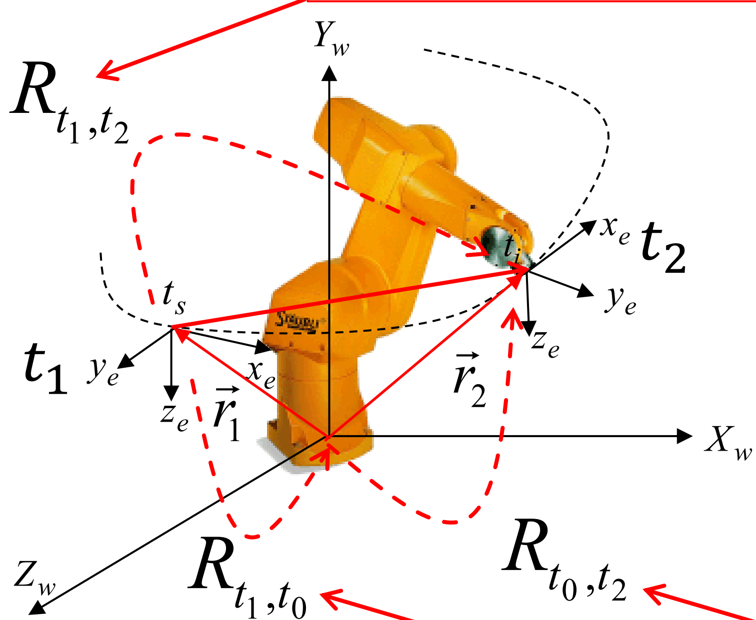
This is the orientation at time t, as the result of the change of orientation from time t0 to time t.

Exercise

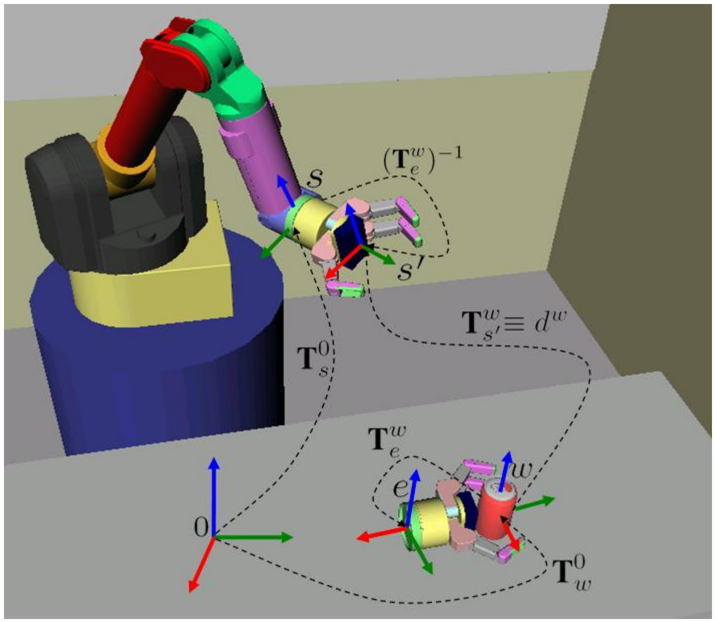
- Describe the orientation of the tool's coordinate system at t2 with respect to the tool's coordinate system at t1?

- Answer:

This is the orientation at time t2, as the result of the change of orientation from time t1 to time t2.

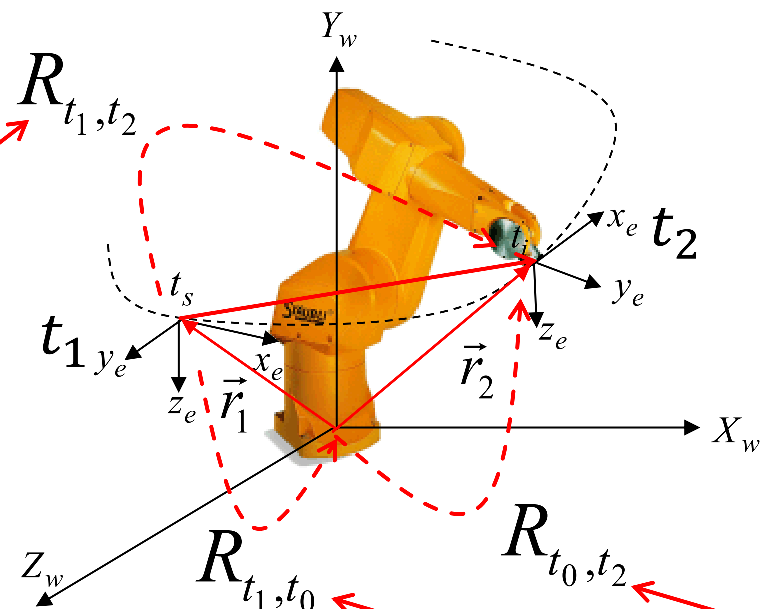


Another Notation



This is the orientation at time t0, as the result of the change of orientation from time t1 back to time t0.

This is the orientation at time t2, as the result of the change of orientation from time t0 to time t2.



This is the orientation at time t2, as the result of the change of orientation from time t1 to time t2.

This is the orientation at time t0, as the result of the change of orientation from time t1 back to time t0.

This is the orientation at time t2, as the result of the change of orientation from time t0 to time t2.

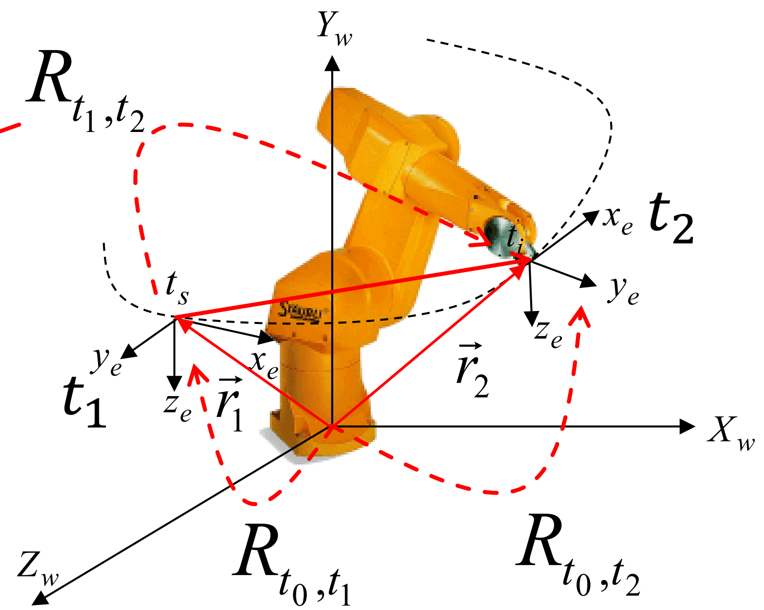
$$R_{t_1,t_2} = R_{t_1,t_0} \bullet R_{t_0,t_2} \quad \Leftrightarrow \quad R_{t_1,t_2} = (R_{t_0,t_1})^{-1} \bullet R_{t_0,t_2}$$

Exercise

- What is the equivalent axis of rotation and the equivalent angle of rotation, if we are given a rotation matrix?

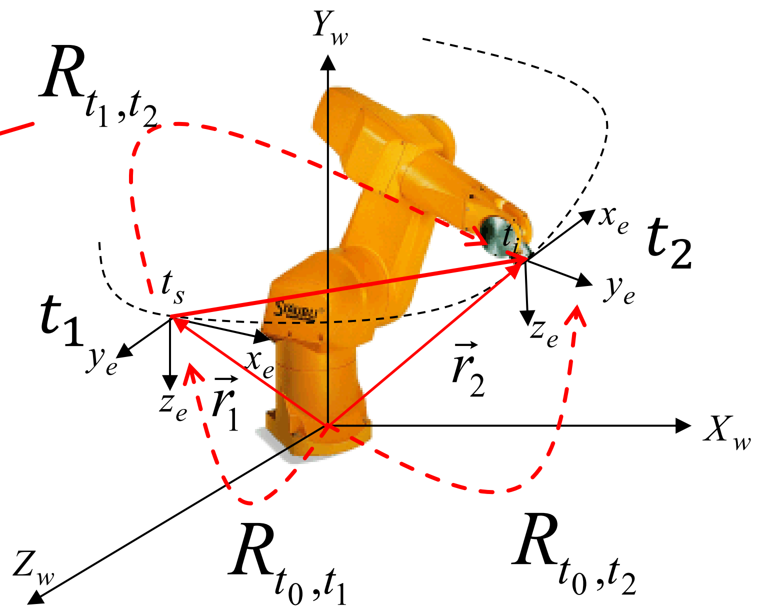
$$R_{t_1,t_2} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$$

This is the orientation at time t2, as the result of the change of orientation from time t1 to time t2.



Answer

$$R_{t_1,t_2} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$$



This is the orientation at time t2, as the result of the change of orientation from time t1 to time t2.

Equivalent Axis of Rotation :

$$\vec{r}\Big|_{t_1,t_2} = \begin{bmatrix} c_{32} - c_{23} \\ c_{13} - c_{31} \\ c_{21} - c_{12} \end{bmatrix}$$

Equivalent Angle of Rotation :

$$\theta_{t_1,t_2} = \arccos\left(\frac{c_{11} + c_{22} + c_{33} - 1}{2}\right)$$

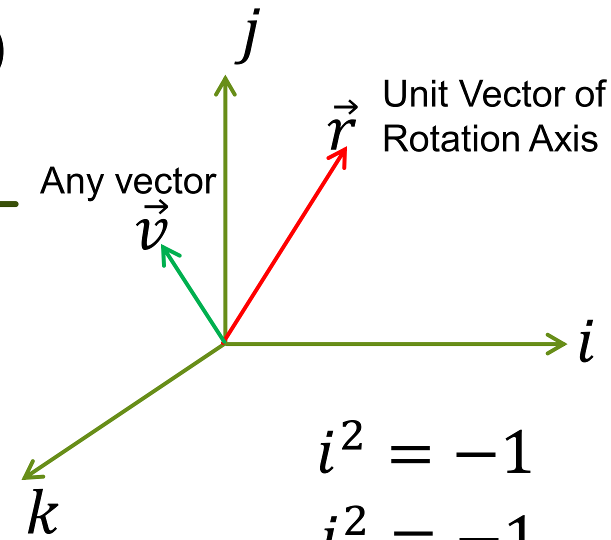
Simple and Useful for Adding Time Constraint

Applications Using Equivalent Axis and Angle of Rotation: Motion Planning / Animation

- Euler Equation: $e^{j\omega} = \cos(\omega) + j\,sin(\omega)$

  $j = \sqrt{-1}$

Generalized Euler Equation: Equation of Unit Quaternion



- Unit Vector of Rotation Axis:

$$\vec{r} = r_x i + r_y j + r_z k$$

$$i^2 = -1$$
$$j^2 = -1$$
$$\mathrm{k}^2 = -1$$
$$\mathrm{ijk} = -1$$

- Angle of Rotation: θ

- Unit Quaternion: $q = e^{\theta\vec{r}} = \cos(\theta) + \sin(\theta)\vec{r}$

- Equation of Rotation: $\vec{v}_{after} = q\vec{v}_{before} = (\cos(\theta) + \sin(\theta)\vec{r}) \cdot \vec{v}_{before}$

More About Quaternion

- Non-Unit Quaternion: $q = (a, \vec{v}) = a + b\vec{i} + c\vec{j} + d\vec{k}$
- Unit Quaternion: $q = (\cos(\theta), \sin(\theta)\vec{r}) = \cos(\theta) + \sin(\theta)\vec{r}$

  Unit vector of axis of rotation
- Sum of Two Quaternions: $q_1 = (a_1, \vec{v}_1) \quad q_2 = (a_2, \vec{v}_2)$

$$q_1 + q_2 = (a_1 + a_2, \vec{v}_1 + \vec{v}_2)$$

- Multiplication of Two Quaternions: $q_1 = (a_1, \vec{v}_1) \quad q_2 = (a_2, \vec{v}_2)$

$$q_1 \cdot q_2 = (a_1 a_2 - \vec{v}_1 \cdot \vec{v}_2, \quad a_1\vec{v}_2 + a_2\vec{v}_1 + \vec{v}_1 \times \vec{v}_2)$$

- Equation of Rotating a Vector: $v = (0, \vec{v}) \quad q = (\cos(\theta), \sin(\theta)\vec{r})$

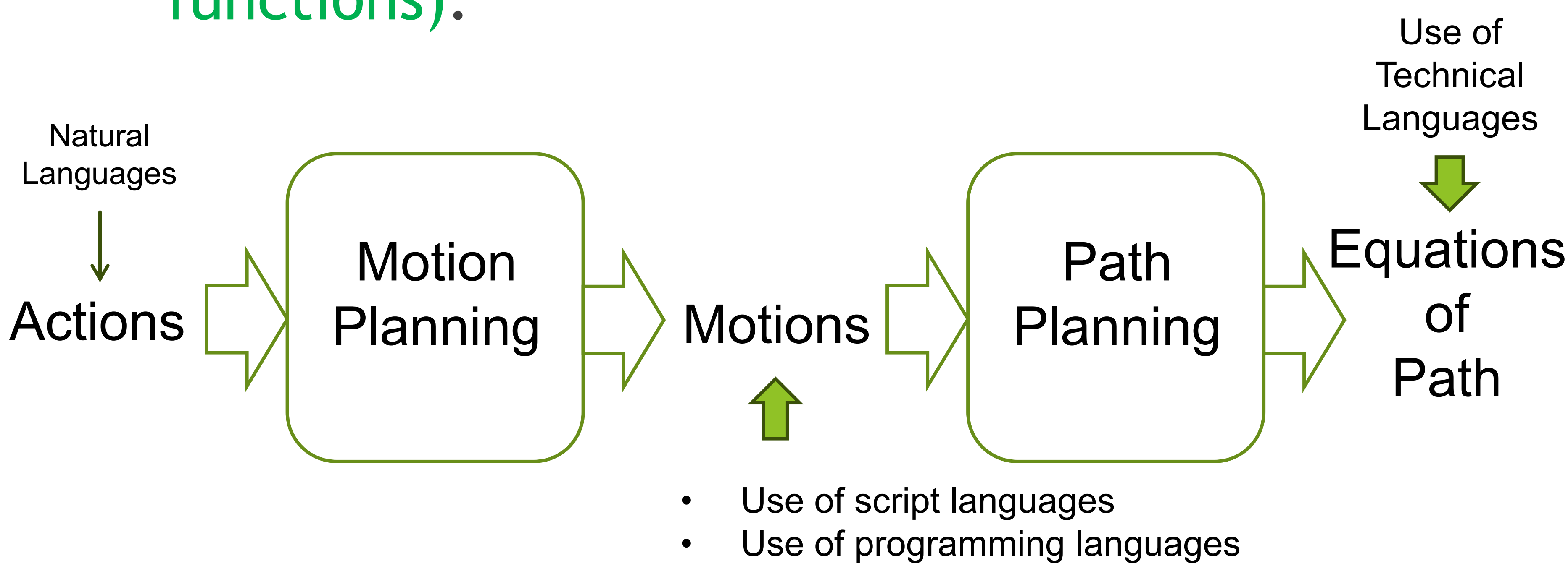
$$v_{new} = q \cdot v$$

Outline of Lecture 4

- Background Knowledge
- Purpose of Path Planning
- Input to Path Planning
- Output from Path Planning
- Process of Autonomous Path Planning
  - Case of Arm Manipulators
  - Case of Car-like Mobile Bases

Input to Path Planning

- The input to path planning is the description of a given motion in the form of script languages (commands) or programming languages (library functions).



Description of Motion

Use of Script Language

- Move To
- Move With
- Speed To
- Accelerate To
- Open/Close Hand

Use of Programming Language

- MoveTo(arguments)
- MoveWith(arguments)
- SpeedTo(arguments)
- AccelerateTo(arguments)
- HandOpenClose()

Example of Motion Description with the Use of Script Language

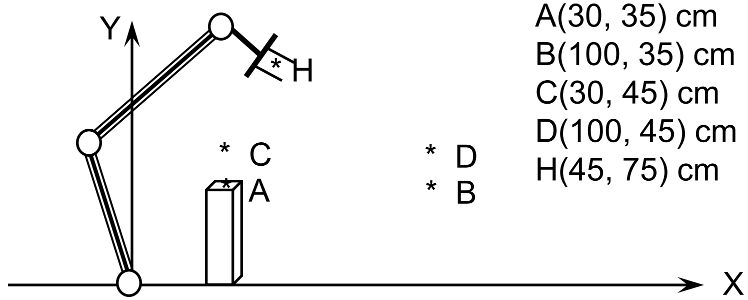
Arm Manipulators

- "Move To" or MoveTo(): it refers to motion which starts at the current pose and reaches the final pose in consideration of space and time constraints.
- Move With or MoveWith(): it refers to motion which follows equations determined by manual or autonomous planning.

Mobile Bases

- "Forward To", "Reverse To", ForwardTo(), or ReverseTo(): it refers to motion which starts at the current pose and reaches the final pose in consideration of space and time constraints.
- "Forward With", "Reverse With", ForwardWith(), or ReverseWith(): it refers to motion which follows equations determined by manual or autonomous planning.

Comparison between the Use of Script Language and the Use of Programming Language



A(30, 35) cm
B(100, 35) cm
C(30, 45) cm
D(100, 45) cm
H(45, 75) cm

Motion Description with Commands:

Define Point H
Define Point A
Define Point B
Define Point C
Define Point D
Open Clipper
Move To A
Close Clipper
Move To C
Move To D
Move To B
Open Clipper
Move To D
….

Motion Description with Library Functions:

```
setp H = (x1, y1)
setp A = (x2, y2)
setp B = (x3, y3)
setp C = (x4, y4)
setp D = (x5, y5)
open
moveto(A)
close
moveto(C)
moveto(D)
moveto(B)
open
moveto(D)
….
```

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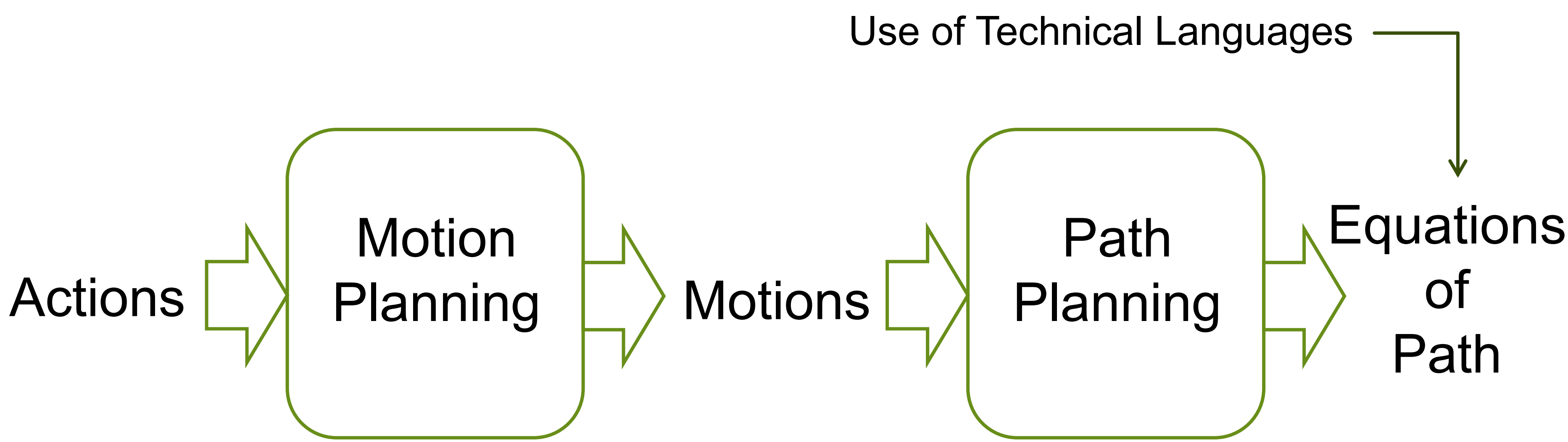
Output from Path Planning

- The output of path planning is a set of equations of motion without time constraints, which will be the input to trajectory planning.



What are typical types of equations of path?

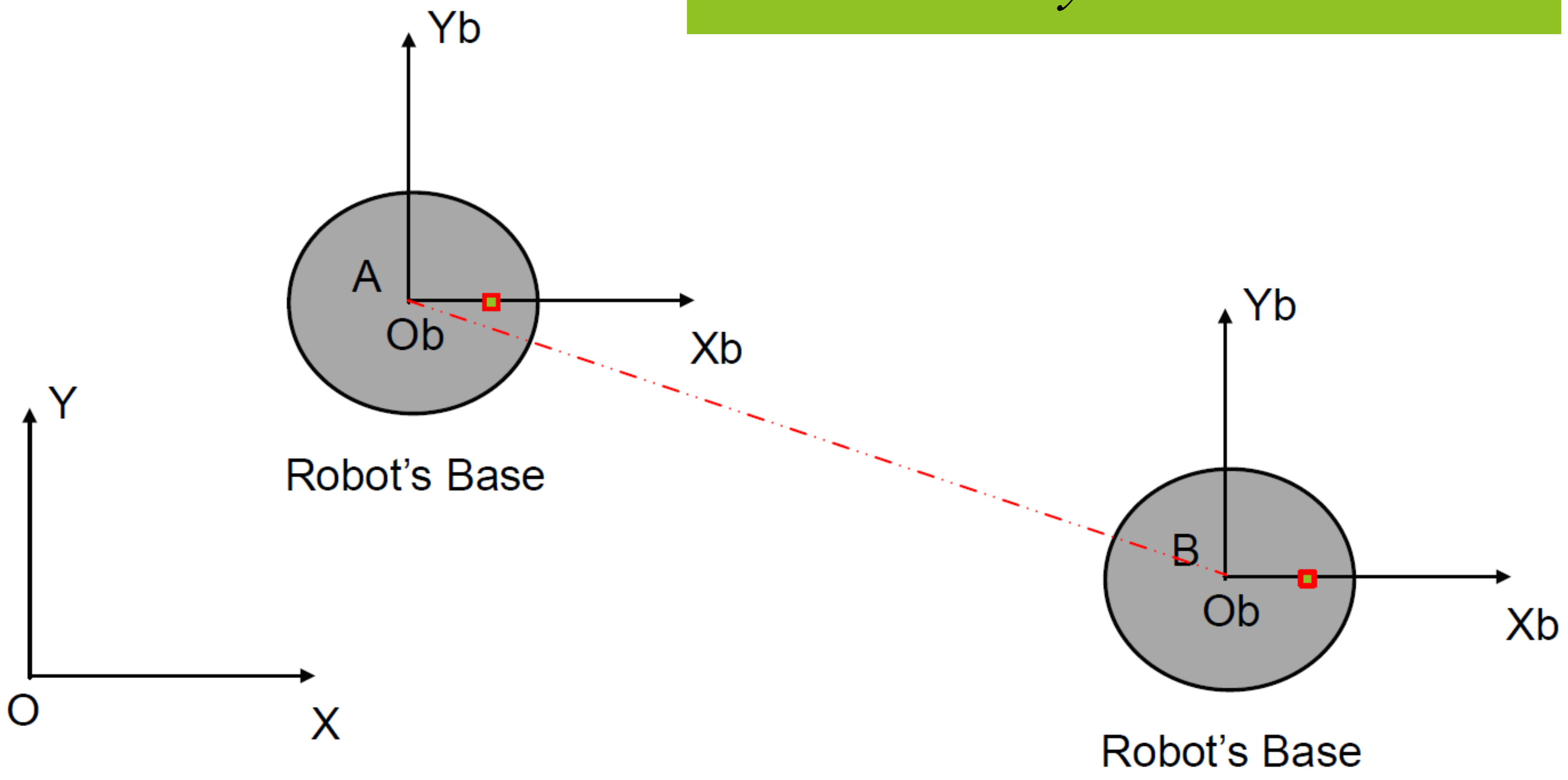
- Polynomial equations of degree 1
- Polynomial equations of degree 2
- Polynomial equations of degree 3
- … etc.

$$y = ax^3 + bx^2 + cx + d$$

$$z = ex^3 + fx^2 + gx + h$$

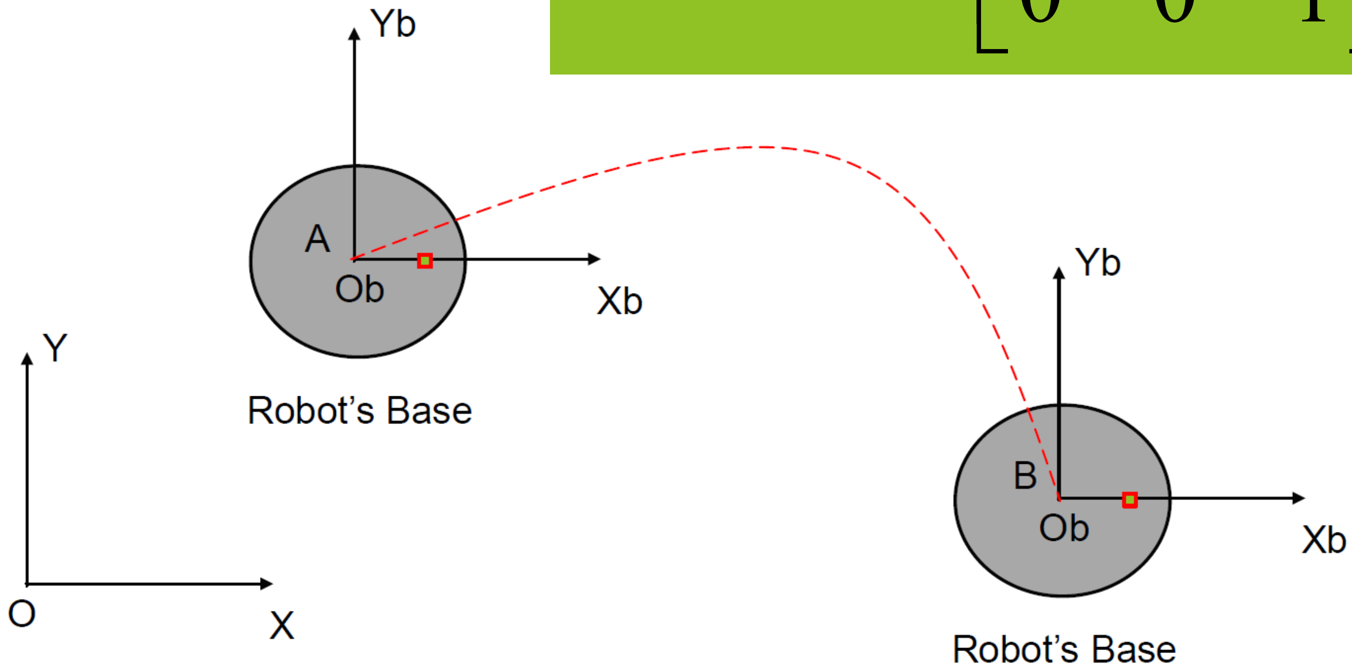
Equations of Path for Mobile Bases (1)

$$a_x X + a_y Y + a_0 = 0$$



Equations of Path for Mobile Bases (2)

$$\begin{bmatrix} x & y & 1 \end{bmatrix} \times \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$



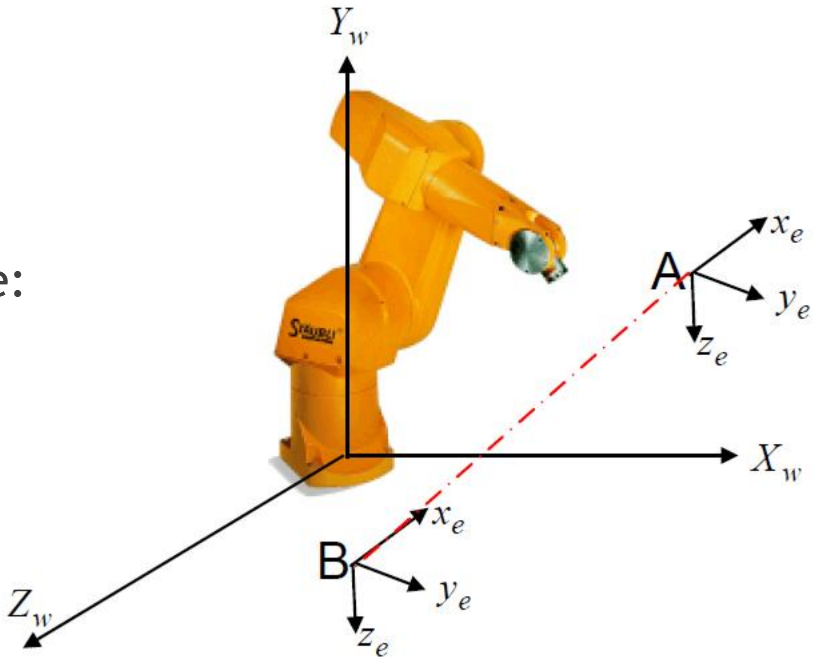
Equations of Path for Arm Manipulator (1)

- Equations of Coordinates in X-Y Plane:

$$Y = a_1 X + a_0$$

- Equations of Coordinates in X-Z Plane:

$$Z = b_1 X + b_0$$



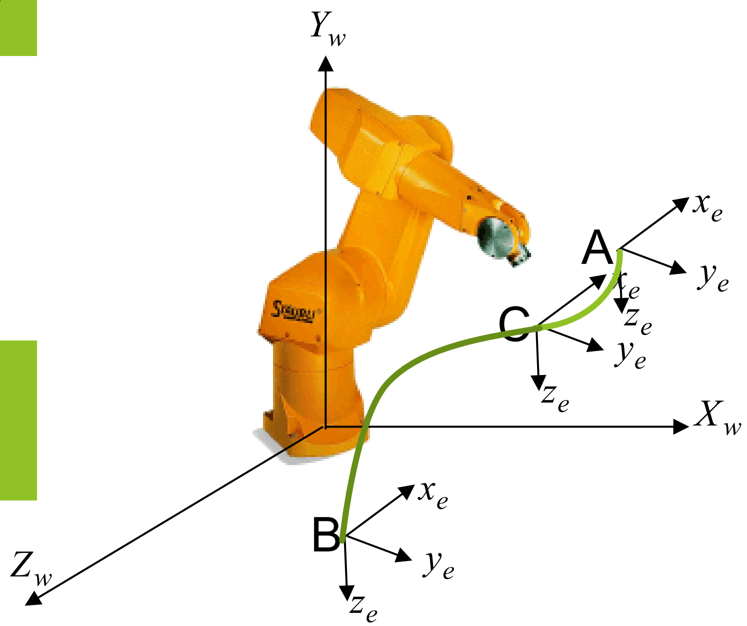
Equations of Path for Arm Manipulator (2)

- Equations of Coordinates in X-Y Plane:

$$Y = a_2X^2 + a_1X + a_0$$

- Equations of Coordinates in X-Z Plane:

$$Z = b_2X^2 + b_1X + b_0$$



Exercise

- A mobile robot moves in a floor and follows a curved path which is described by the following equation. What is the matrix form of this equation?

$$3.2x^2 - 4.5xy - 2.1y^2 + 9.8 = 0$$

- Answer:

$$3.2x^2 + (-2.5xy) + (-2.0xy) + (-2.1)y^2 + 9.8 = 0$$

$$\begin{bmatrix} x & y \end{bmatrix} \times \begin{bmatrix} 3.2 & -2.5 \\ -2.0 & -2.1 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} + 9.8 = 0$$

Exercise

- Express the following matrix equation into the form of polynomials.

$$\begin{bmatrix} x & y \end{bmatrix} \times \begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} + f = 0$$

- Answer:

$$\begin{bmatrix} x & y \end{bmatrix} \times \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix} + f = 0$$

$$ax^2 + bxy + cxy + dy^2 + f = 0$$

Exercise

- Express the following matrix equation into the form of polynomials.

$$\begin{bmatrix} x & y & z \end{bmatrix} \times \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & l \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} + m = 0$$

- Answer:

$$\begin{bmatrix} x & y & z \end{bmatrix} \times \begin{bmatrix} ax + by + cz \\ dx + ey + fz \\ gx + hy + lz \end{bmatrix} + m = 0$$

$$ax^2 + bxy + cxz + dxy + ey^2 + fyz + gxz + hyz + lz^2 + m = 0$$

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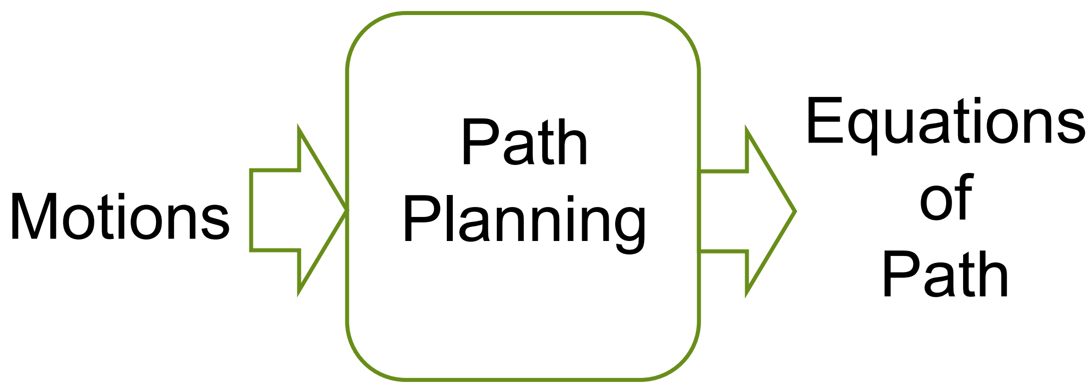
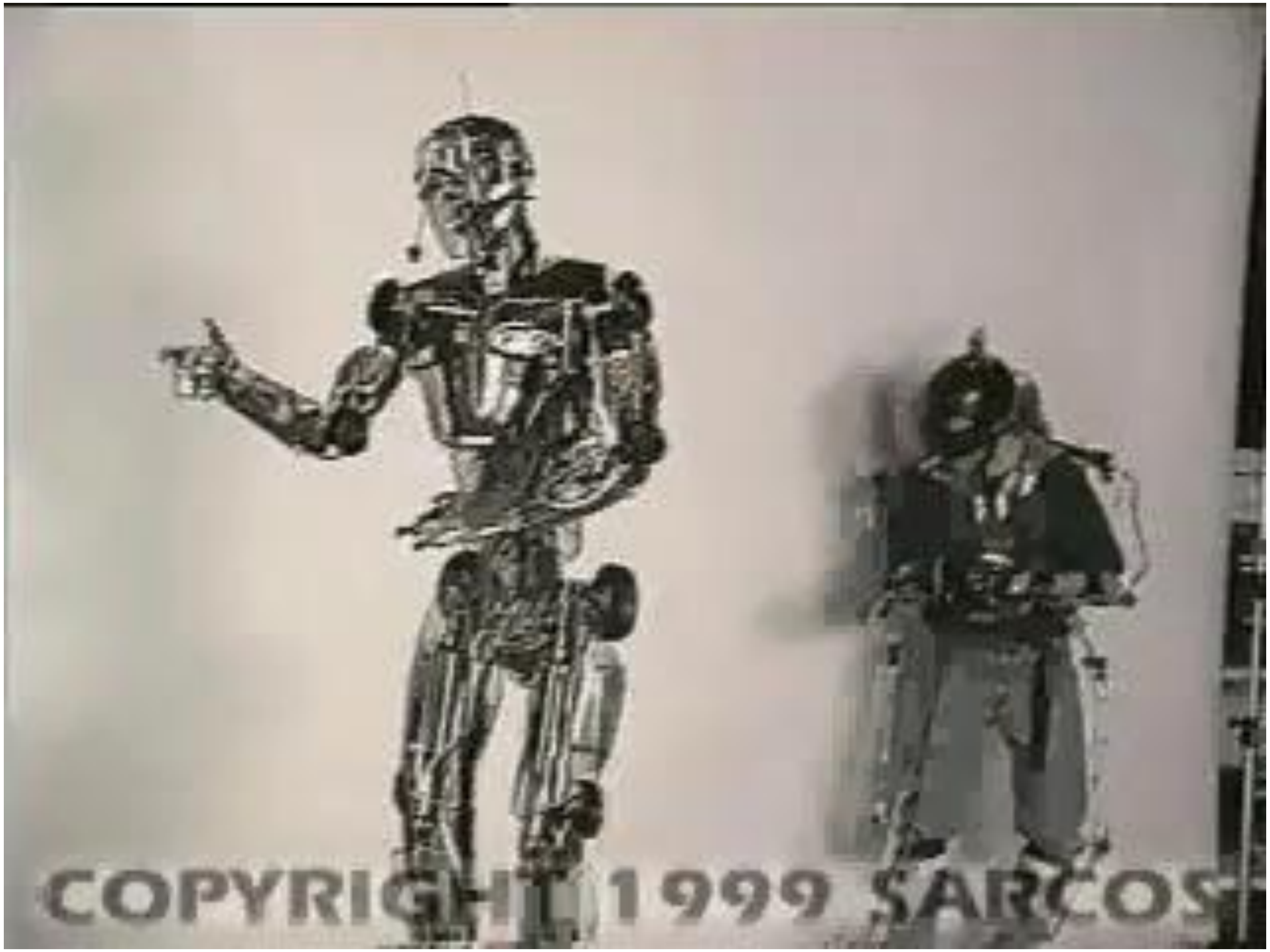
How to do path planning?

- Scenario 1: You are the buyers or users
  - Human-Assisted Path Planning
    - Path Planning by Demonstration or Teaching
    - Path Planning by Intuitive/Professional Programming
- Scenario 2: You are the designers of robots
  - Autonomous Path Planning by Robots

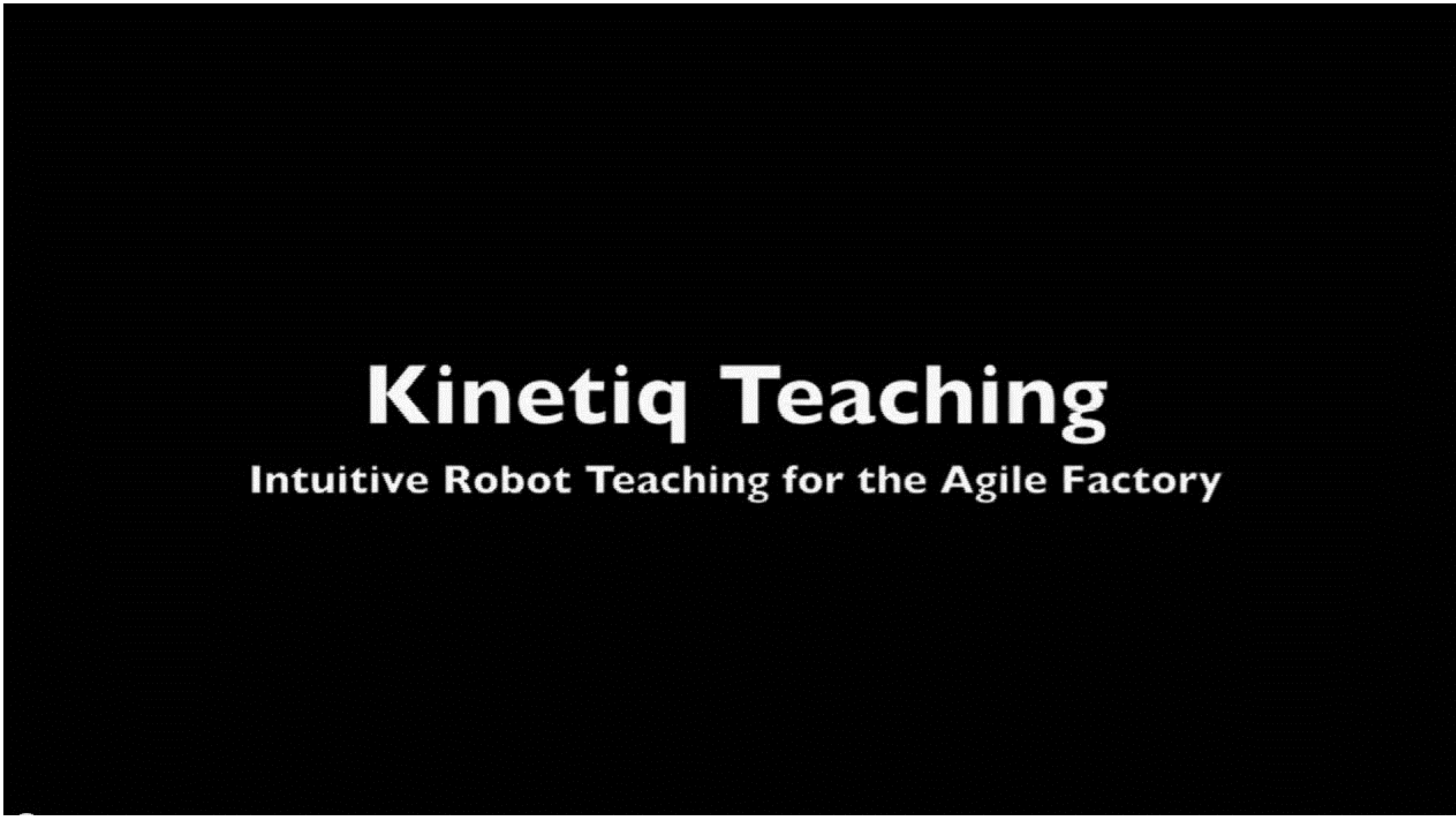
    Level 1 of Intelligence Readiness



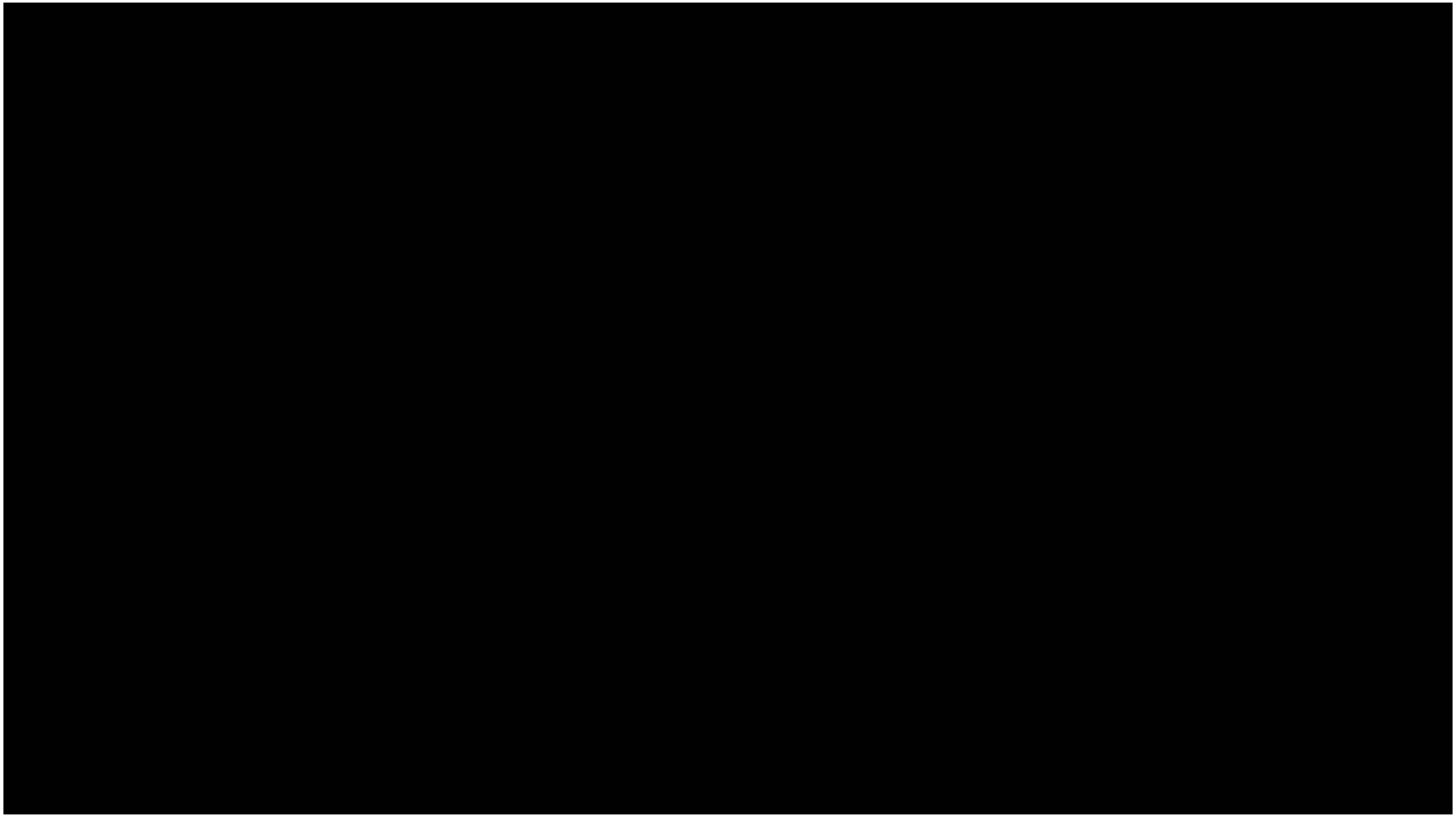
Assistance by Demonstration



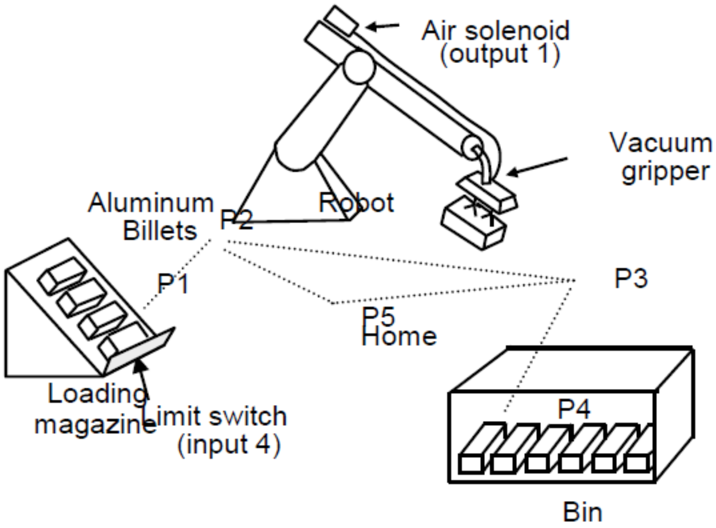
Assistance by Teaching



Assistance by Intuitive Programming

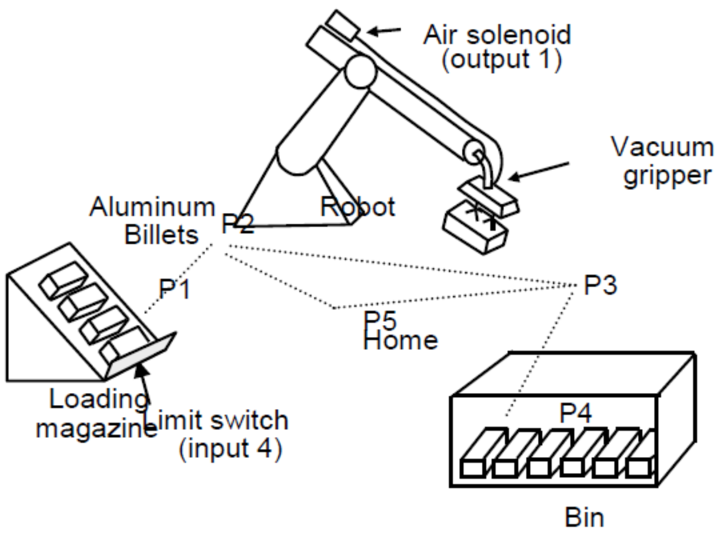


Assistance by Professional Programming



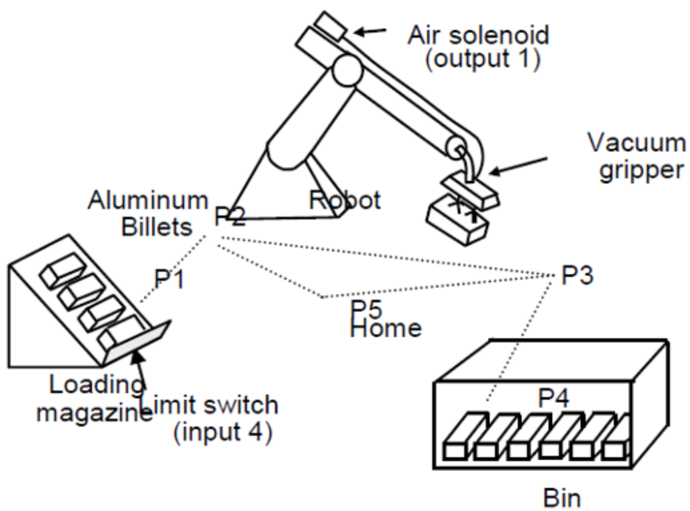
Example:

Aluminum billets are loaded in the loading magazine. The limit switch sensor detects the presence of any billet. The robot is programmed to pick and place the billets into the Bin, one at a time. A vacuum gripper is equipped at the tip of robot arm. The “open” and “close” actions are under control of the robot.




```
Program Part_Handling
VAR
   P1, P2, P3, P4, P5:  POSITION
BEGIN
   Plan_Path ;
   Execute_Path;
END Part_Handling
```

```
Program Plan_Path
BEGIN
         set P1 = (X1, Y1, Z1);
         set P2 = (X2, Y2, Z2);
         set P3 = (X3, Y3, Z3);
         set P4 = (X4, Y4, Z4);
         set P5 = (X5, Y5, Z5);
END Plan_Path ;
```

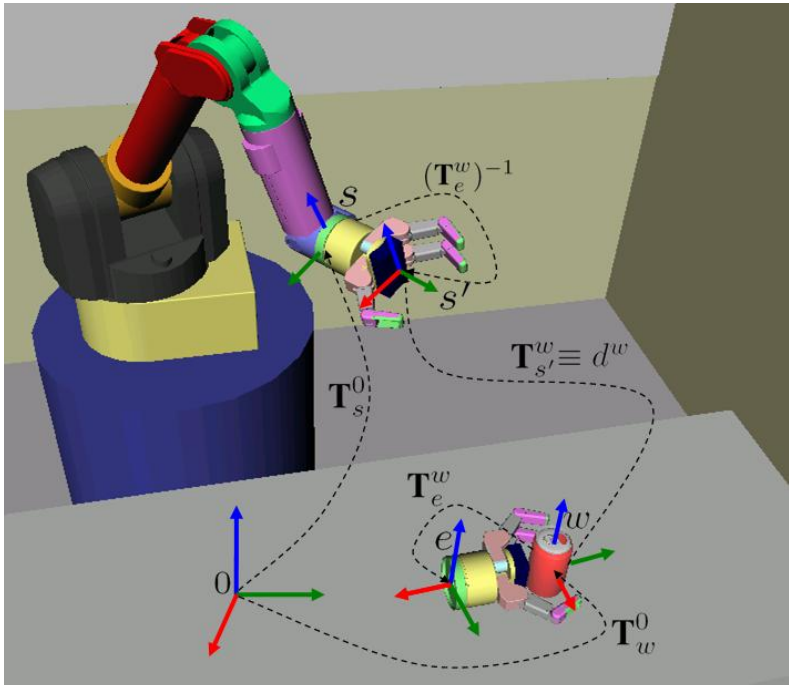



```
Program Execute_Path
BEGIN
        While (Limit-Swith = On)
        BEGIN
                Open()
                MoveTo(P2);
                SpeedTo(0.2);
                MoveTo(P1)
                Close();
                SpeedTo(0.9);
                MoveTo(P2):
                MoveTo(P3);
                SpeedTo(0.2);
                MoveTo(P4);
                Shift_Bin_Left(1);
                Open();
                SpeedTo(0.9);
                MoveTo(P3)
                MoveTo(P5):
         END
END
```

Outline of Lecture 4

- Background Knowledge
- Purpose of Path Planning
- Input to Path Planning
- Output from Path Planning
- Process of Autonomous Path Planning
  - Case of Arm Manipulators
  - Case of Car-like Mobile Bases

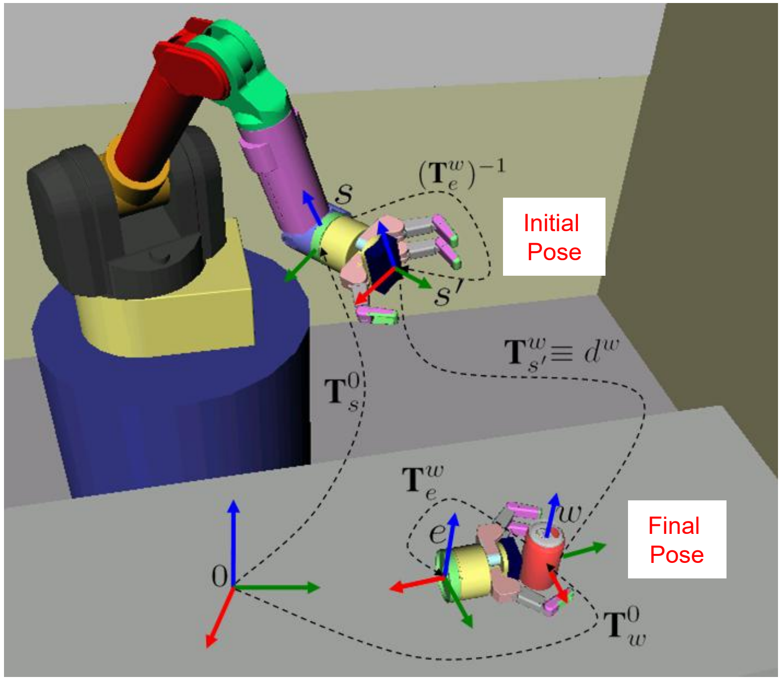
Basic Questions



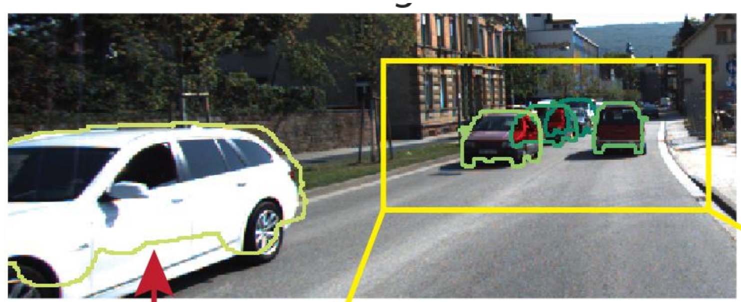
- Question 1: Input
  - What are available inputs?
- Question 2: Output
  - What is the goal or output?
- Question 3: Autonomous Path Planning Algorithm
  - What is the algorithm which leads to the achievement of the output from the input?

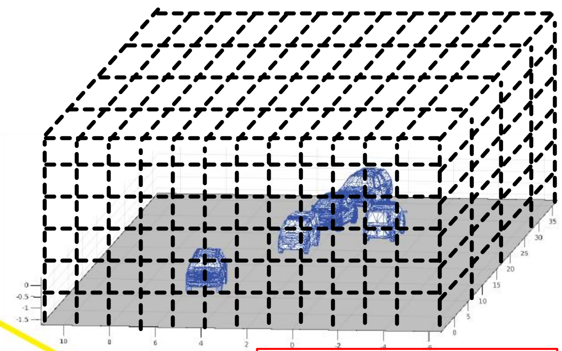
Answer to Question 1: Input

- Initiate State of Position and Orientation (Pose)
- Final State of Position and Orientation (Pose)
- Geometry of Task Space
  - How to represent a task space?
  - How to define and represent a workspace (i.e. a sub-space of task space) ?



Representation of Task Space with a Matrix of Voxels (e.g. Volume Elements)





- 1: Occupied
- 0: Empty

Cloud of 3D Points

3D Discrete Scene :

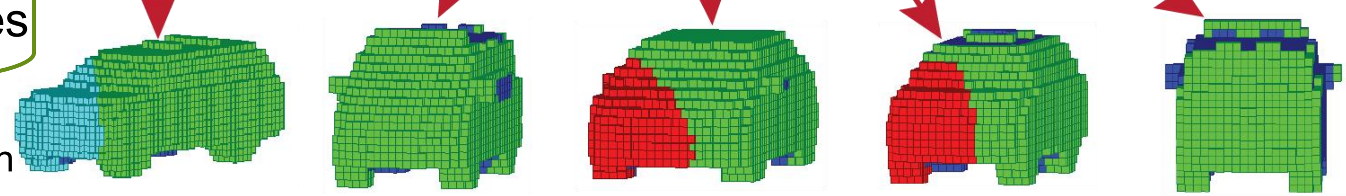
$$\{(X_i, Y_i, Z_i), i = 0,1,2,...\}$$

with :

$$Z_i = Z_{\min} + i \bullet \Delta Z$$

Voxels — Volume Representation

3D Triangles — Surface Representation



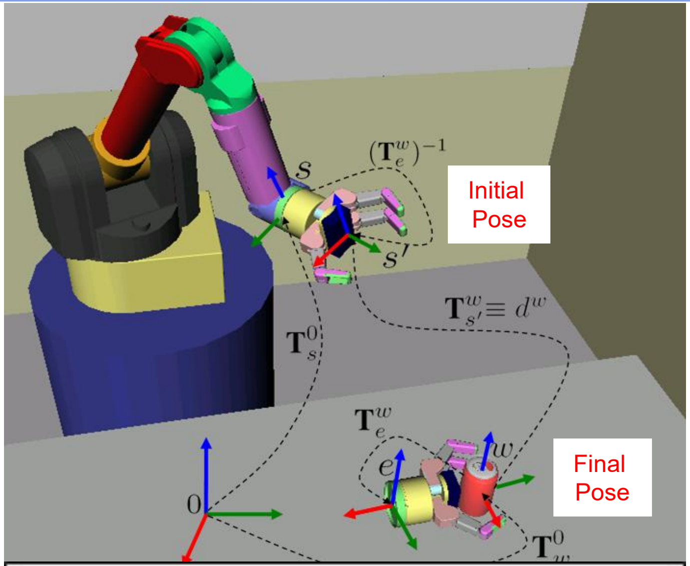
3D voxel patterns

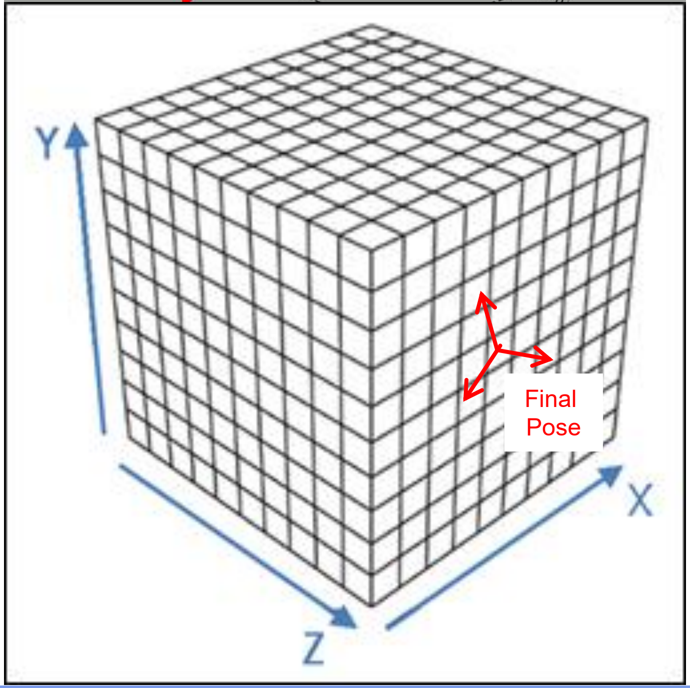
Definition and Representation of Workspace

- Draw a line from the initial pose to the final pose.
- Let this line to be the Z axis of the new workspace.
- Let the X axis to be horizontal.
- Let the Y axis to be perpendicular to Z-X plane.
- Choose a workspace to be within:
  - X: [-Xsize/2, +Xsize/2]
  - Y: [-Ysize/2, +Ysize/2]
  - Z: [0, Zsize]

(Note: Zsize is the distance between the two poses)

- Digitize the workspace into a 3D matrix of voxels which is called Normalized Workspace.



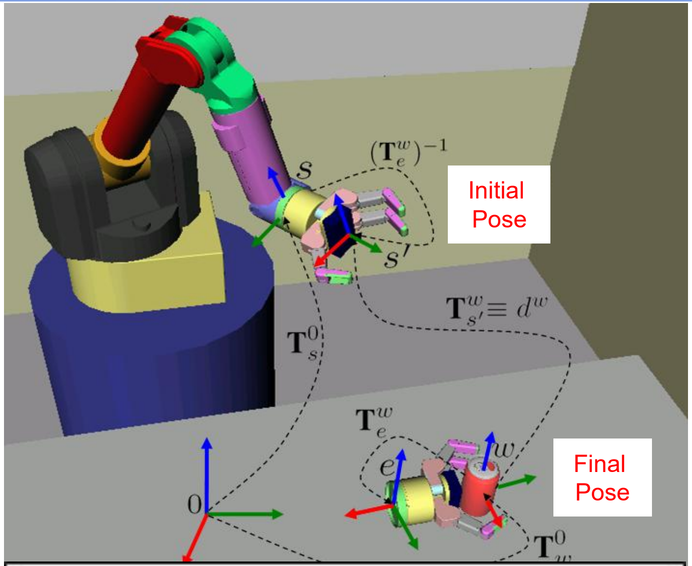


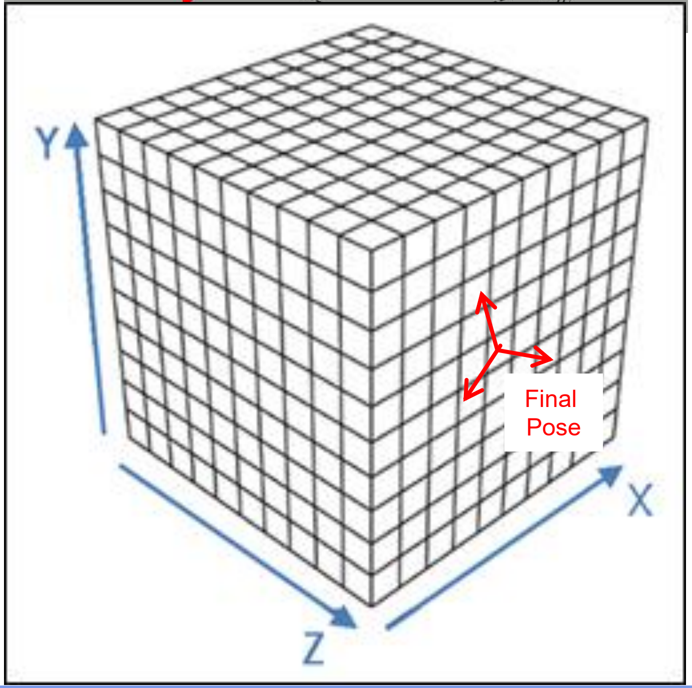
Answer to Question 2: Output

- It could be the next waypoint to reach
- It could be the series of waypoints to reach

Is it possible to determine the output autonomously?

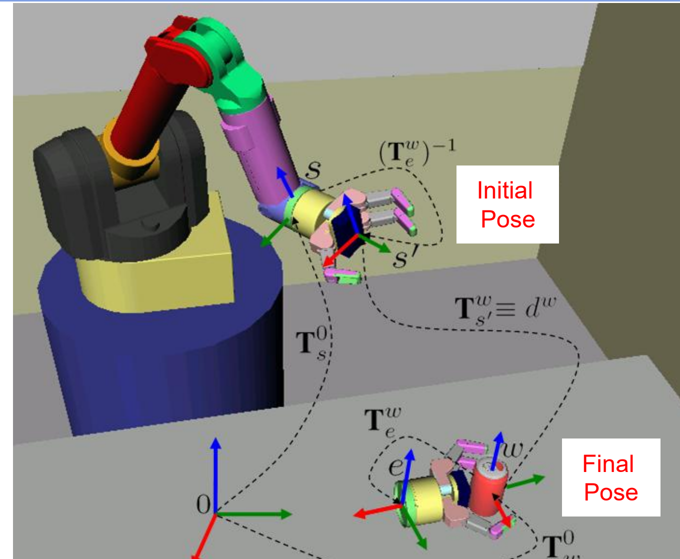
If yes, how?





Answer to Question 3: Autonomous Path Planning Algorithms

- Input to the Algorithm:
  - The initial pose is known
  - The final pose is known
  - The task space is known



Cloud of 3D Points

3D Discrete Scene :

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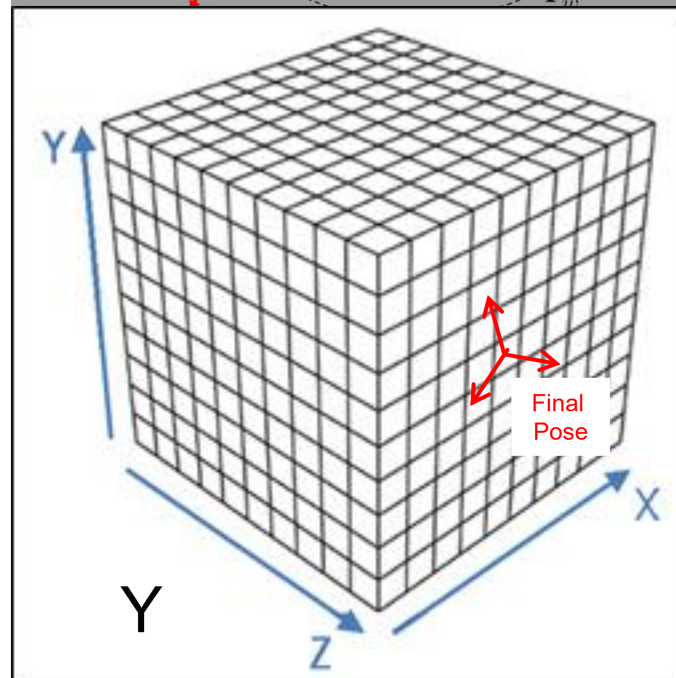
with :

$$Z_i = Z_{\min} + i \bullet \Delta Z$$

Voxels

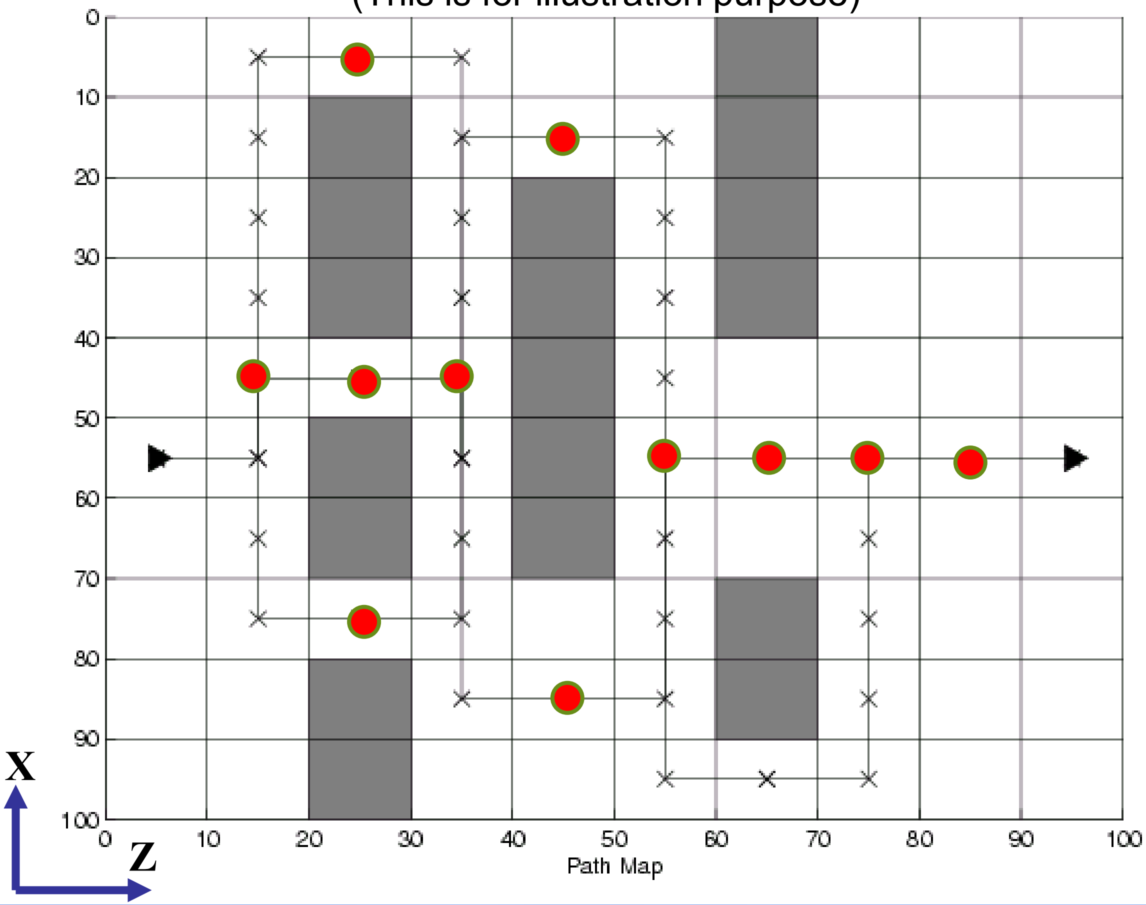
3D Triangles

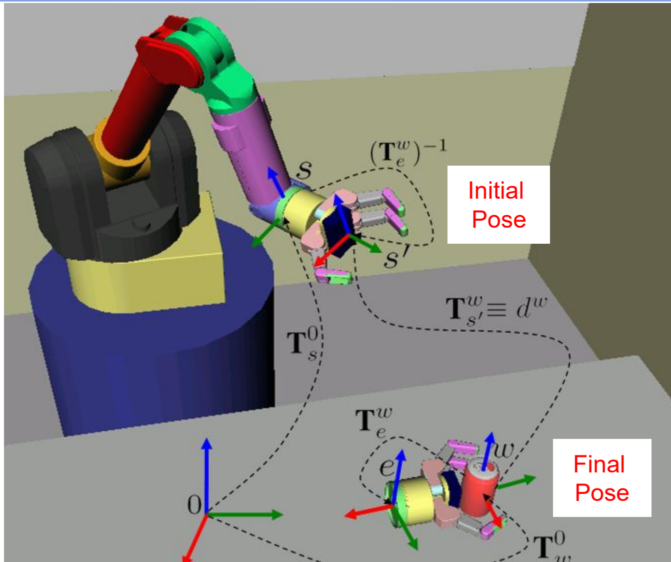


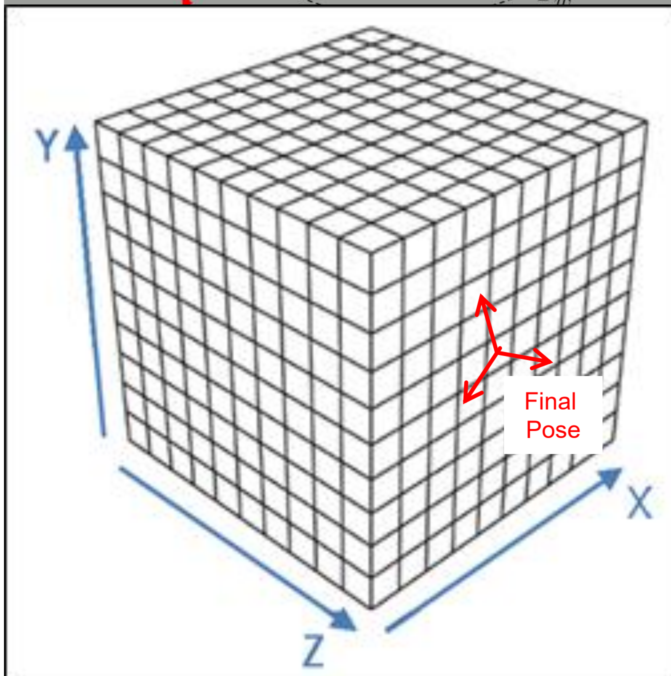
A General Algorithm:

- Output from the Algorithm:
  - A series of waypoints to reach

(This is for illustration purpose)



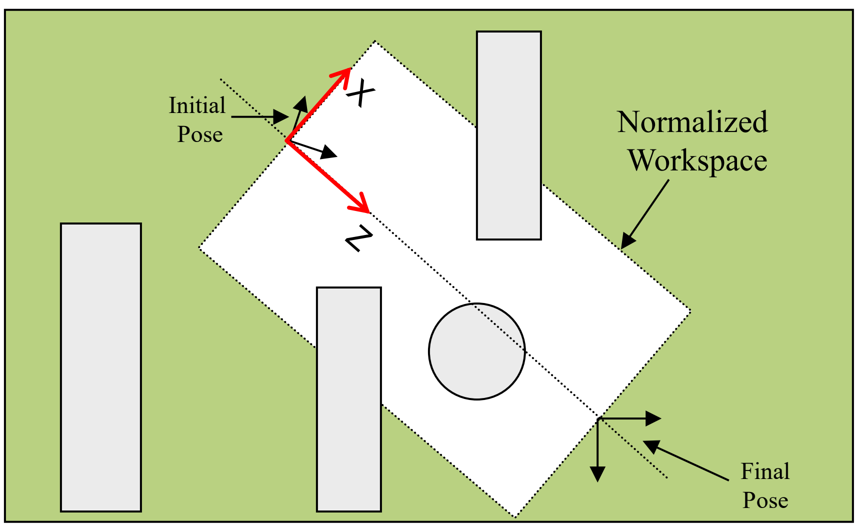


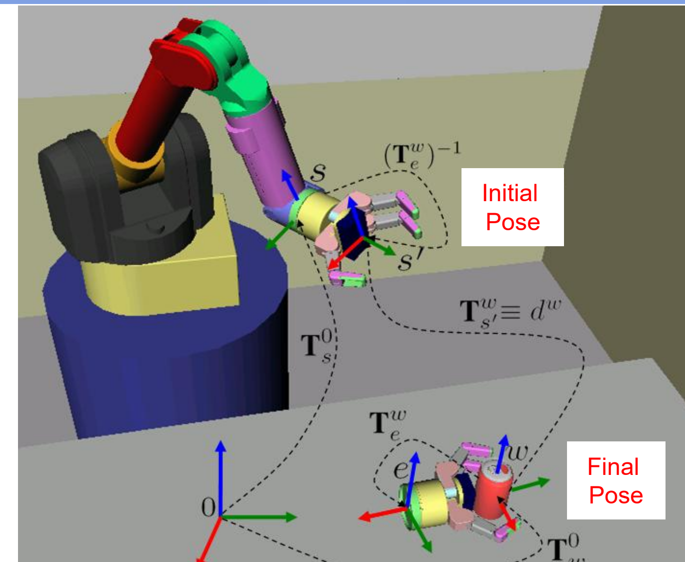


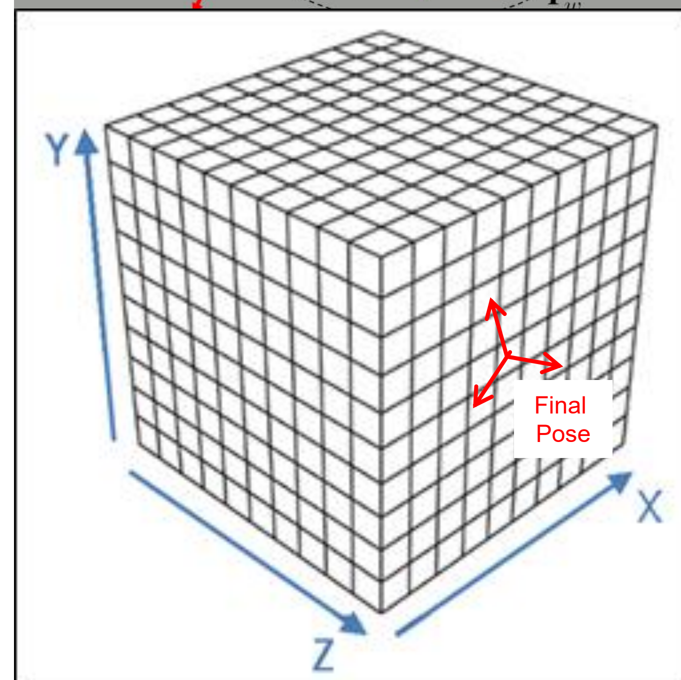
A General Algorithm:

- Details of Steps (Illustration in 2D)
  - Step 1: Choose a workspace

Z-X plane when Y = 0 for illustration purpose

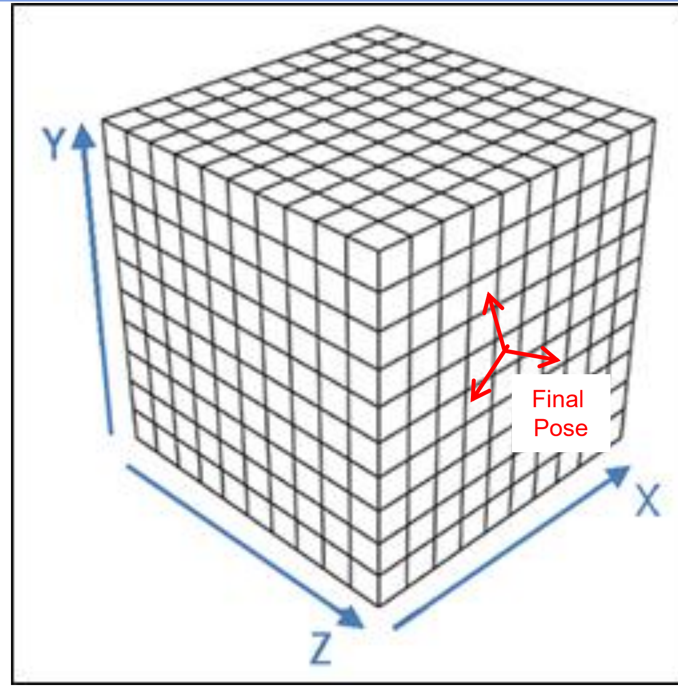




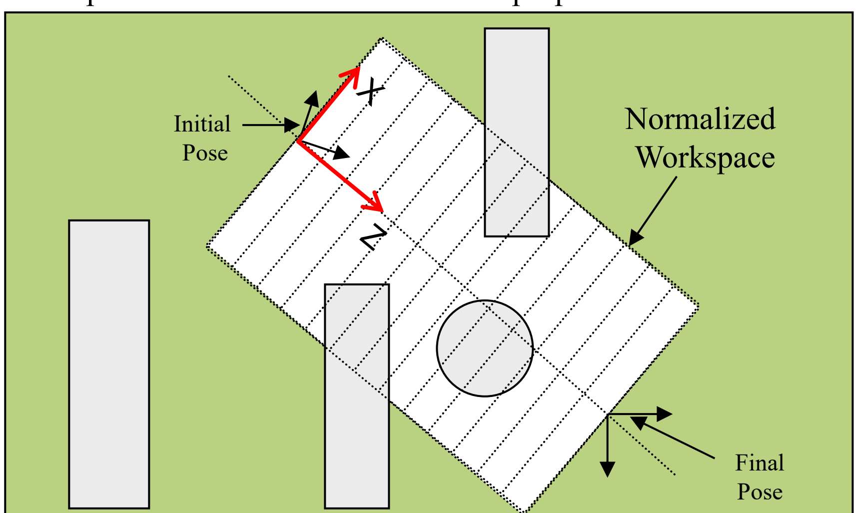


A General Algorithm:

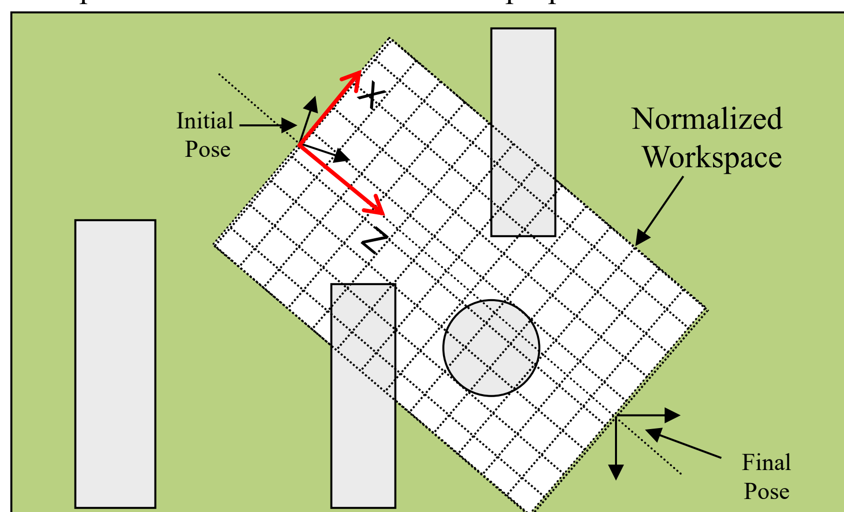
- Details of Steps (Illustration in 2D)
    - Step 2: Digitize the workspace



Z-X plane when Y = 0 for illustration purpose

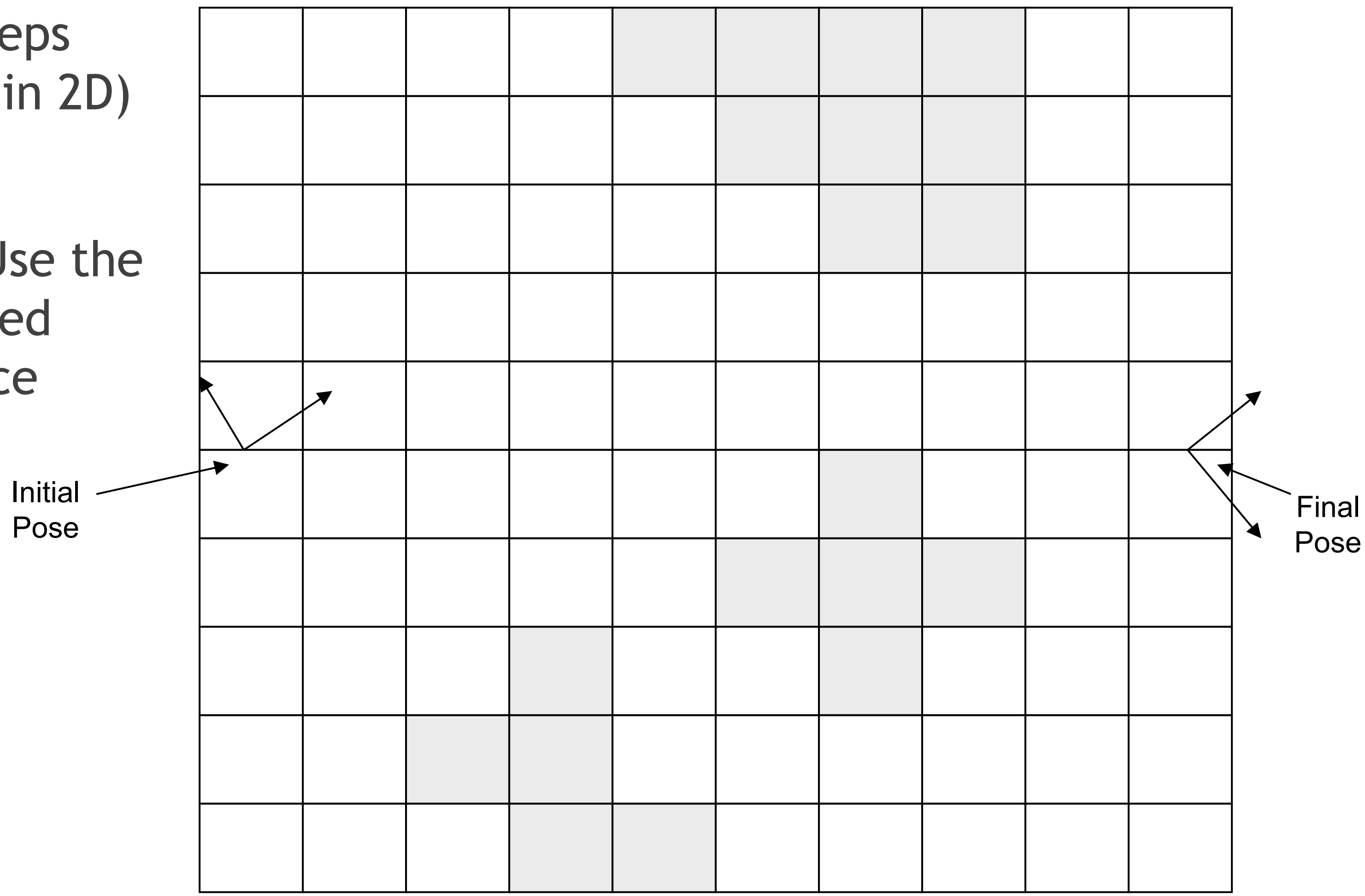


Z-X plane when Y = 0 for illustration purpose



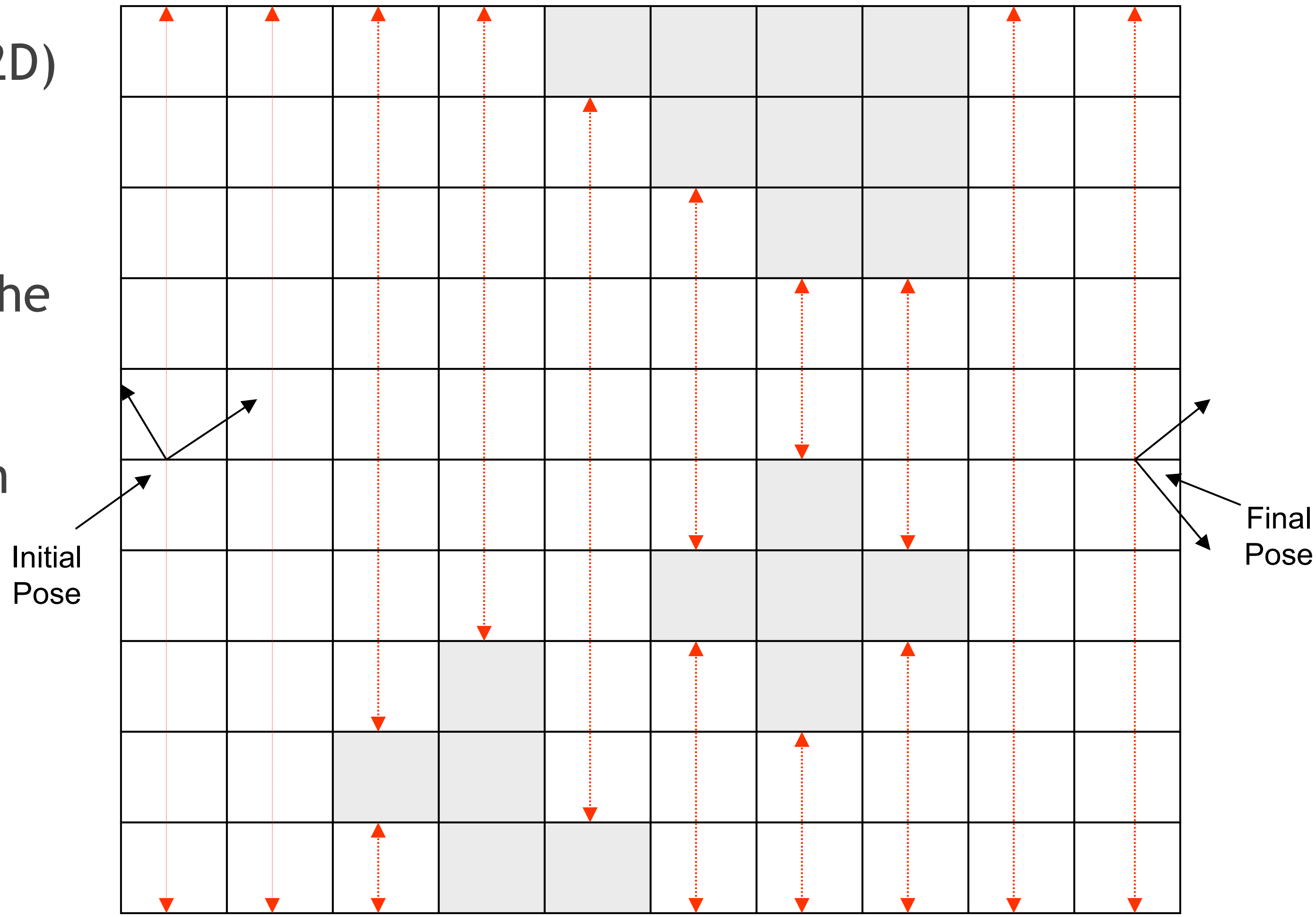
A General Algorithm:

- Details of Steps (Illustration in 2D)
  - Step 3: Use the normalized workspace



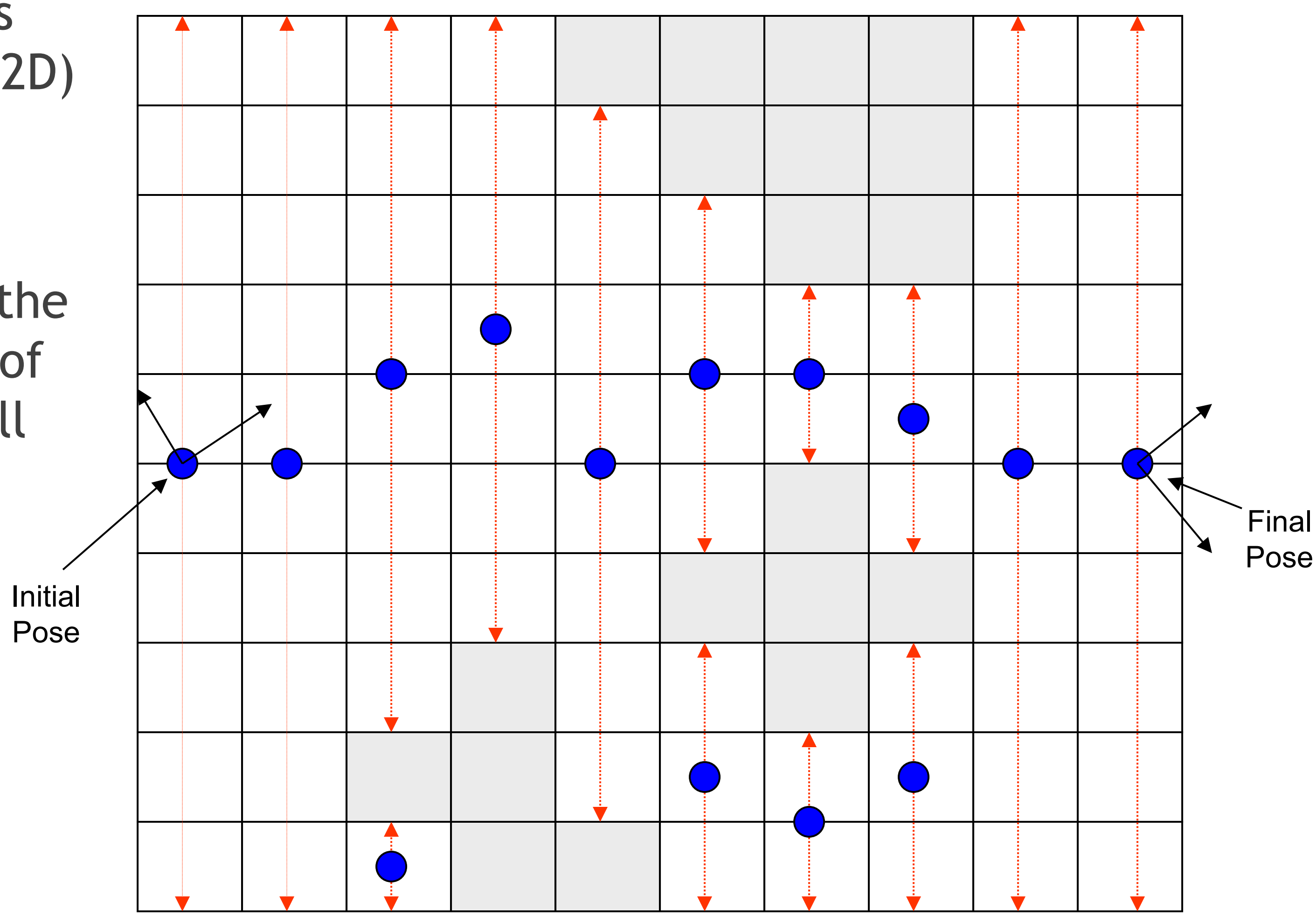
A General Algorithm:

- Details of Steps (Illustration in 2D)
  - Step 4: Determine the free-cell segments in each column



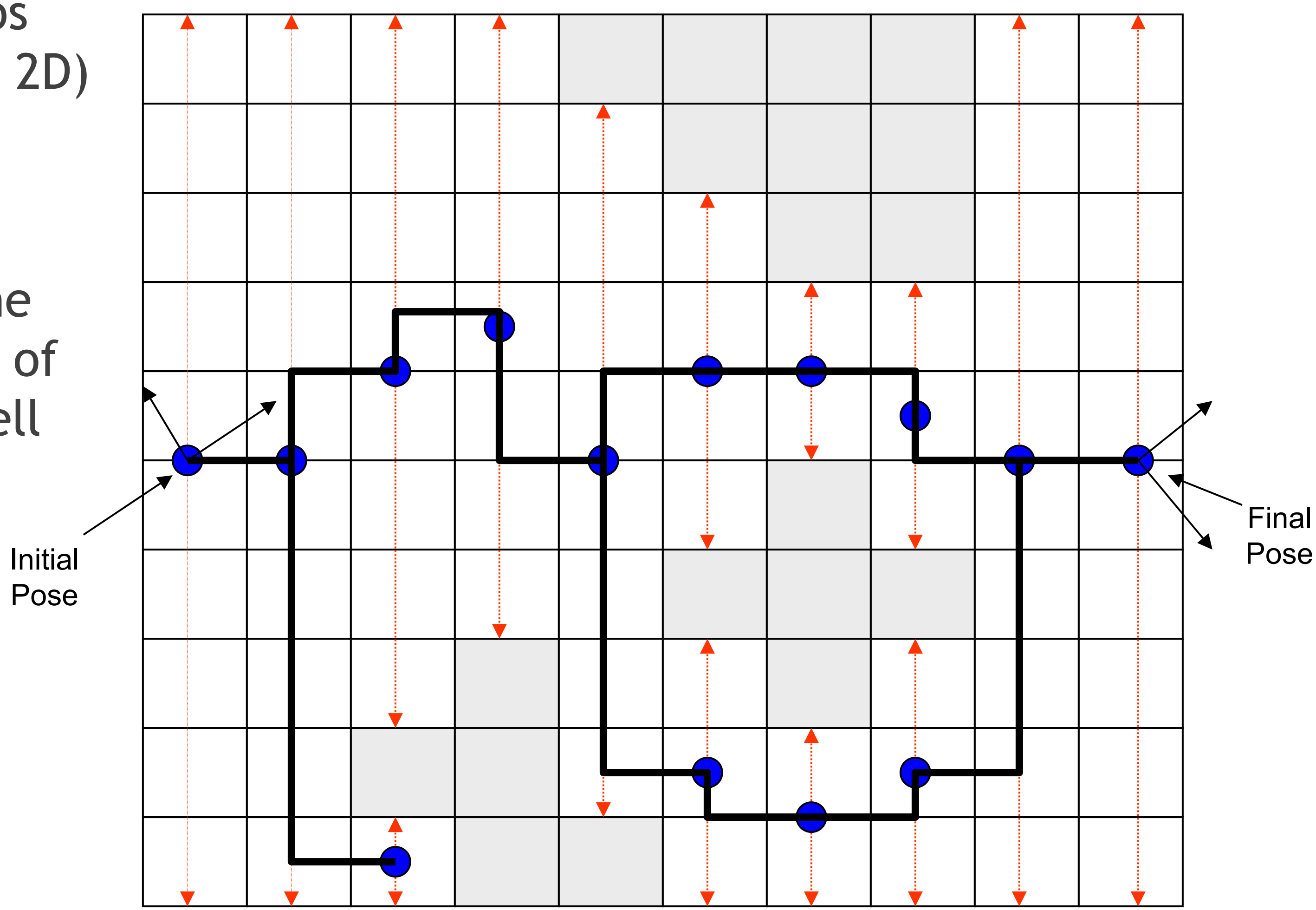
A General Algorithm:

- Details of Steps (Illustration in 2D)
  - Step 5: Determine the mid-points of the free-cell segments



A General Algorithm:

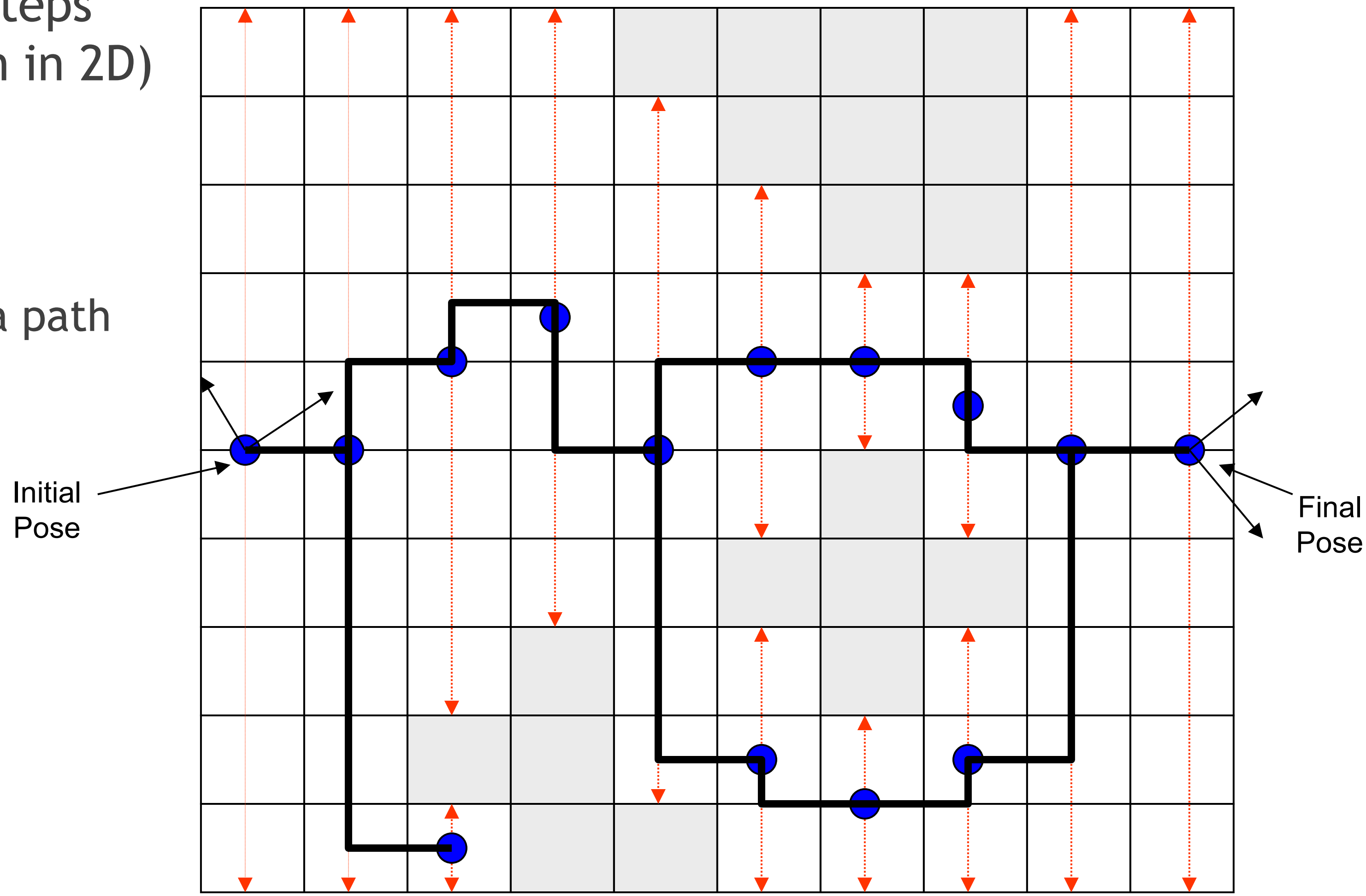
- Details of Steps (Illustration in 2D)
  - Step 6: Connect the mid-points of the free-cell segments



A General Algorithm:

- Details of Steps (Illustration in 2D)
  - Step 7:

    Choose a path



Exercise

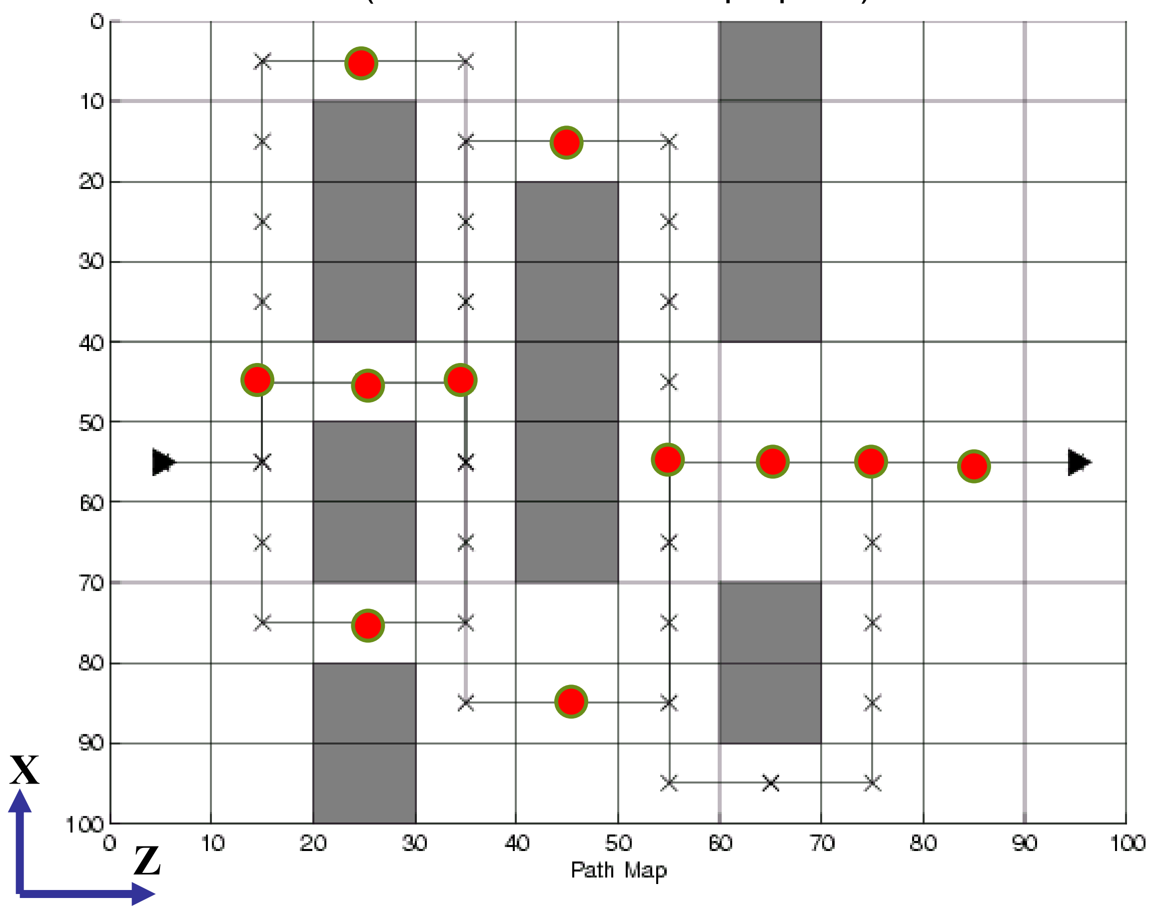
Question 1:

What is the traveled distance along Y axis for a chosen path?

Question 2:

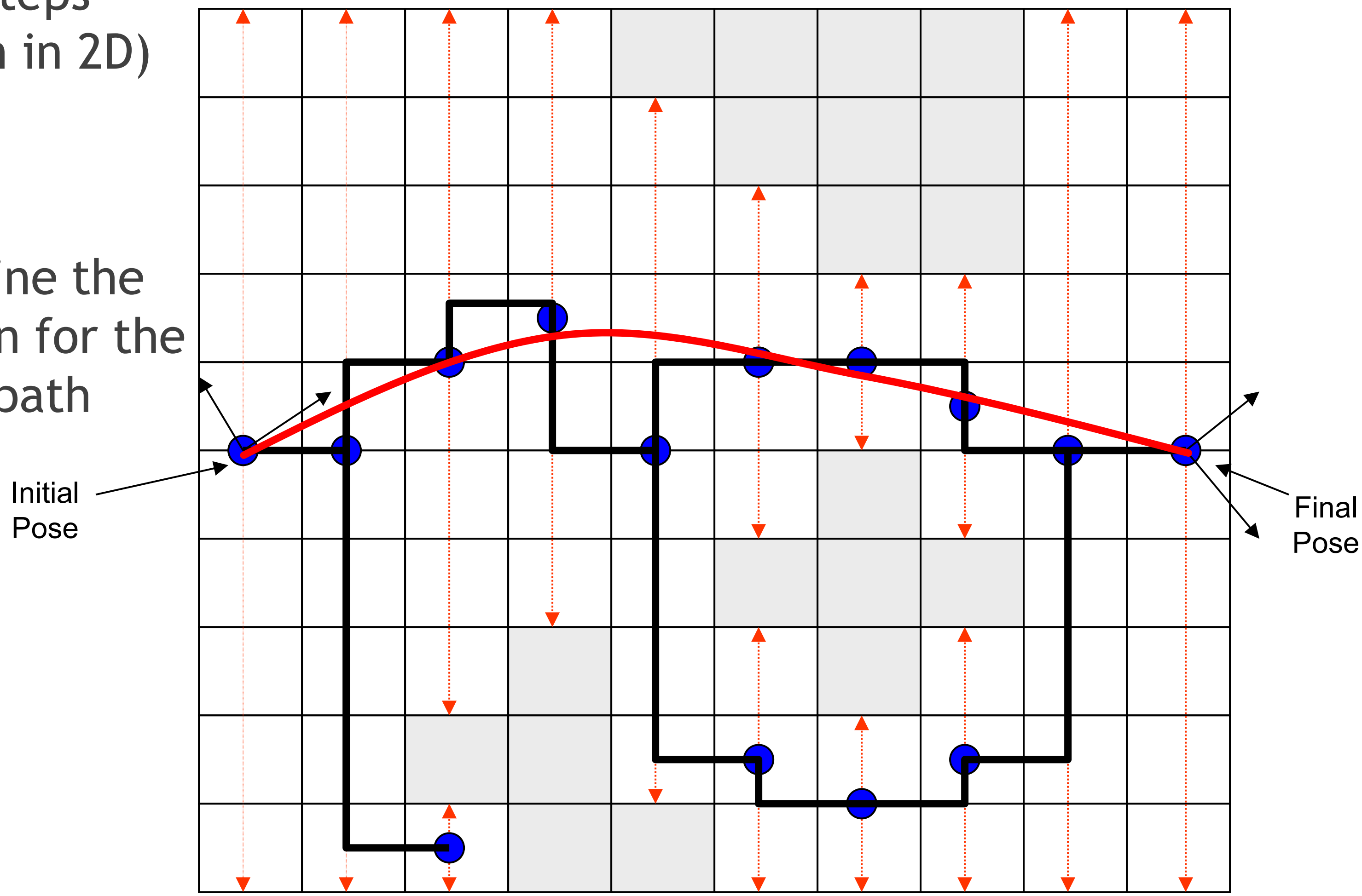
What is the traveled distance along X axis for a chosen path?

(This is for illustration purpose)



A General Algorithm:

- Details of Steps (Illustration in 2D)
  - Step 8: Determine the equation for the chosen path



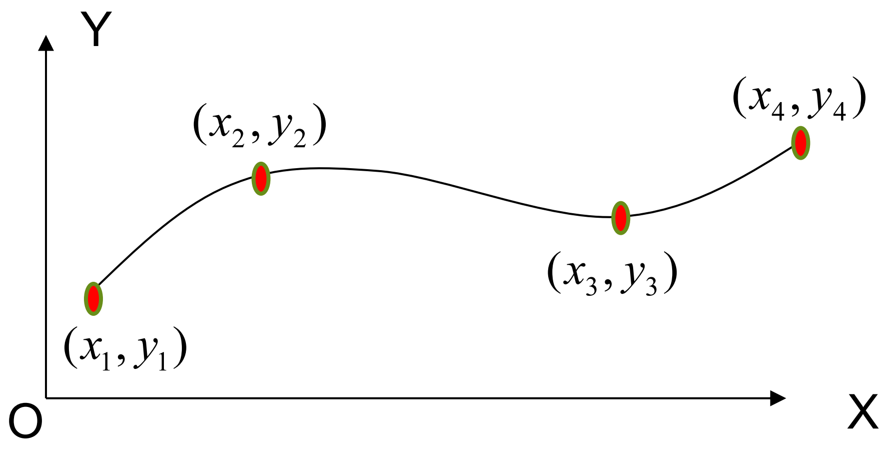
Exercise

Given a set of four points $\{(x_{i,} y_i),\ i=1,2,3,4\}$,

which satisfy the equation below :

$$y = ax^3 + bx^2 + cx + d$$

What is the solution of determining the coefficients?

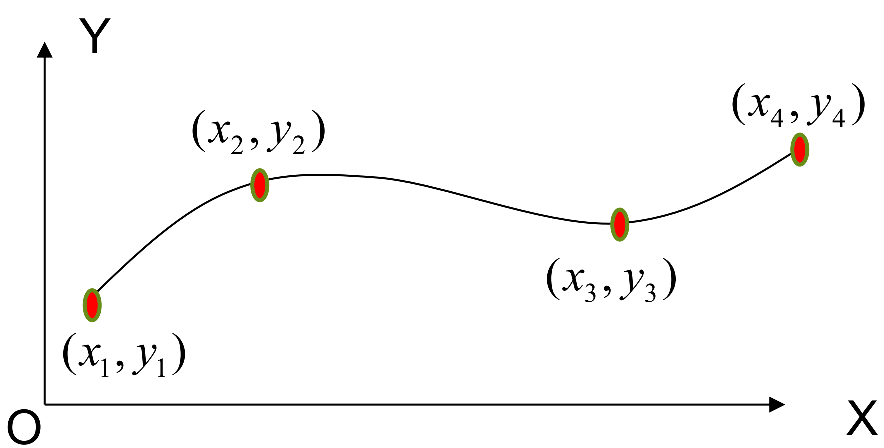


Solution

- Step 1: Establish equations

$$y = ax^3 + bx^2 + cx + d$$

$$\begin{aligned} y_1 &= ax_1^3 + bx_1^2 + cx_1 + d \\ y_2 &= ax_2^3 + bx_2^2 + cx_2 + d \\ y_3 &= ax_3^3 + bx_3^2 + cx_3 + d \\ y_4 &= ax_4^3 + bx_4^2 + cx_4 + d \end{aligned}$$

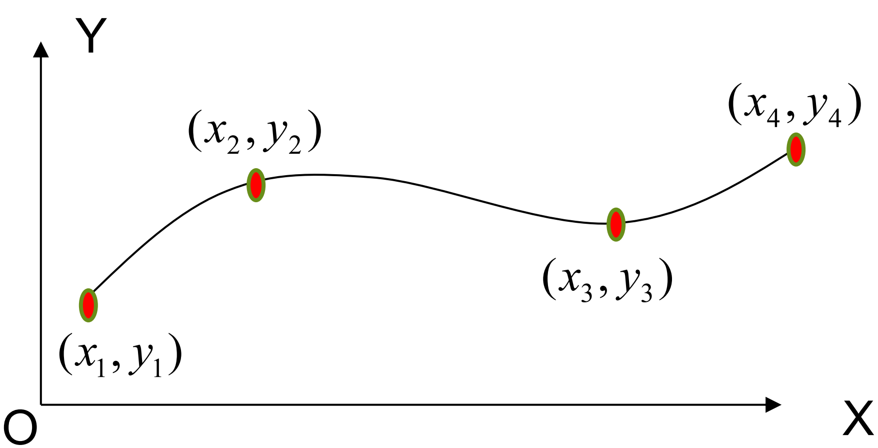


Solution (continued)

- Step 2: Write equations into matrix form

$$y_1 = ax_1^3 + bx_1^2 + cx_1 + d$$
$$y_2 = ax_2^3 + bx_2^2 + cx_2 + d$$
$$y_3 = ax_3^3 + bx_3^2 + cx_3 + d$$
$$y_4 = ax_4^3 + bx_4^2 + cx_4 + d$$

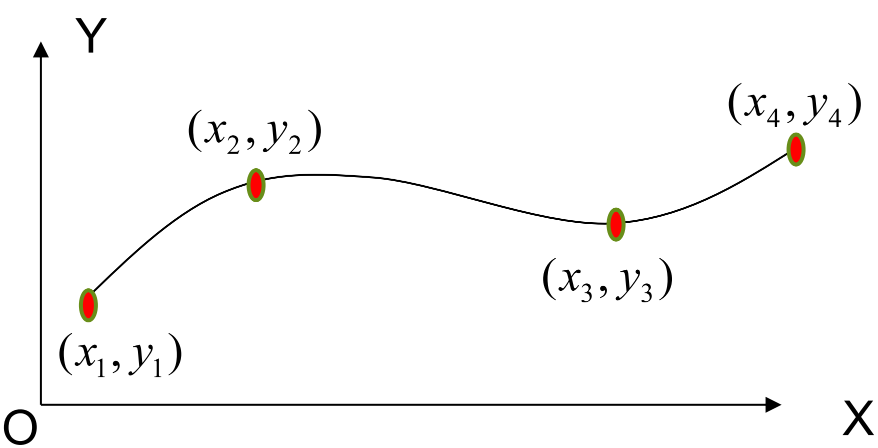
$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} x_1^3 & x_1^2 & x_1 & 1 \\ x_2^3 & x_2^2 & x_2 & 1 \\ x_3^3 & x_3^2 & x_3 & 1 \\ x_4^3 & x_4^2 & x_4 & 1 \end{bmatrix} \bullet \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$



Solution (continued)

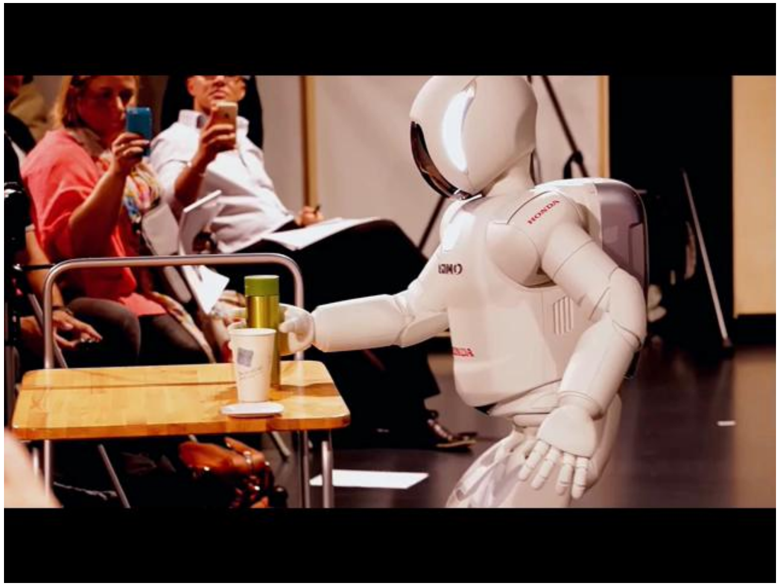
- Step 3: Solve the equation of the matrix form

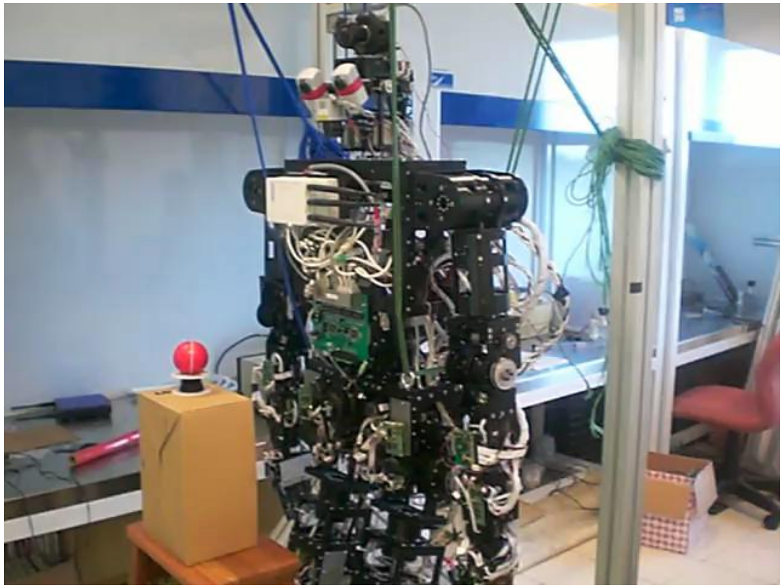
$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} x_1^3 & x_1^2 & x_1 & 1 \\ x_2^3 & x_2^2 & x_2 & 1 \\ x_3^3 & x_3^2 & x_3 & 1 \\ x_4^3 & x_4^2 & x_4 & 1 \end{bmatrix} \bullet \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$



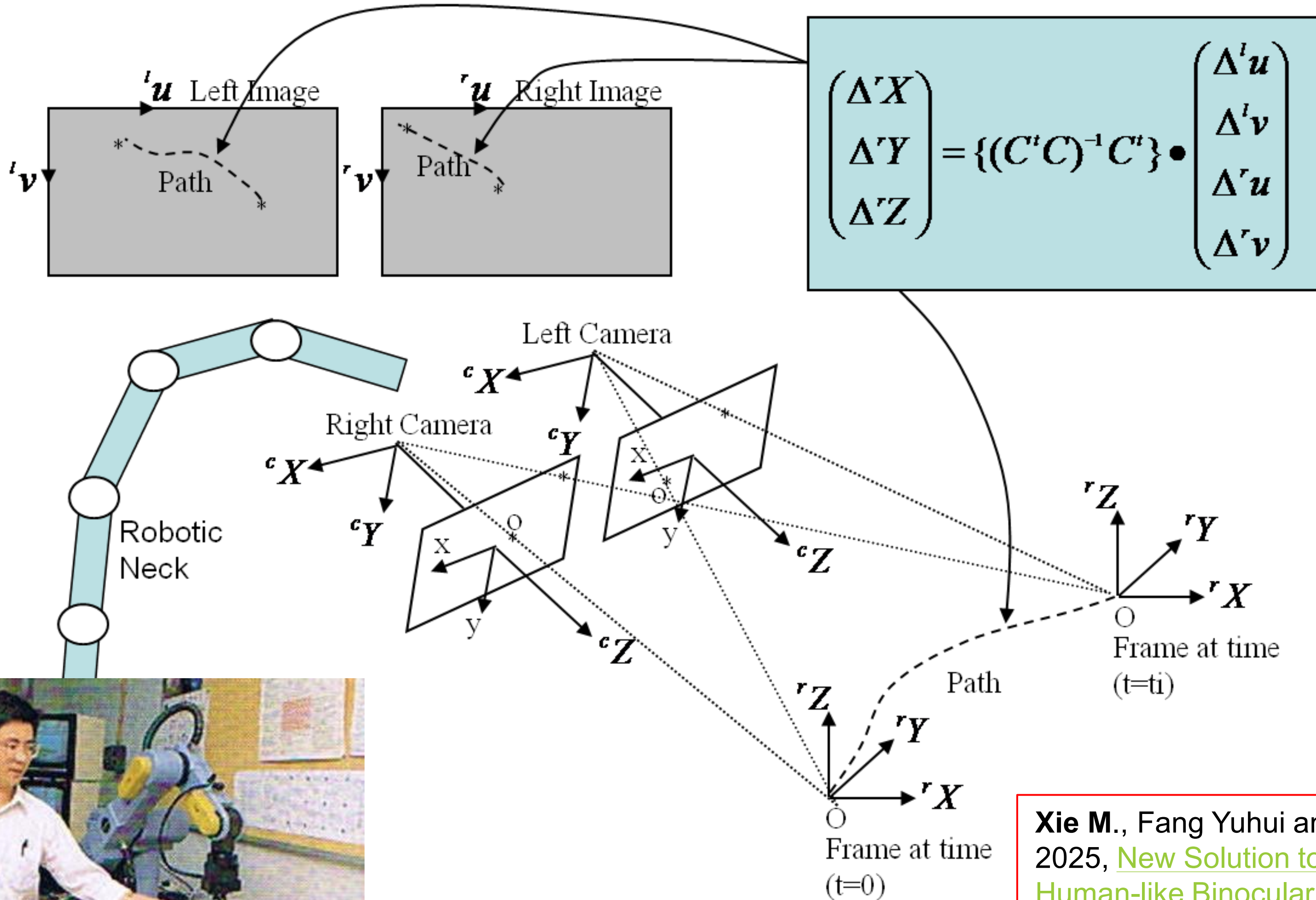
$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} x_1^3 & x_1^2 & x_1 & 1 \\ x_2^3 & x_2^2 & x_2 & 1 \\ x_3^3 & x_3^2 & x_3 & 1 \\ x_4^3 & x_4^2 & x_4 & 1 \end{bmatrix}^{-1} \bullet \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

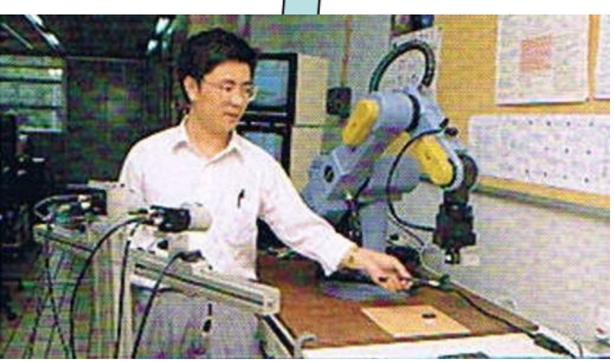
Examples of Vision-Guided Grasping and Manipulation





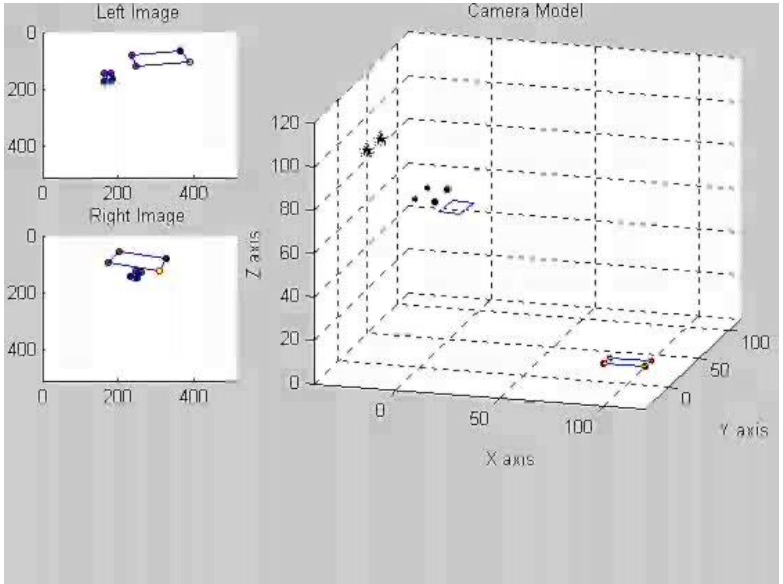
More Solution: Hand-Eye Coordination

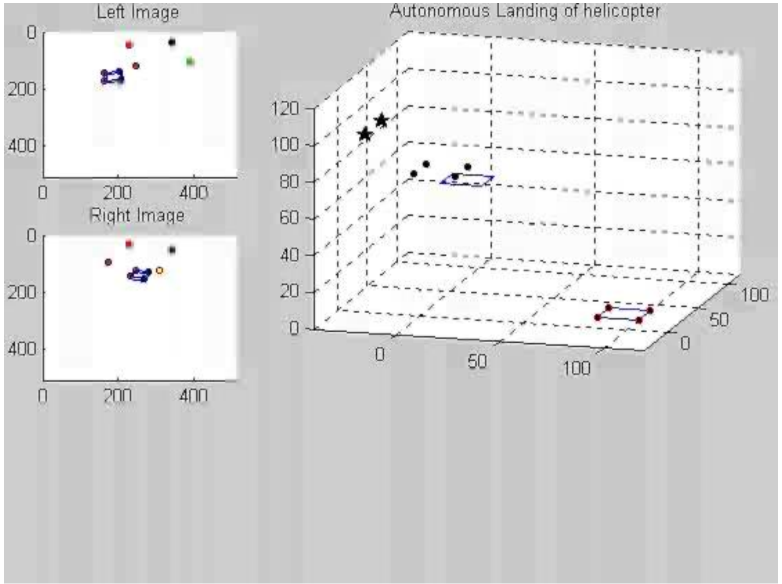


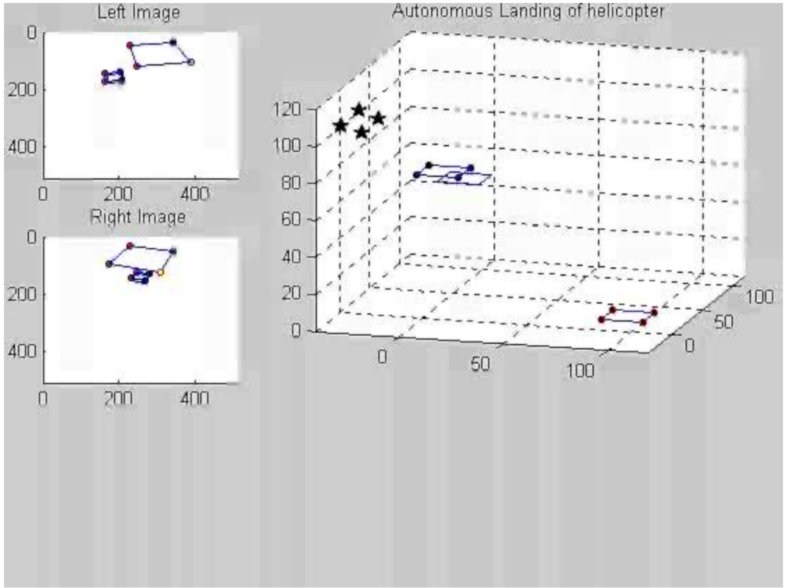


Xie M., Fang Yuhui and Lai Tingfeng, 2025, New Solution to 3D Projection in Human-like Binocular Vision, International Journal of Humanoid Robotics.

More Solution: Head-Eye Coordination







Outline of Lecture 4

- Background Knowledge
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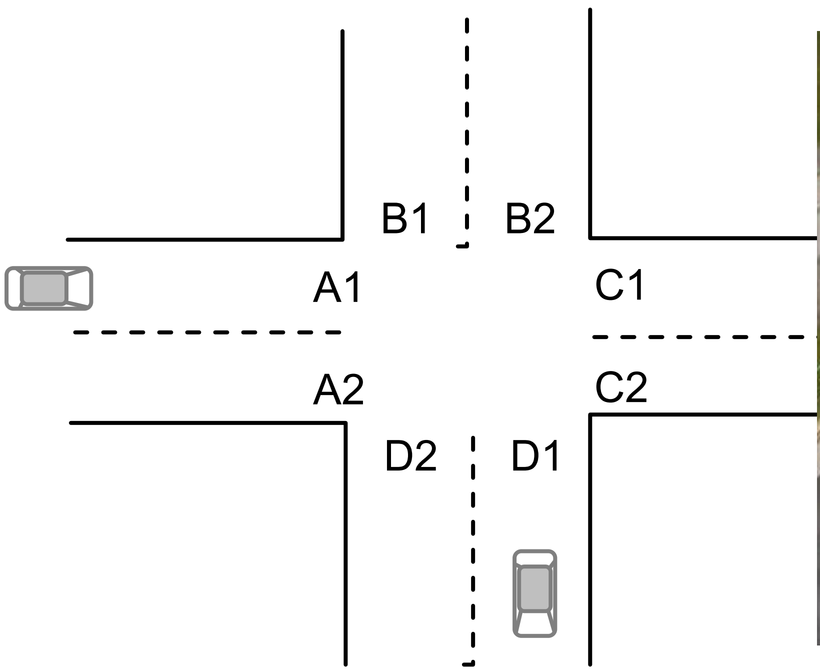
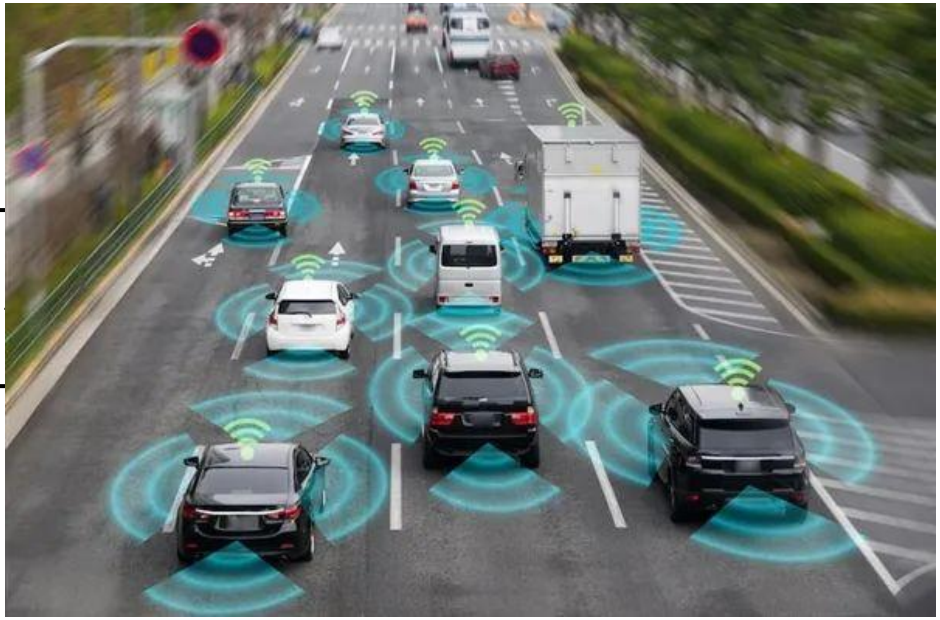
Method 1: Forward Planning

- There is a junction connecting four roads, each of which has two lanes as shown in the figure. A vision-guided smart car is approaching the junction from lane A1. Assume that the geometry of the roads is conform to a national standard. How will the smart car self-find the solution (e.g. motion) for it to turn into lane D1 or B2?





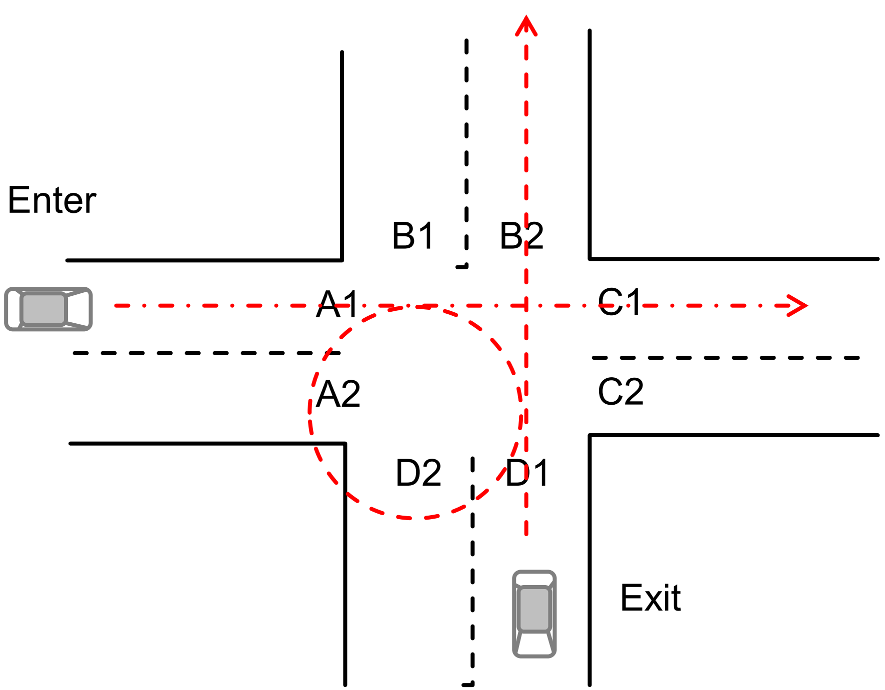
Possible Design Solutions …

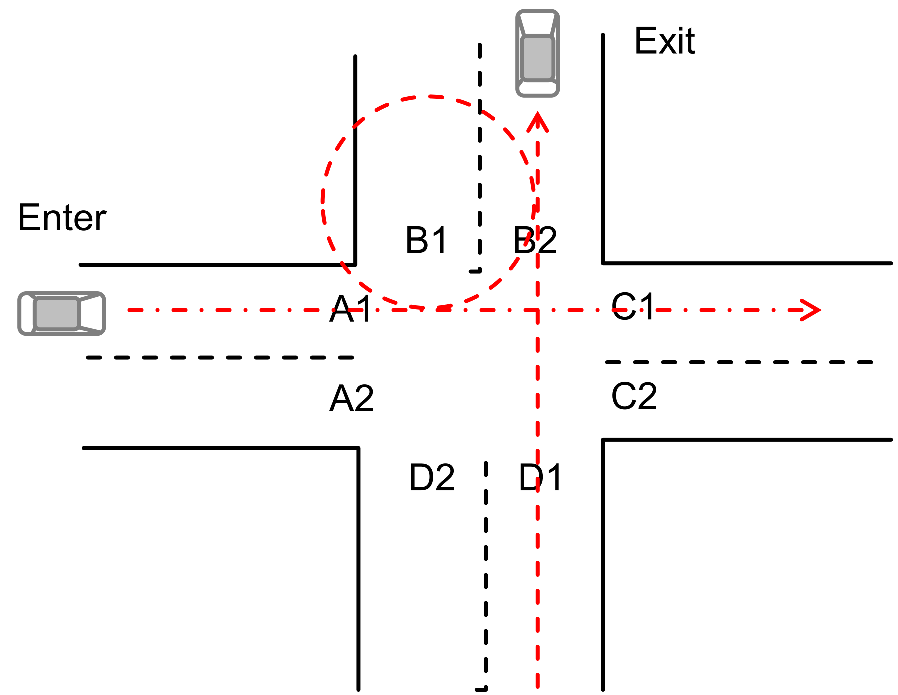
When a car-like vehicle follows a circular trajectory, its steering angle is a constant.

Design algorithm for motions with two phases:

Step 1: Choose the horizontal lane's central line as X axis, and the vertical lane's central line as Y axis.

Step 2: Position a circle with the radius r_R at $(-r_R, -r_R)$ or $(-r_R, +r_R)$ where r_R is the car-like vehicle's minimum value of rotation radius.



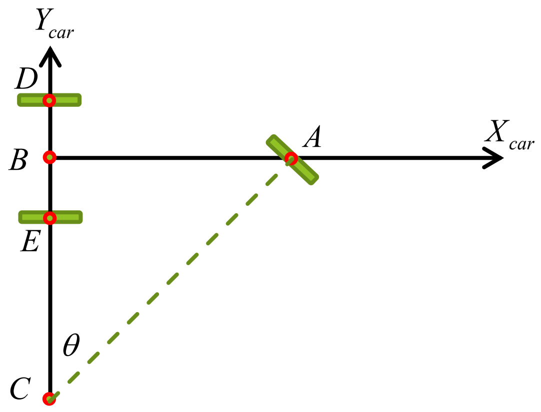


Preparation to Solution

- On a ground plane, a car is considered as a high-speed mobile robot which occupies a rectangular area. We call it a pose of a car.

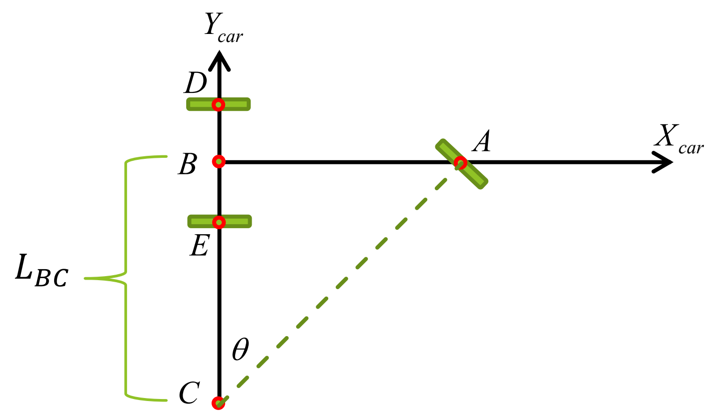
- We assume that a car has two frontal steering wheels. Therefore, when a car rotates, its two rear wheels will follow circular paths, respectively.

- We assign a coordinate system to a car. Its origin is at the center between the two rear wheels. Its X axis is parallel to the wheels, while its Y axis is perpendicular to the wheel



Solution

- Step 1: Describe the capabilities of a car in terms of a car's kinematic model.



$OX_{car}Y_{car}$: local coordinate system assigned to a car

A : the center between two front wheels

B : the center between two rear wheels

C : the center of circular path when steering angle is not zero

D : the position of the left rear wheel

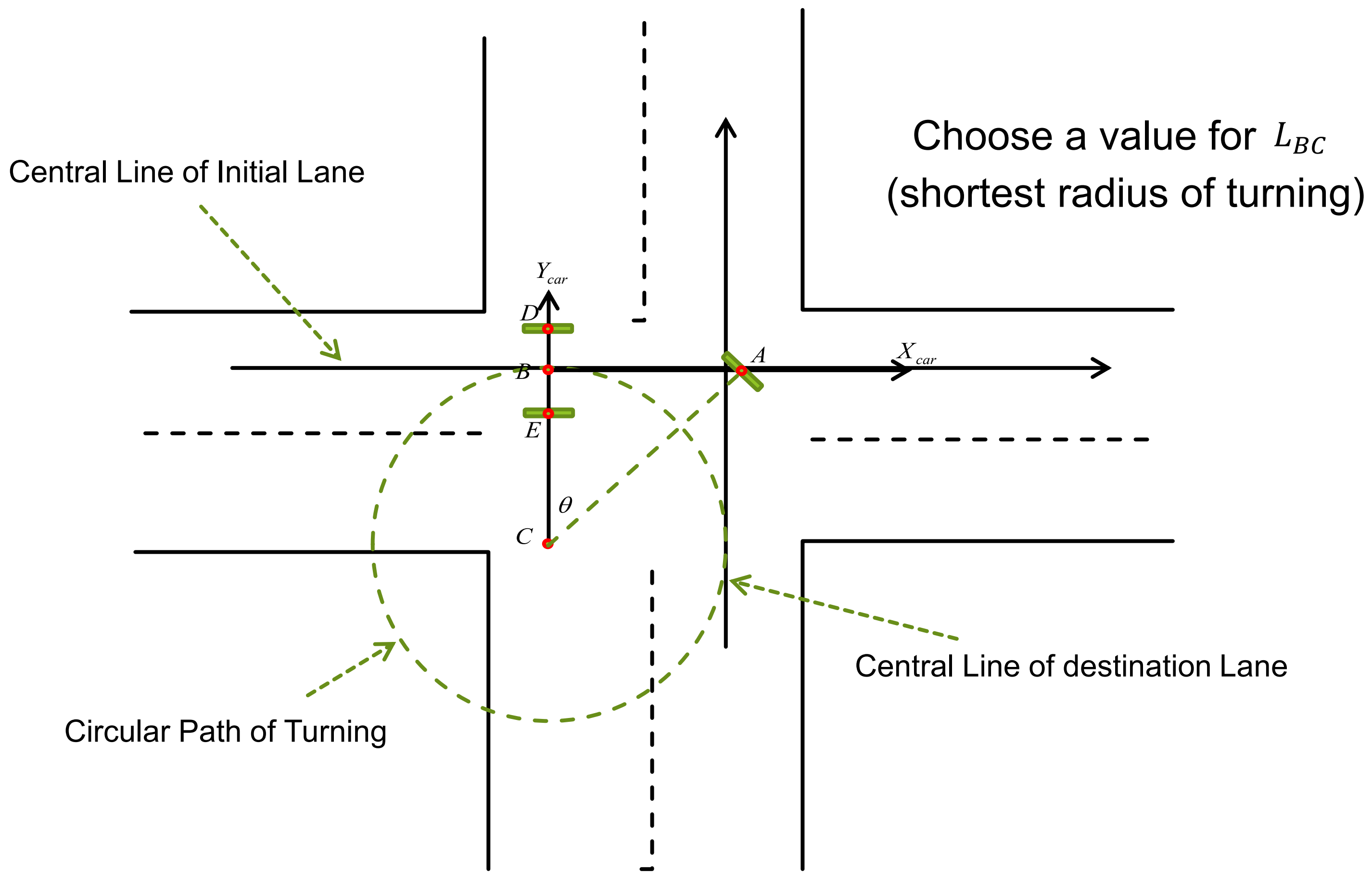
E : the position of the right rear wheel

θ : the steering angle of car, which has a maximum value

L_{BC} : The radius of a car's circular path, which has a minimum value

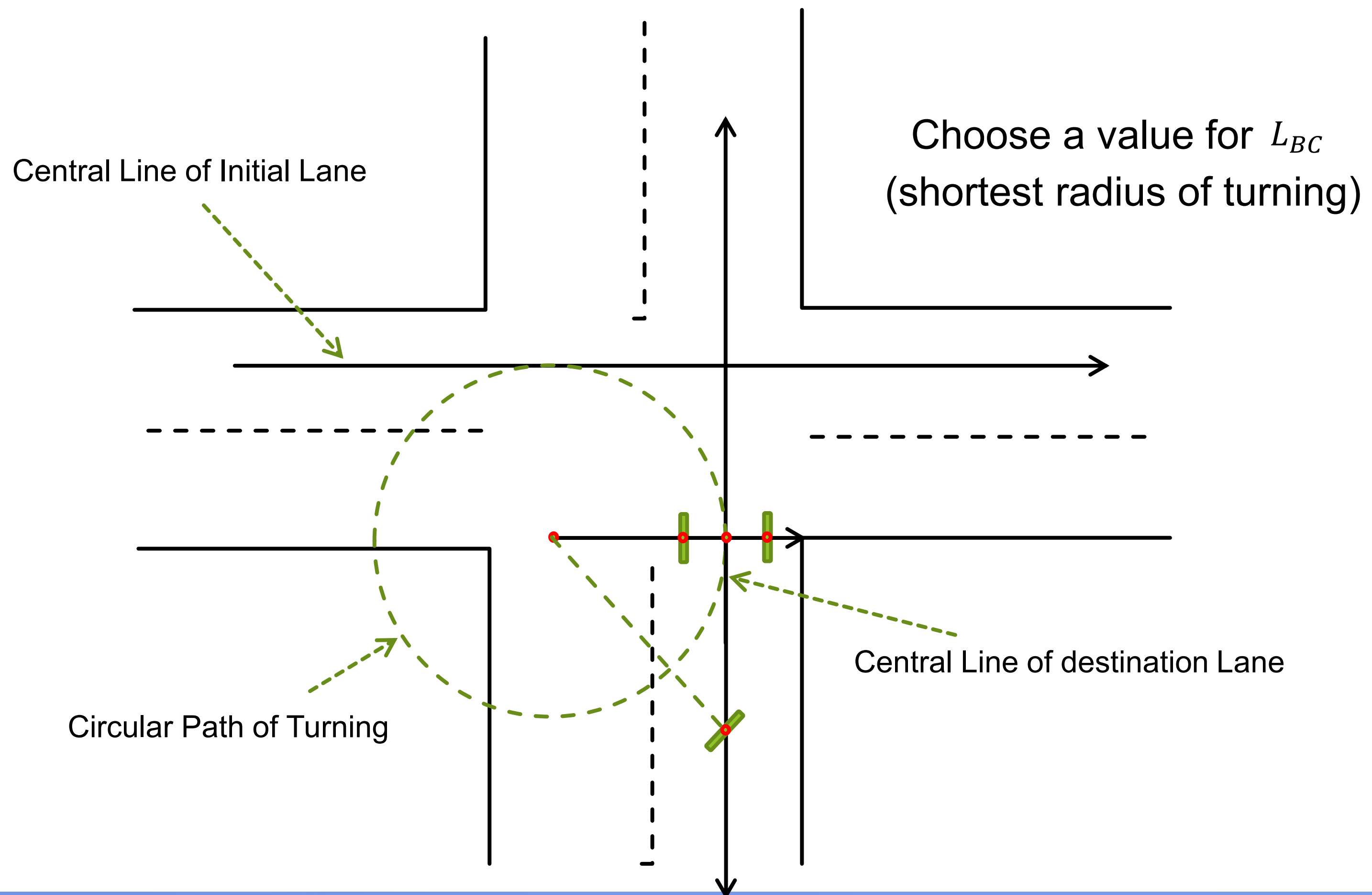
Solution (continued)

- Step 2: Describe the initial state of the car before turning:



Solution (continued)

- Step 3: Describe the final state of the car after the turning:



Solution (continued)

- Step 4: Plan the solution:

1. At time t_s, set the steering angle to be:

$$\theta = \arctan\left(\frac{X_A(t_s) - X_B(t_s)}{Y_B(t_s) - Y_C(t_s)}\right)$$

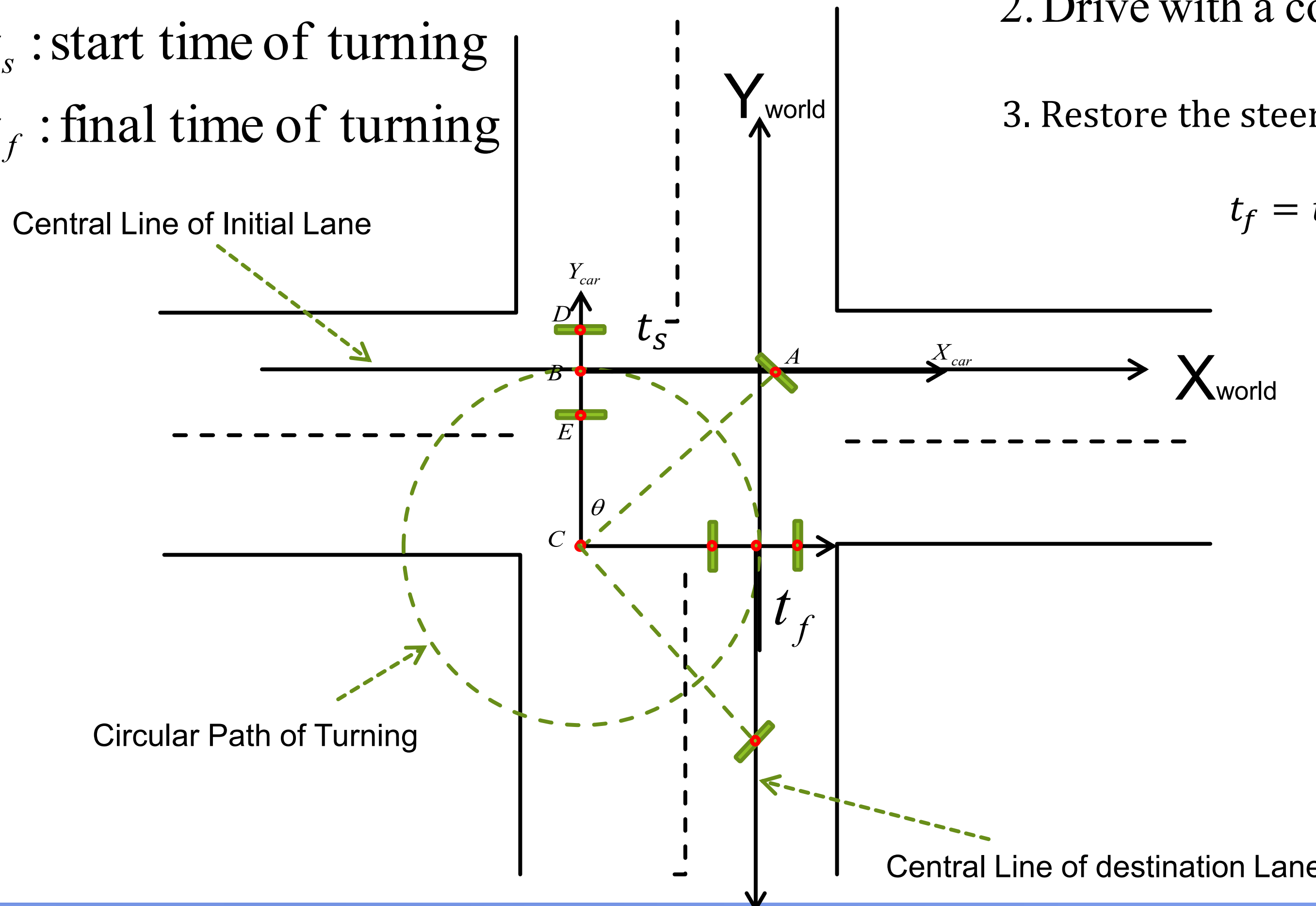
2. Drive with a constant speed of v.

3. Restore the steering angle back to zero *at*:

$$t_f = t_s + \frac{\pi L_{BC}}{2v}$$

t_s : start time of turning

t_f : final time of turning

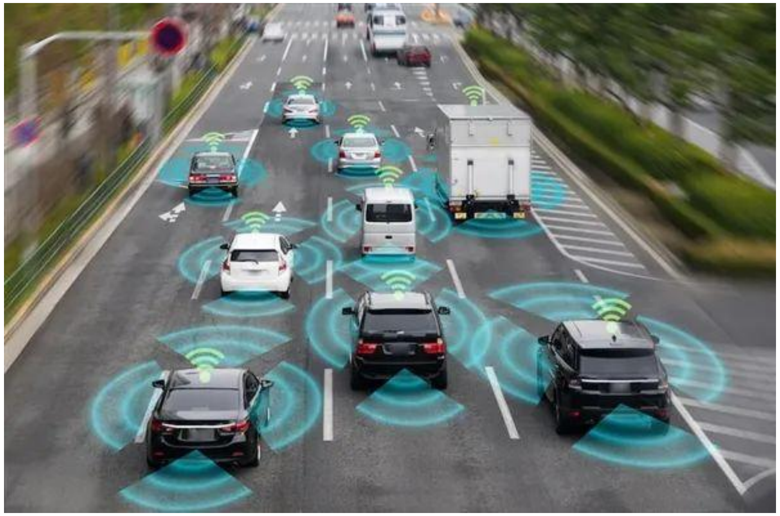


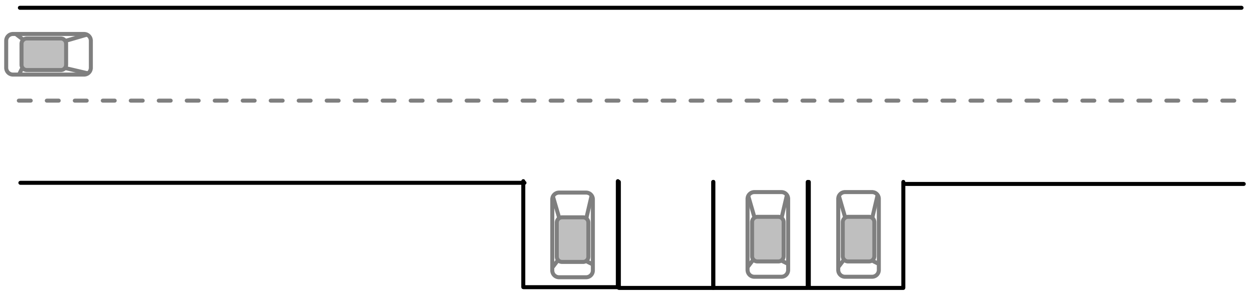
Real Application …

Another Application (with zero radius) …

Method 2: Inverse Planning

- A vision-guided smart car is moving along a street of a city. When it sees an empty lot of vertical parking, it decides to enter the parking lot and to take a rest. What is the sequence of motions which will make the smart car to enter the empty lot of vertical parking as shown in the figure below?





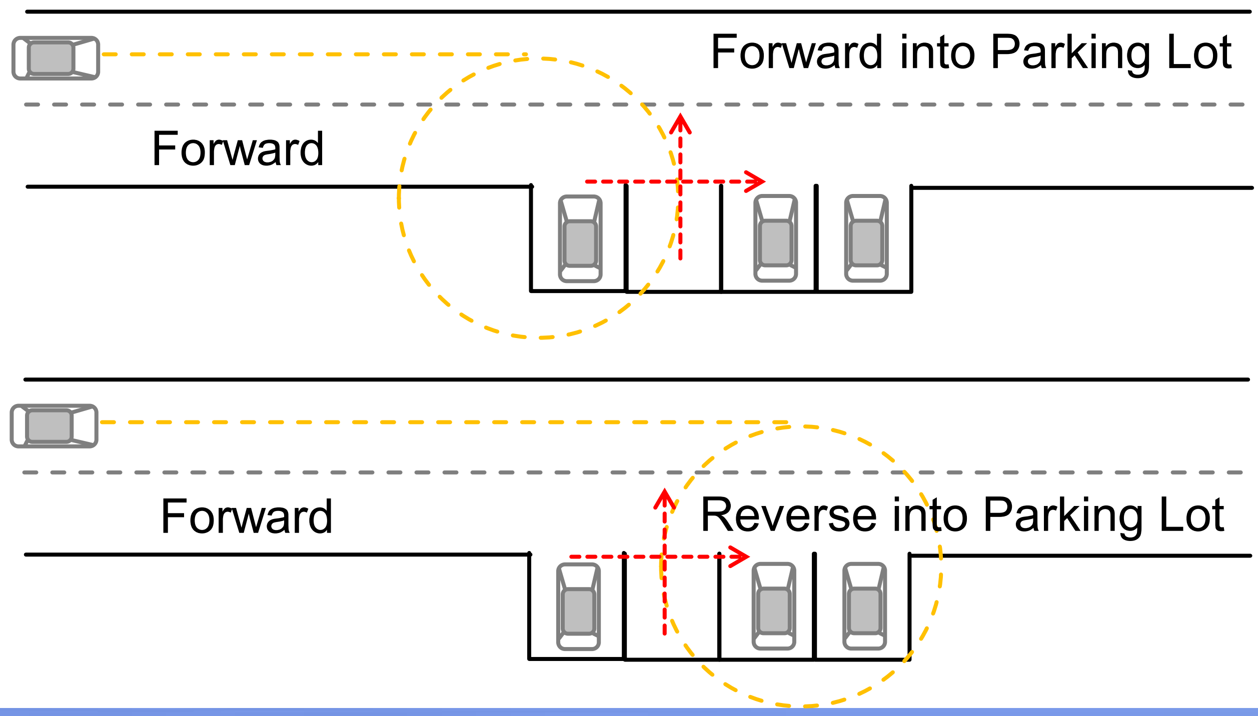
Possible Design Solutions …

When a car-like vehicle follows a circular trajectory, its steering angle is a constant.

Design algorithm for motions with two phases:

Step 1: Position the world coordinate system at the entry of a free parking lot.

Step 2: Position a circle with the radius Y_{car} at $(-Y_{car}, 0)$ or $(+Y_{car}, 0)$ where Y_{car} is the car-like vehicle's Y coordinate with respect to the world coordinate system.

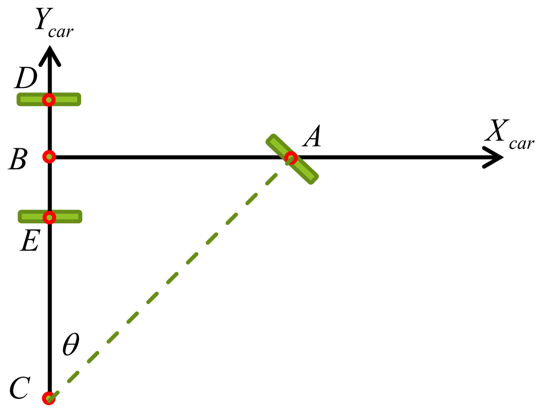


Preparation to Solution

- On a ground plane, a car is considered as a high-speed mobile robot which occupies a rectangular area. We call it a pose of a car.

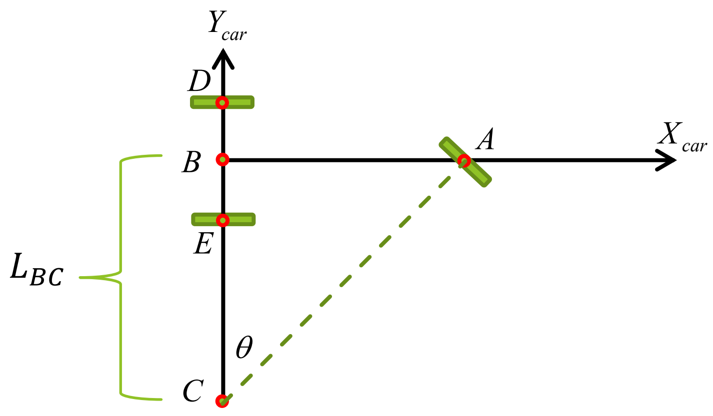
- We assume that a car has two frontal steering wheels. Therefore, when a car rotates, its two rear wheels will follow circular paths, respectively.

- We assign a coordinate system to a car. Its origin is at the center between the two rear wheels. Its X axis is parallel to the wheels, while its Y axis is perpendicular to the wheel.



Solution

- Step 1: Understand the capabilities of a car in terms of a car's kinematic model.



$OX_{car}Y_{car}$: local coordinate system assigned to a car

A : the center between two front wheels

B : the center between two rear wheels

C : the center of circular path when steering angle is not zero

D : the position of the left rear wheel

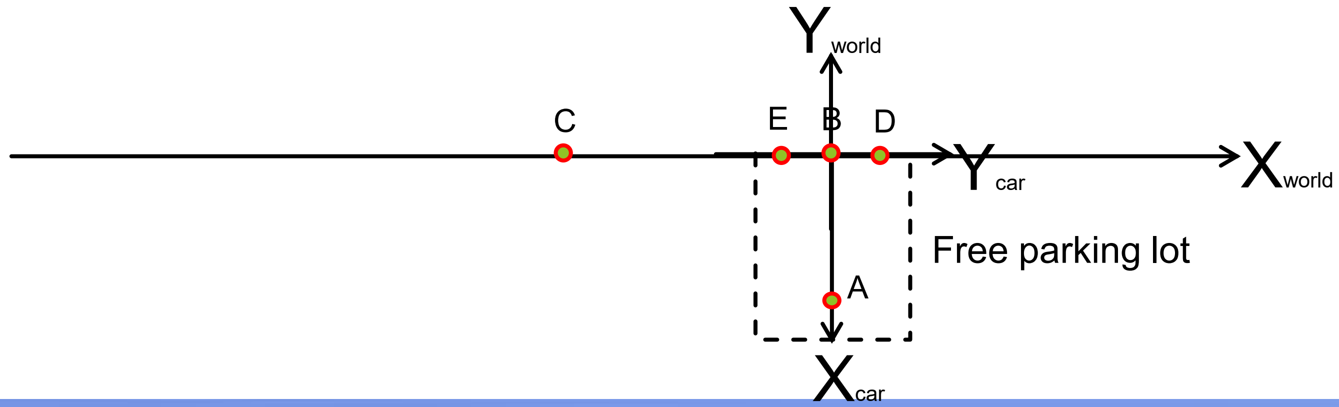
E : the position of the right rear wheel

θ : the steering angle of car, which has a maximum value

L_{BC} : The radius of a car's circular path, which has a minimum value

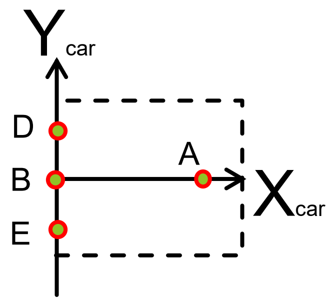
Solution (continued)

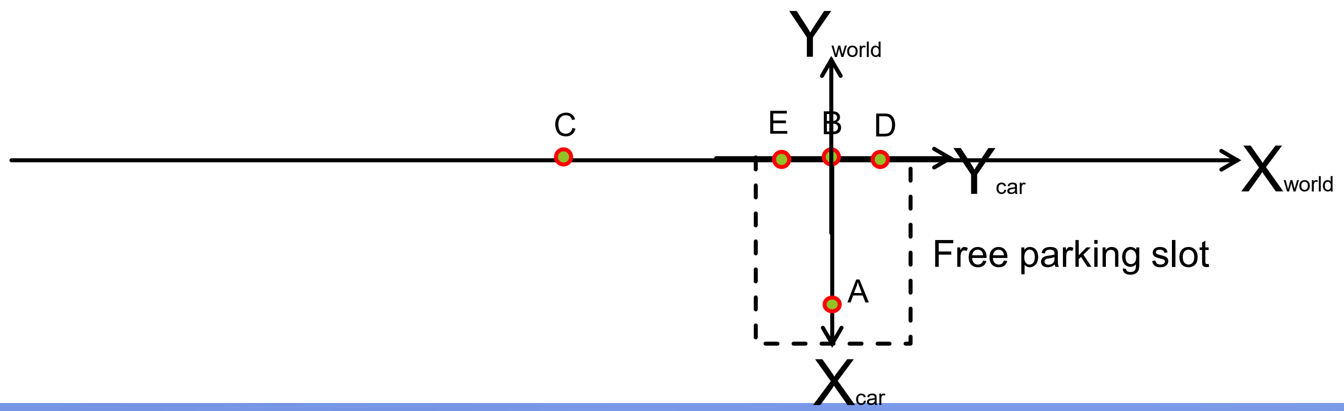
- Step 2: Describe the goal pose (i.e. expected final pose)



Solution (continued)

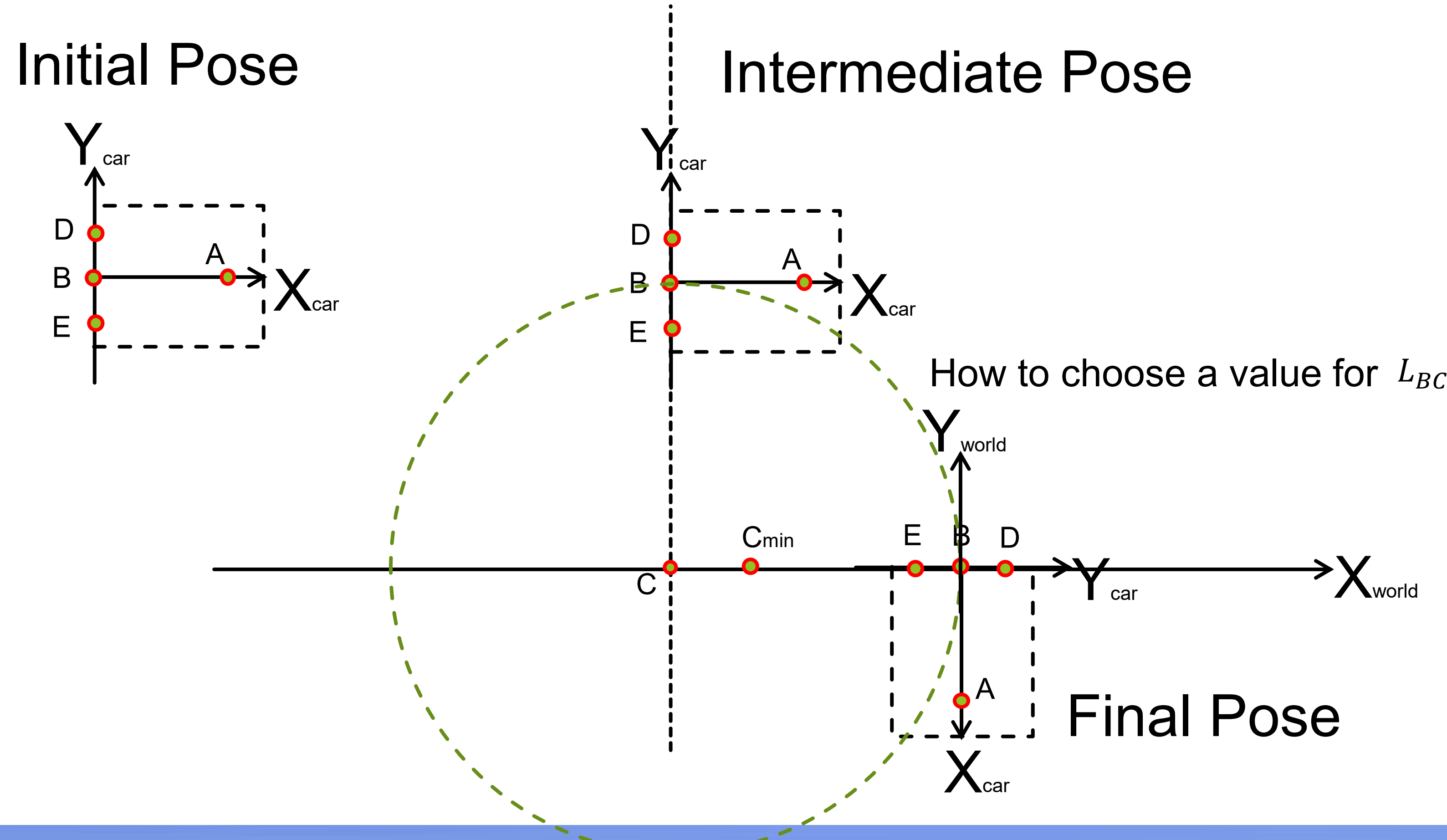
- Step 3: Describe the initial pose (i.e. an ideal initial pose)





Solution (continued)

- Step 4: Describe the intermediate pose (i.e. reverse motion from the final pose to the intermediate pose)

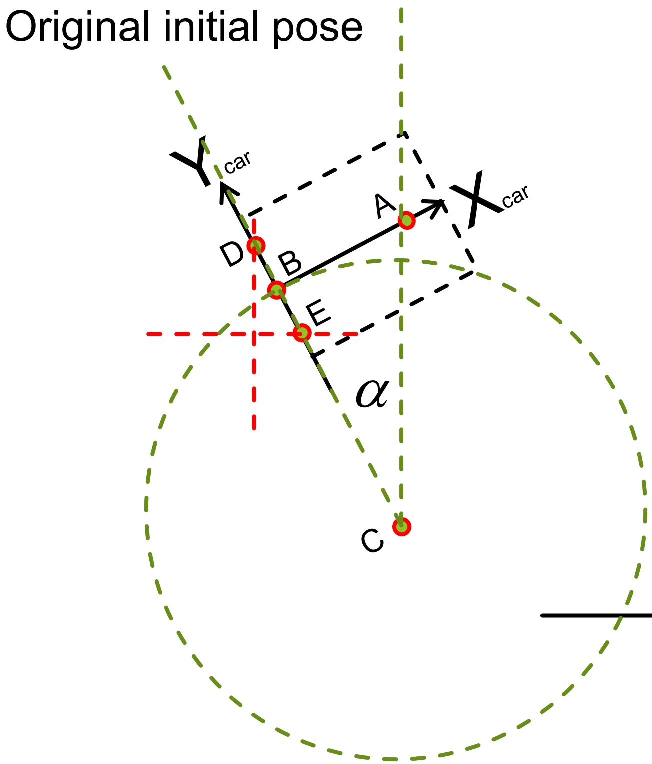


Solution (continued)

- Step 5: Hence, the motion sequence to be performed is:
  - Motion 1:
    - Drive the car from its initial pose to a new initial pose (e.g. all axes are parallel).
  - Motion 2:
    - Drive the car from the new initial pose to an intermediate pose.
  - Motion 3:
    - Drive the car from the intermediate pose to the final pose.

Solution (continued)

- Step 6: Motion 1:



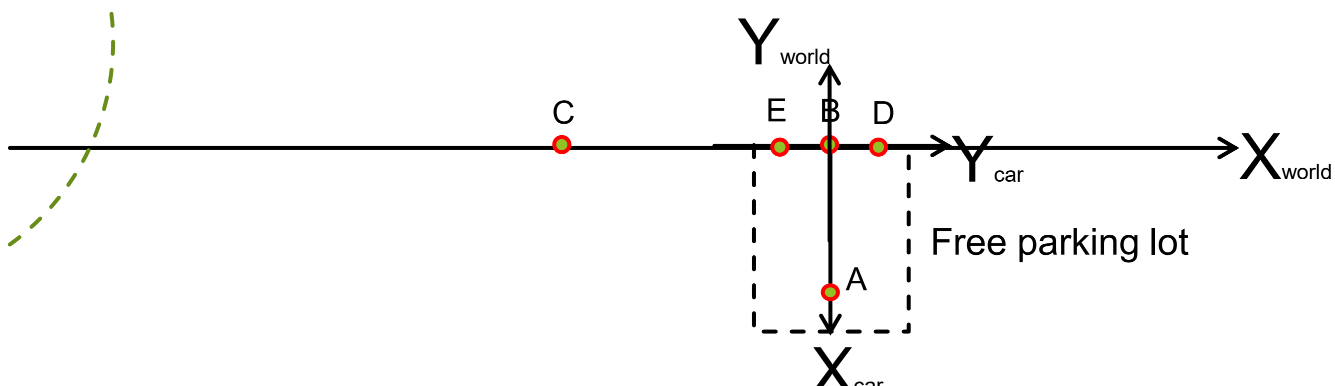
1. Set steering angle to:

$$\alpha = \frac{\pi}{2} - \arctan\left(\frac{Y_D - Y_E}{X_D - X_E}\right)$$

2. Drive the car for a distance of:

$$d = L_{BC} \bullet \alpha$$

with a constant speed of v.



Solution (continued)

- Step 7: Motion 2:

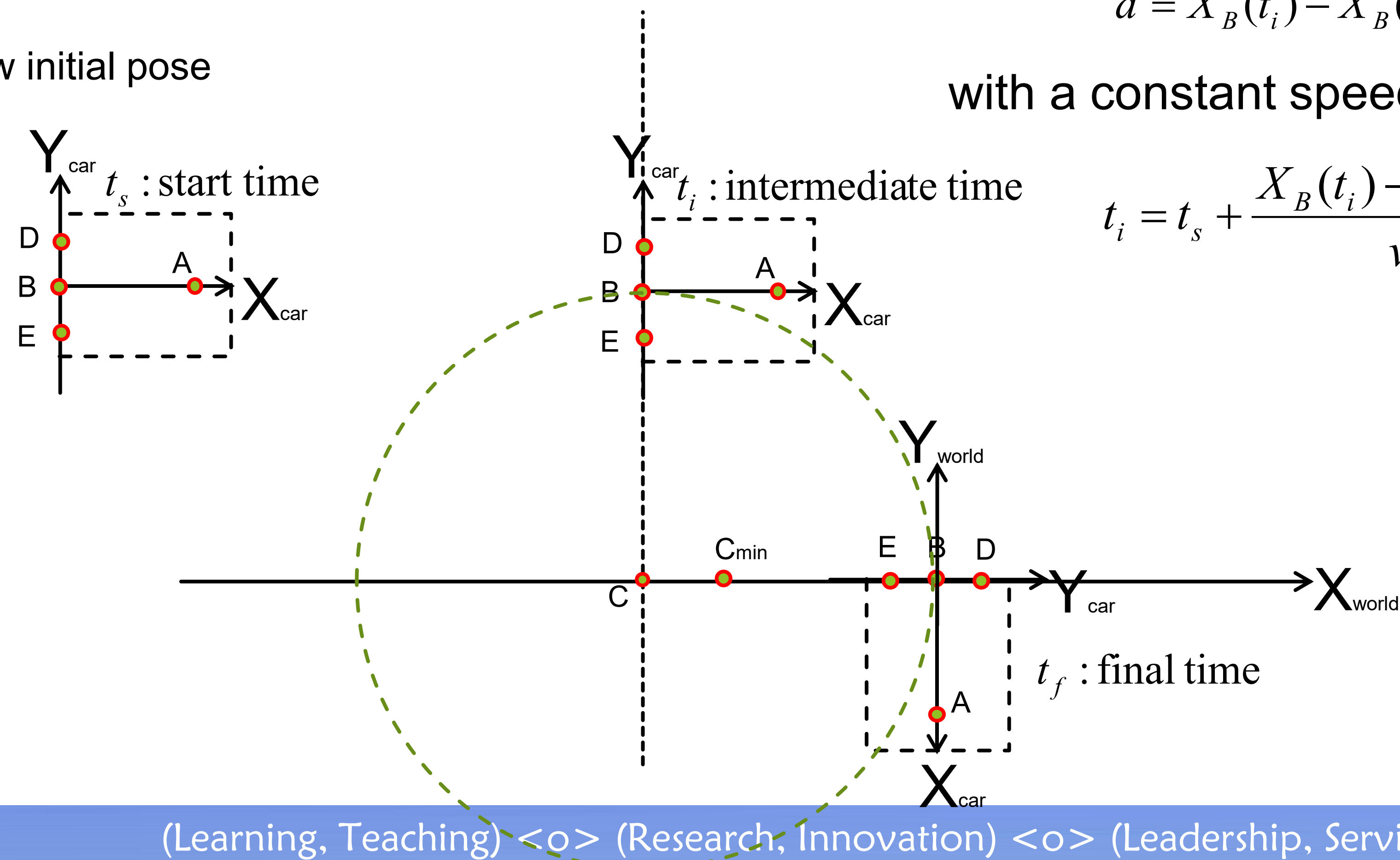
1. Restore steering angle to zero.
2. Drive the car for a distance of:

$$d = X_B(t_i) - X_B(t_s)$$

with a constant speed of *v* until:

$$t_i = t_s + \frac{X_B(t_i) - X_B(t_s)}{v}$$

New initial pose



Solution (continued)

- Step 8: Motion 3:

1. Set steering angle to:

$$\alpha = \arctan\left(\frac{X_A(t_i) - X_B(t_i)}{Y_B(t_i) - Y_C(t_i)}\right)$$

2. Drive the car for a distance of:

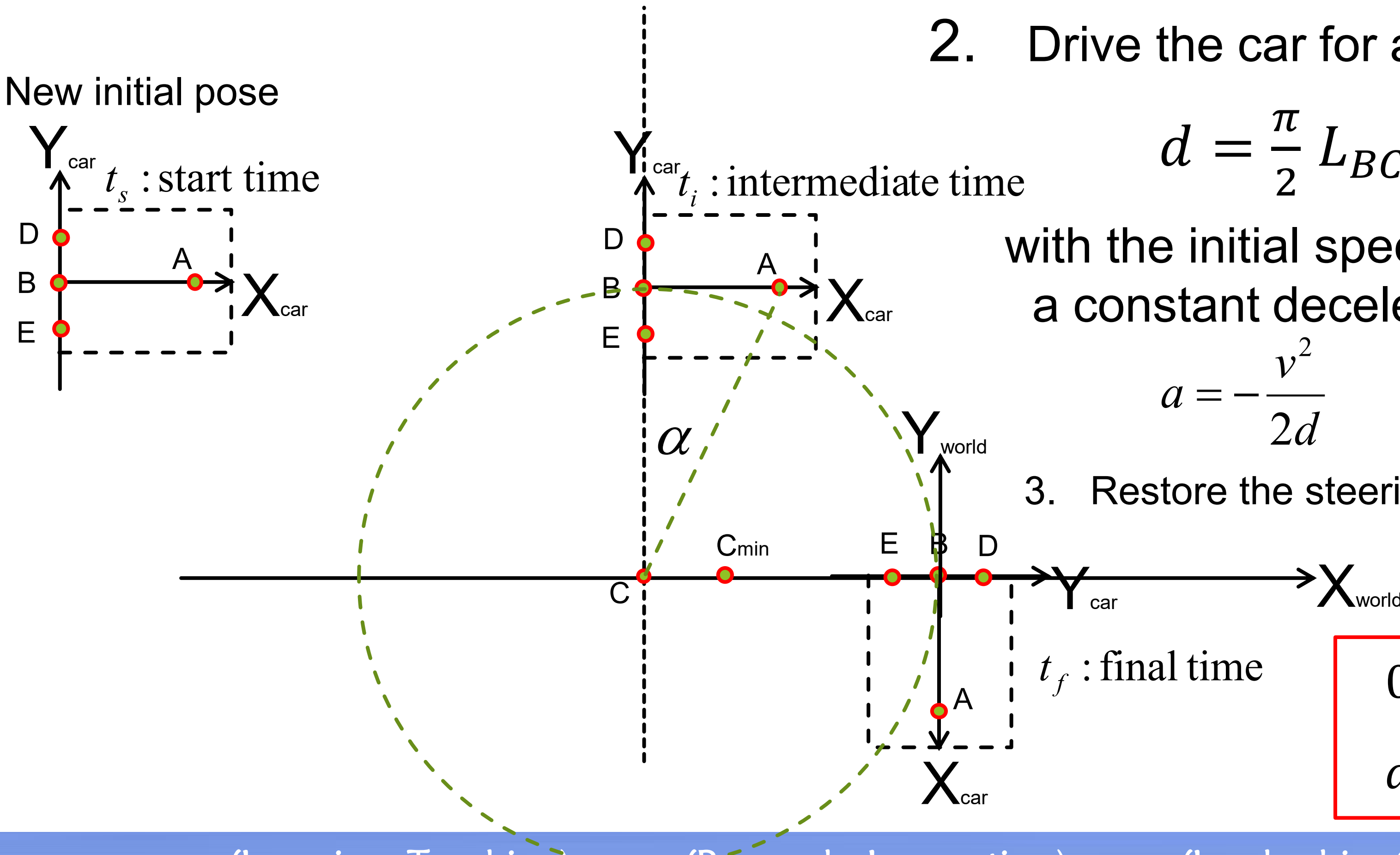
$$d = \frac{\pi}{2} L_{BC}$$

with the initial speed of *v* and a constant deceleration of:

$$a = -\frac{v^2}{2d}$$

3. Restore the steering angle to zero.

New initial pose

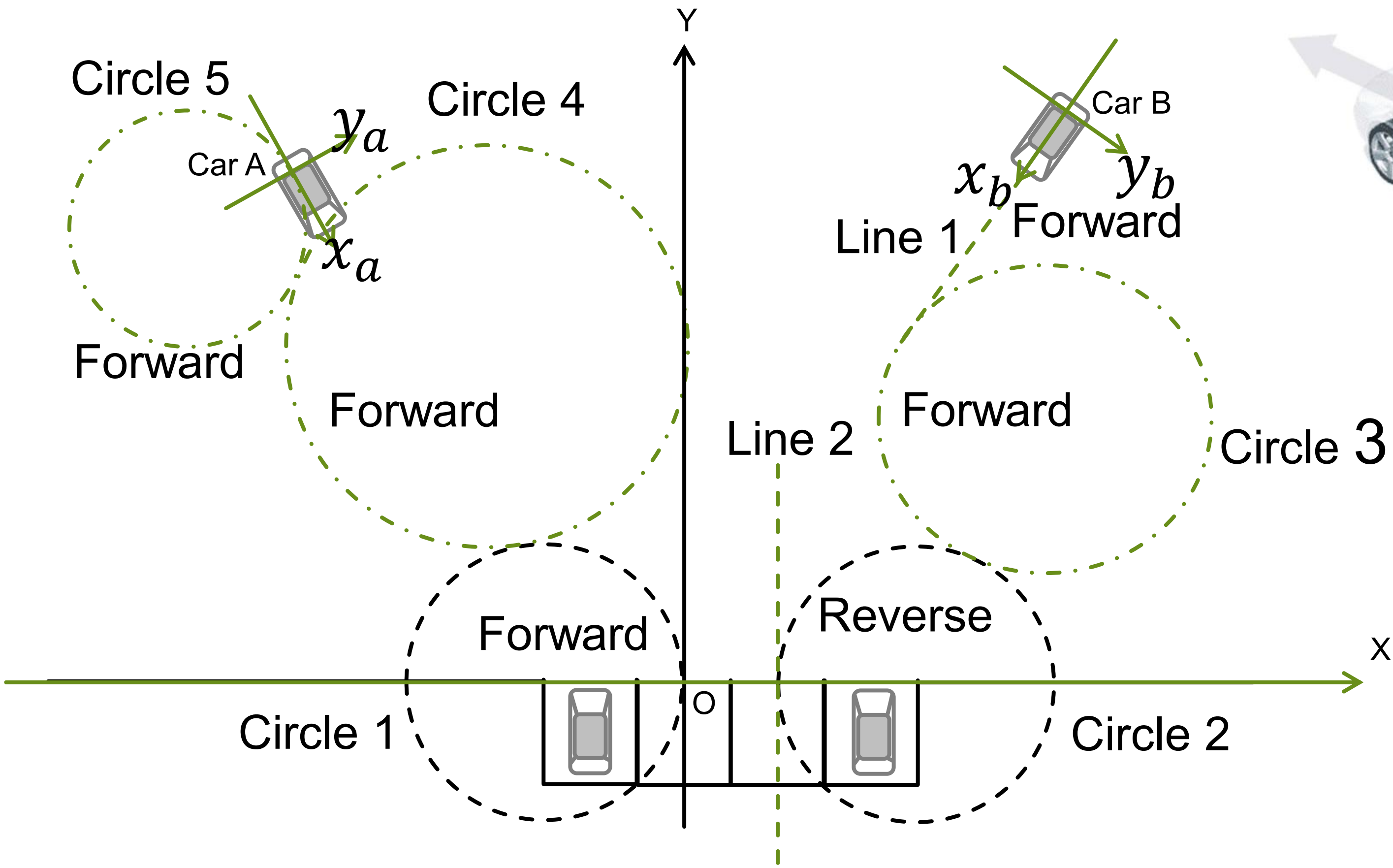


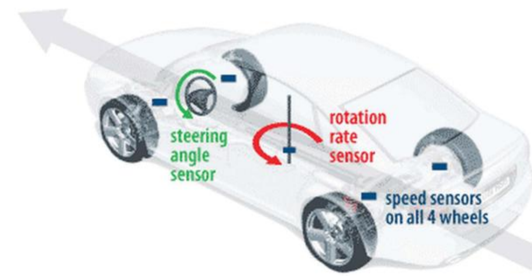
$$0 - v = at$$

$$d = vt + \frac{1}{2}at^2$$

Other Design Solutions …

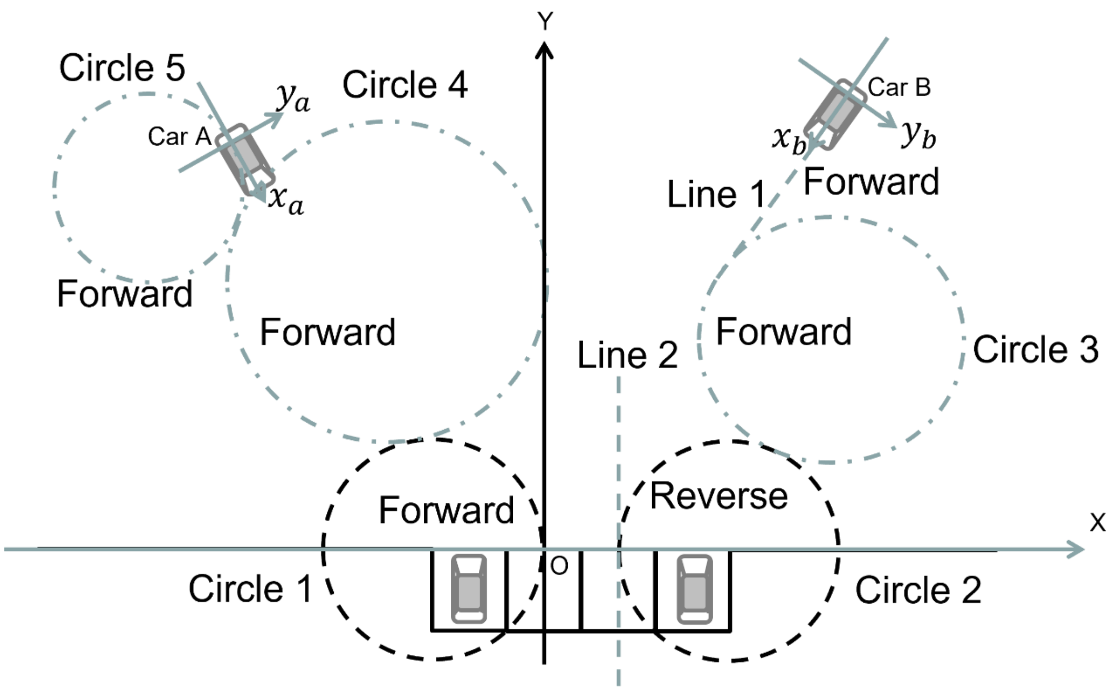
- A carpark has two free parking lots. Each of the two cars is equipped with a monocular vision to measure the pose of chosen free parking lot. What should be the solutions of paths (i.e. spatial locations) for the two cars to follow?





Solution for Car A …

When a car-like vehicle follows a circular trajectory, its steering angle is a constant.



Design algorithm for car A's paths with three motions:

Step 1: Position the world coordinate system at the entry of a free parking lot.

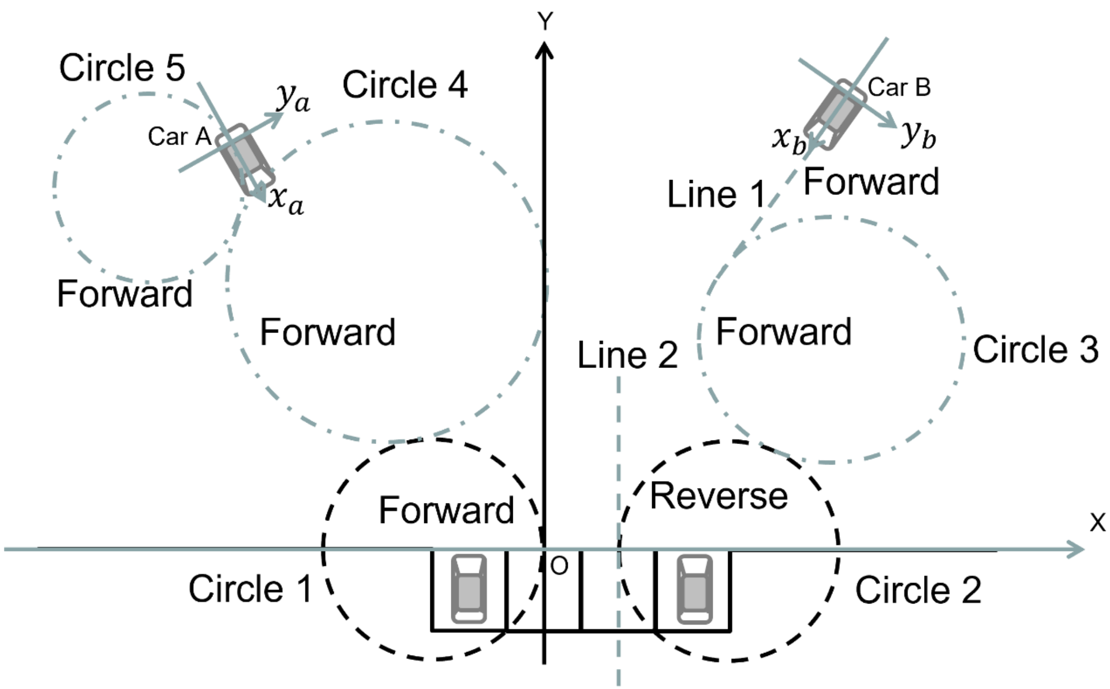
Step 2: Choose and position circle 1.

Step 3: Choose and position circle 5.

Step 4: Choose and position circle 4 which is tangent to circle 1 and circle 5.

Solution for Car B …

When a car-like vehicle follows a circular trajectory, its steering angle is a constant.



Design algorithm for car B's paths with four motions:

Step 1: Position the world coordinate system at the entry of a free parking lot.

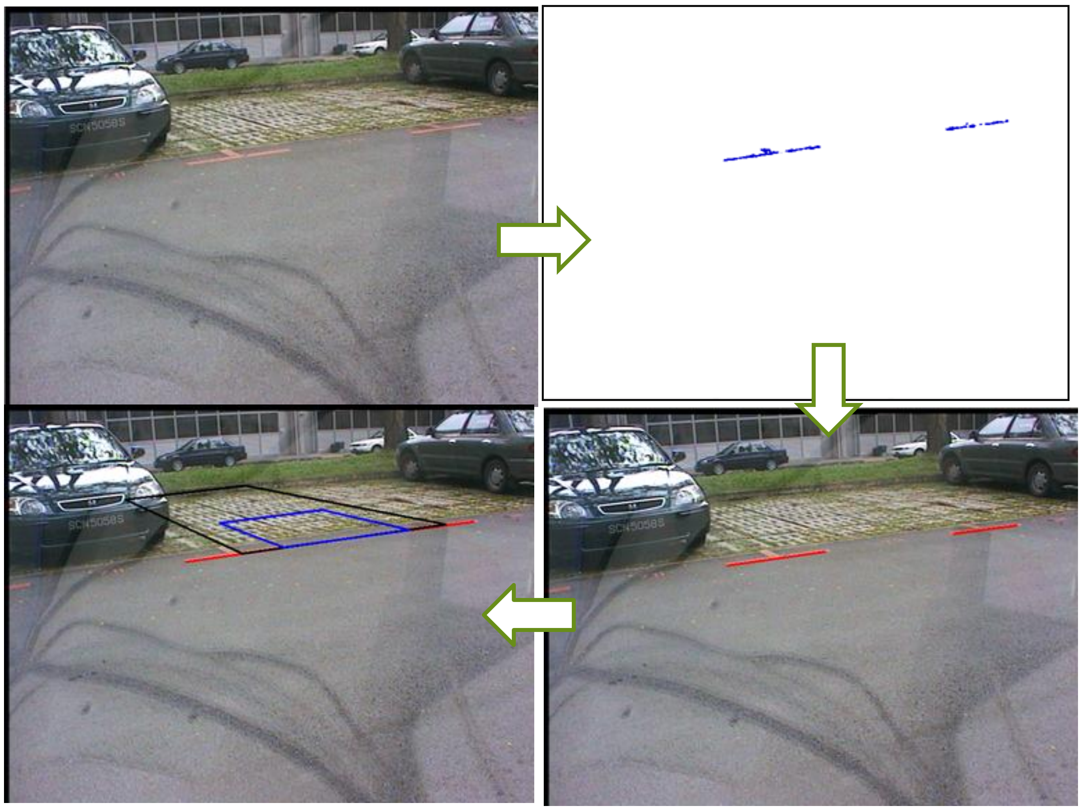
Step 2: Choose and position line 2.

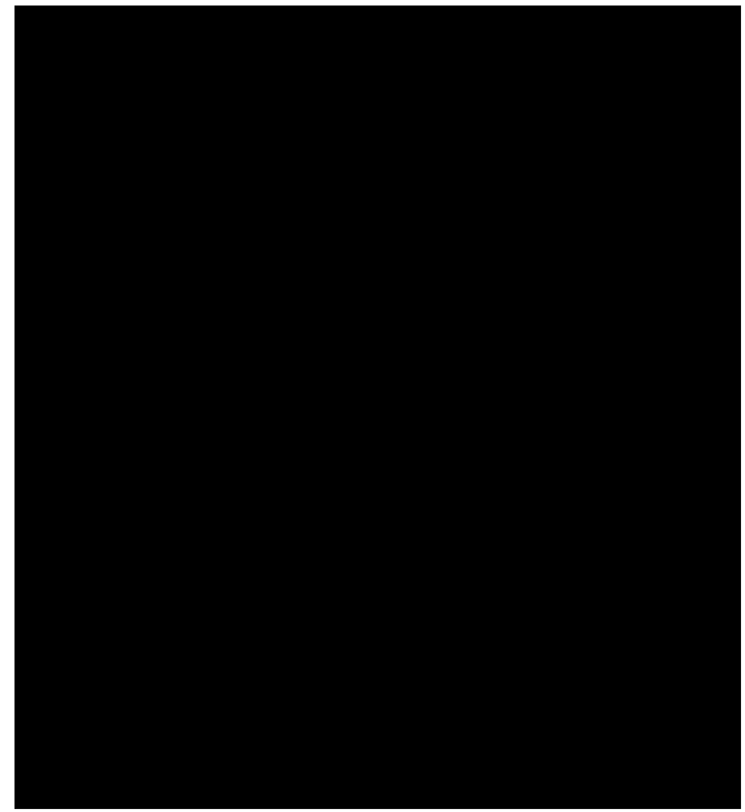
Step 3: Choose and position circle 2 .

Step 4: Choose and position line 1.

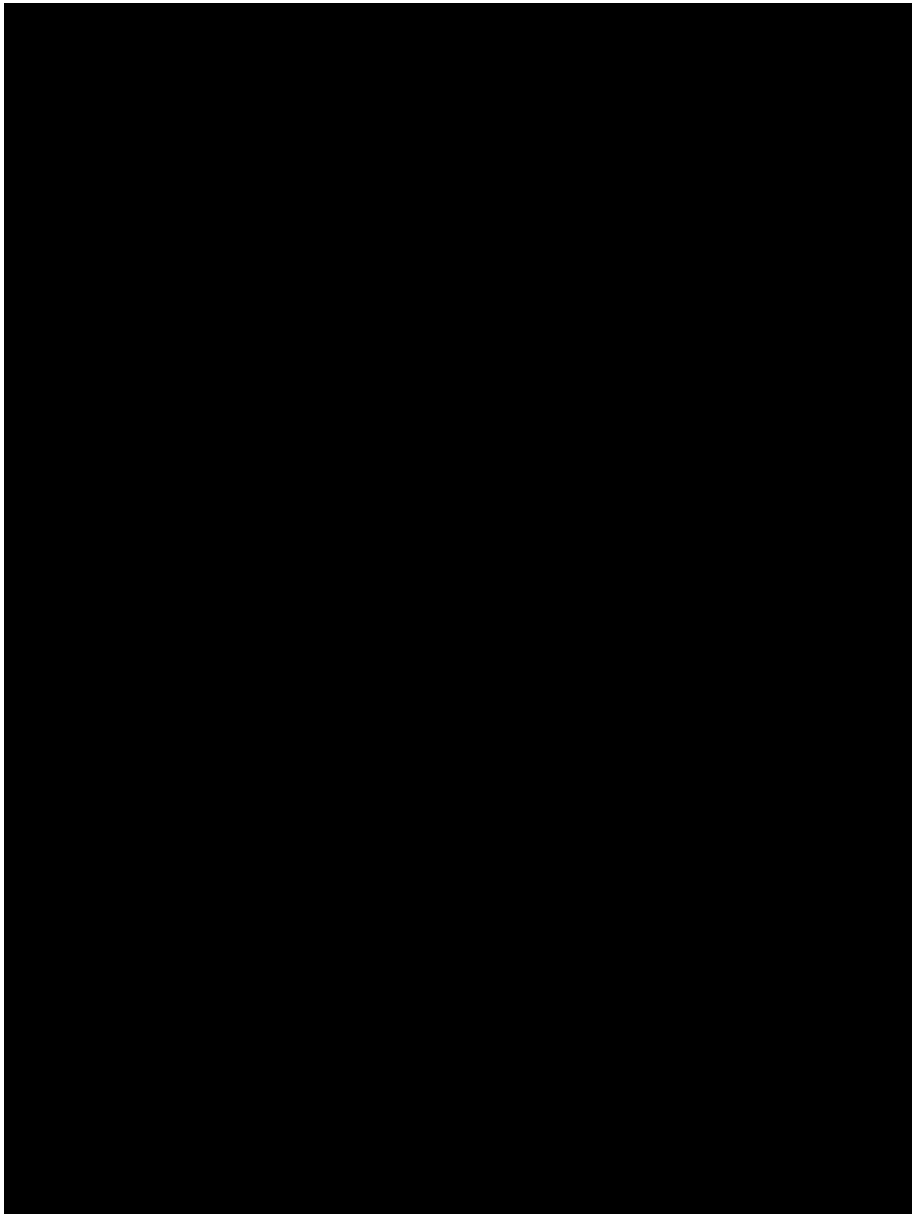
Step 5: Choose and position circle 3 which is tangent to line 1 and circle 2.

Experiments





More experiments …



Summary of Lecture 4

- Background Knowledge
- Purpose of Path Planning
- Input to Path Planning
- Output from Path Planning
- Process of Autonomous Path Planning
  - Case of Arm Manipulators
  - Case of Car-like Mobile Bases

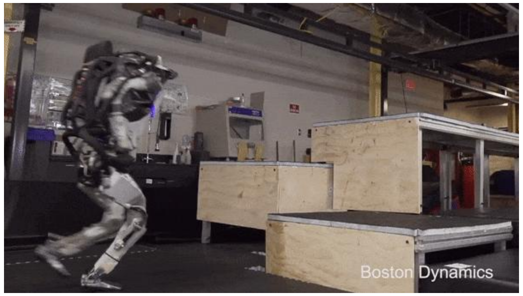
Outline of Module 3

Conceptual World

- Task Planning
- Action Planning
- Motion Planning

Physical World

- Path Planning
- Trajectory Planning



School of Mechanical & Aerospace Engineering

Design, Machine, Control, Intelligence

Module 3

MA4825 Robotics

Lecture 5

Trajectory Planning

Xie Ming, PhD (France)

http://personal.ntu.edu.sg/mmxie

Outline of Lecture 5

Start-Move-Stop Motions

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

- Purpose of Trajectory Planning
- Input to Trajectory Planning
- Output from Trajectory Planning
- Process of Autonomous Trajectory Planning

Outline of Lecture 5

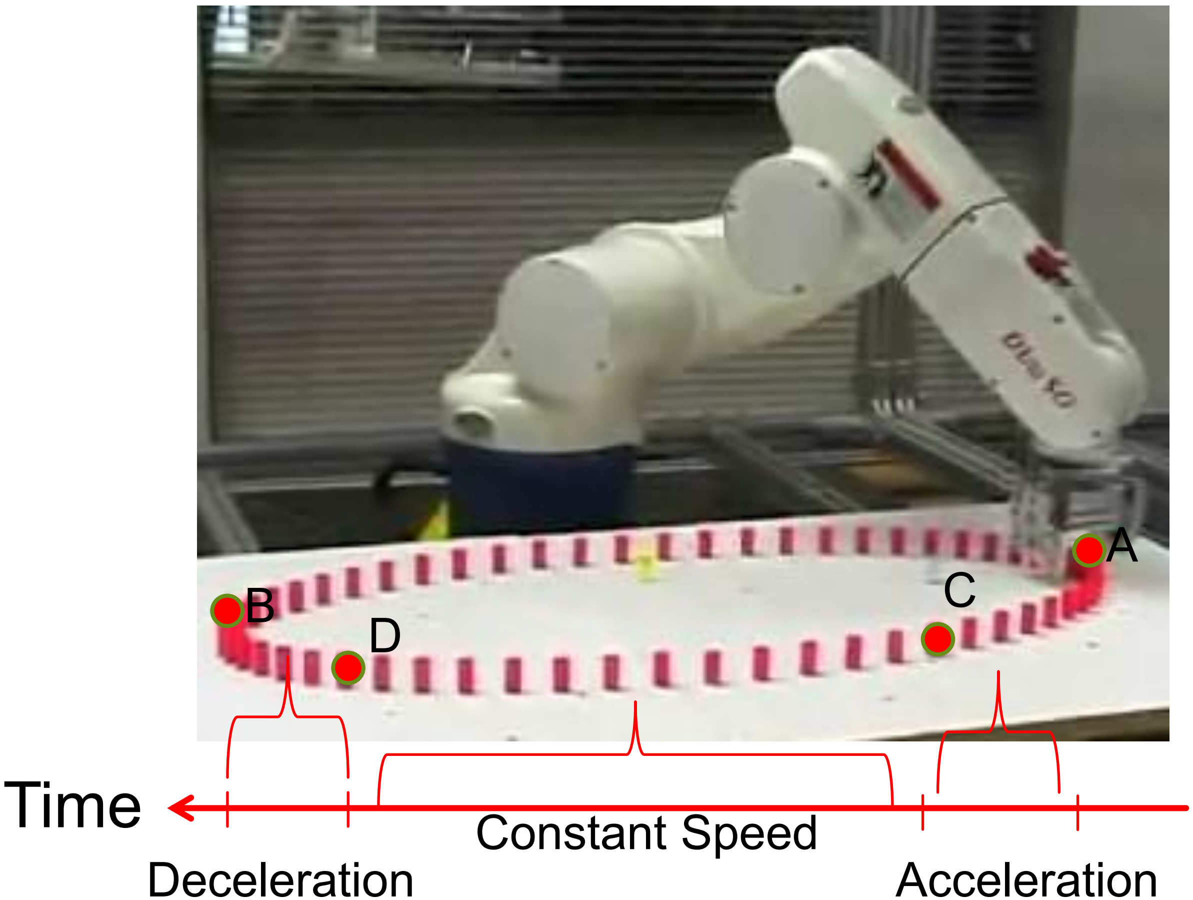
Start-Move-Stop Motions

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

- Purpose of Trajectory Planning
- Input to Trajectory Planning
- Output from Trajectory Planning
- Process of Autonomous Trajectory Planning

Definition of Trajectory

- Trajectory refers to spatial positions and orientations with time constraints. (Note: Non-stop Motions versus End-stopped Motions)



Two Design Schemes:

1. Design with Two Phases (Speed-up and Slow-down)

2. Design with Three Phases (Speed-up, Cruise, and Slow-down)

Rest-to-Velocity

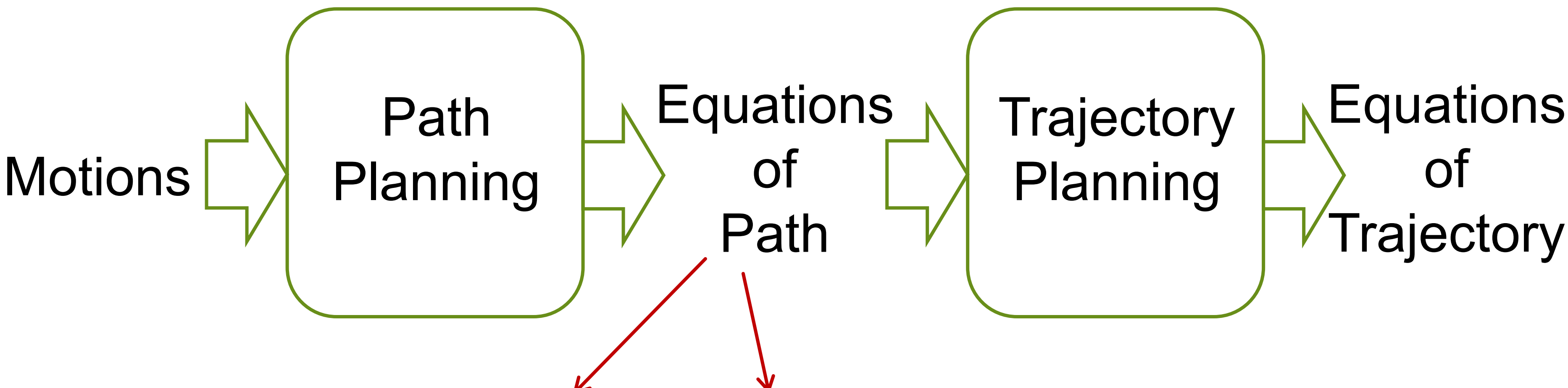
Velocity-to-Velocity

Velocity-to-Rest

Start-Move-Stop Motions

Definition of Trajectory Planning

- Trajectory planning is a process which takes equations of a given path as input and produces equations of trajectory as output.



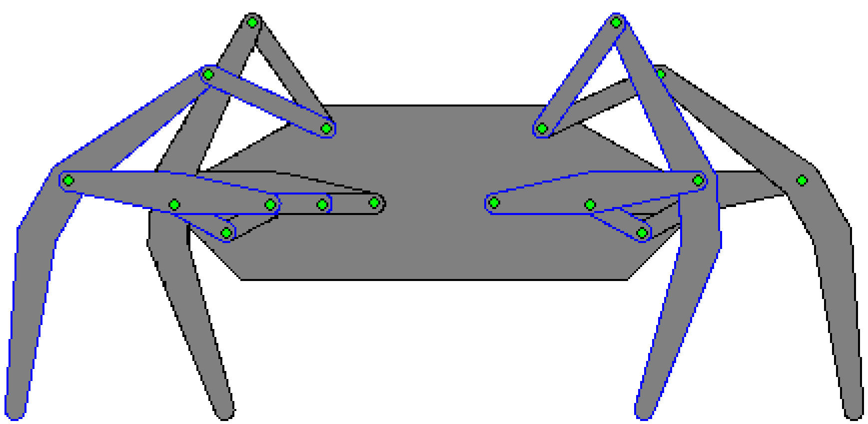
$$y = ax^3 + bx^2 + cx + d$$

$$z = ex^3 + fx^2 + gx + h$$

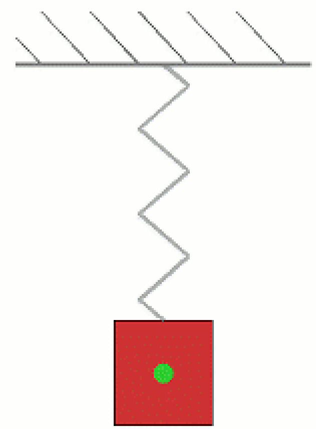
Example of Performing Trajectories

Example of Continuous Motions

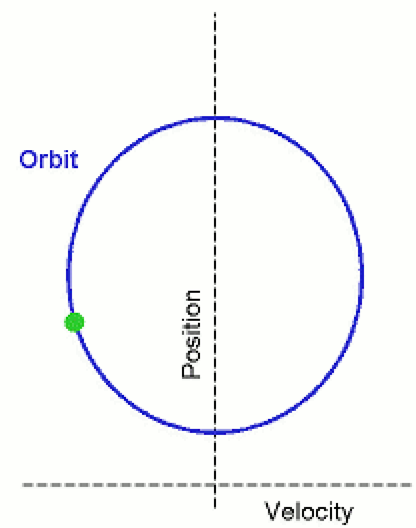
Robot Body's Motion



Robot Hand's Motion



Mobile Base's Motion



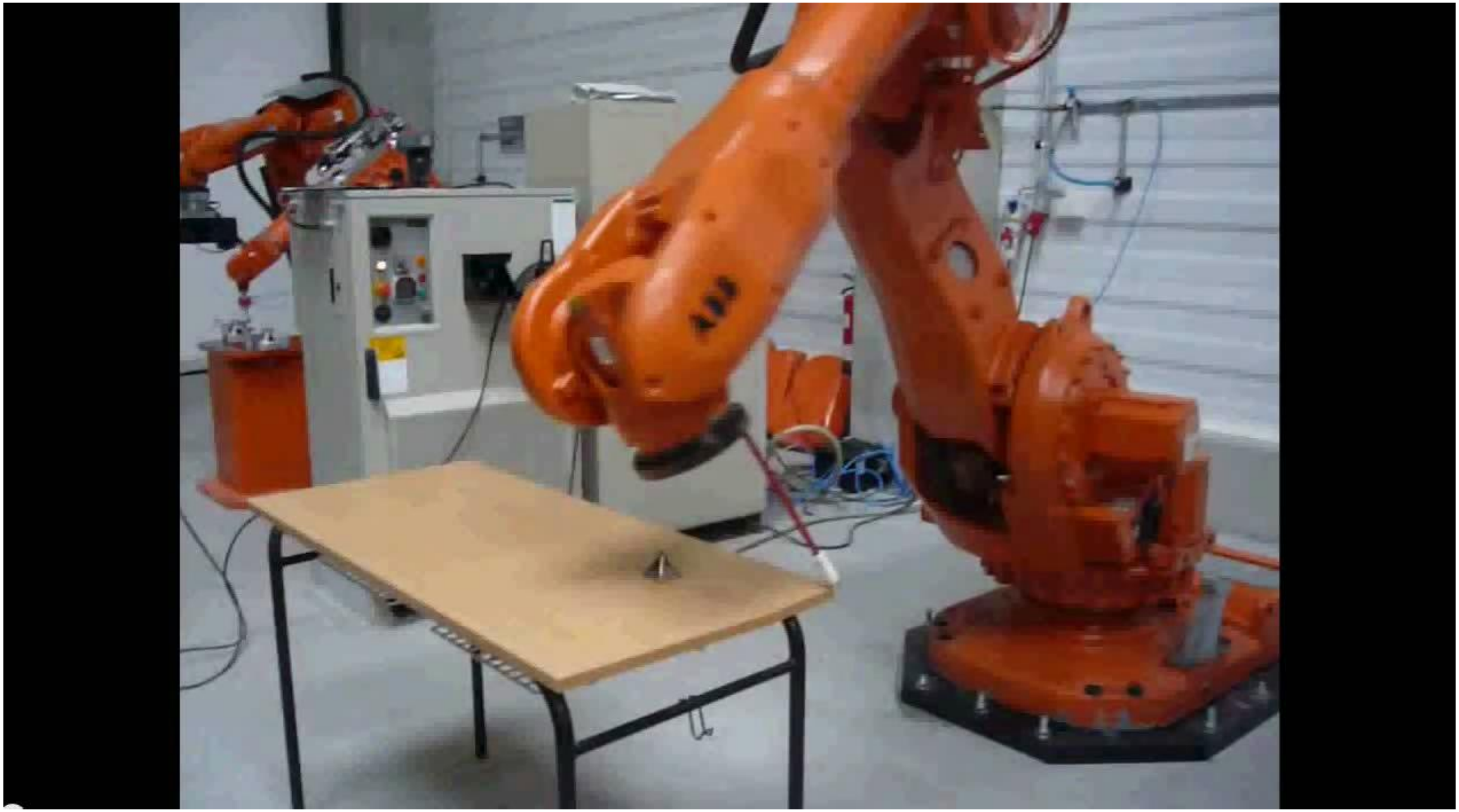
$[x(t), y(t), z(t), \theta(t)]$

Industrial Robot's
Motion Coordinates

$[v(t)\ \theta(t)]$ or $[v_l(t)\ v_r(t)]$

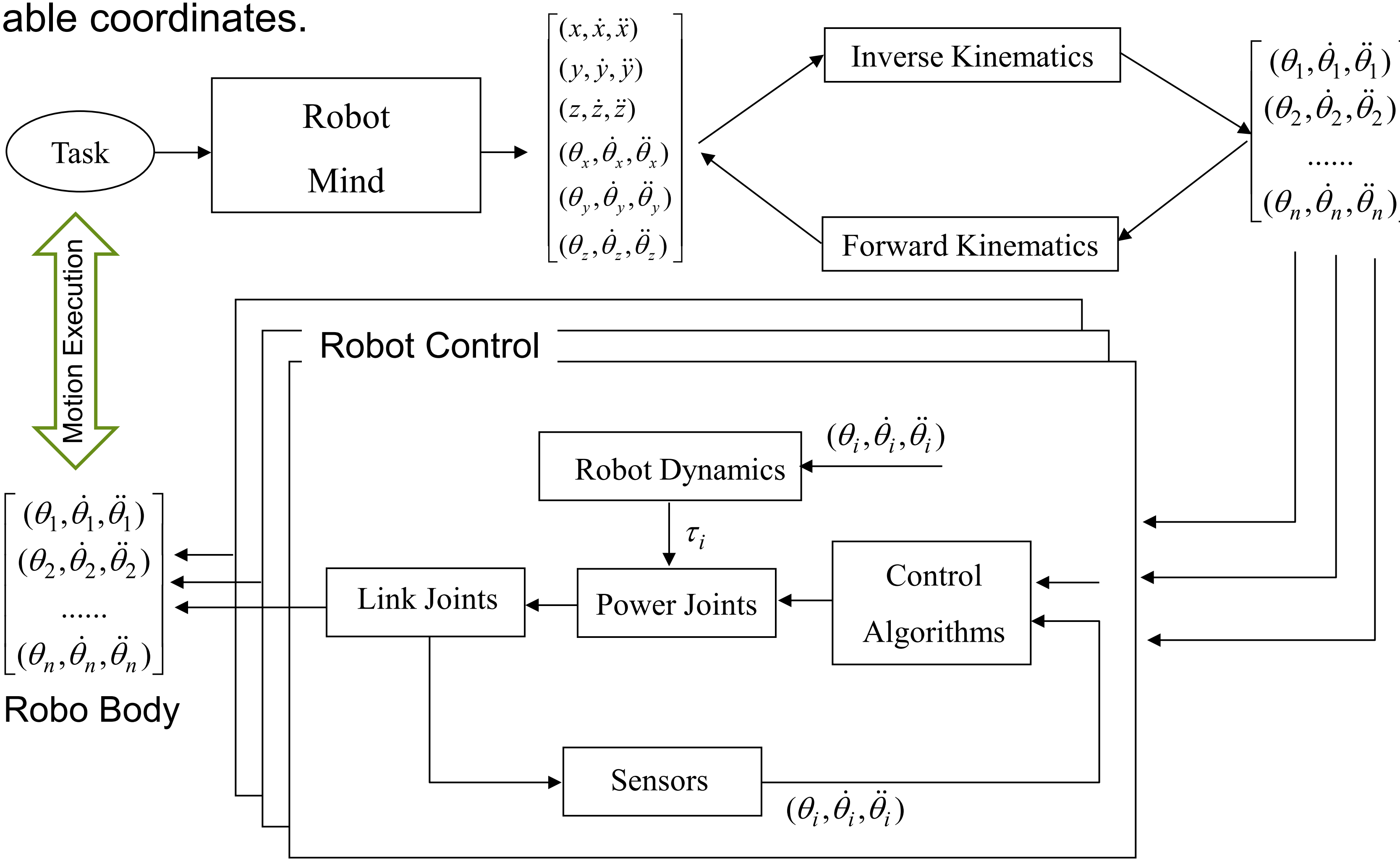
Mobile Robot's
Motion Coordinates

Example of Start-Move-Stop Motions



Why is trajectory planning important?

Answer: To have time functions of controllable coordinates.



Equations of Trajectory of 2D Linear Motion

- Case 1: Constant velocity (i.e., Rest to Velocity)

$$x(t) - x_0 = v_x \bullet (t - t_0)$$
$$y(t) - y_0 = v_y \bullet (t - t_0)$$

$$x(t) = x_0 + v_x \bullet (t - t_0)$$
$$y(t) = y_0 + v_y \bullet (t - t_0)$$

$$v_x = \frac{d}{dt} x(t)$$
$$v_y = \frac{d}{dt} y(t)$$

Equivalent Axis of Rotation:

$$\vec{r}\Big|_{t_1,t_2} = \begin{bmatrix} c_{32} - c_{23} \\ c_{13} - c_{31} \\ c_{21} - c_{12} \end{bmatrix}$$

Between Two Adjacent Poses

Equivalent Angle of Rotation:

$$\theta_{t_1,t_2} = \arccos\left(\frac{c_{11} + c_{22} + c_{33} - 1}{2}\right)$$

$$\theta(t) - 0 = \omega \times (t - t_1)$$

Example of Mobile Base

- A mobile robot moves at a constant speed on a floor. It is observed that the robot passes through point (10.0, 20.0) (cm) at t=1.5 seconds. And, it passes through point (20.0,50.0) (cm) at t=3.5 seconds. What is the equation of the robot's trajectory?
- Answer:

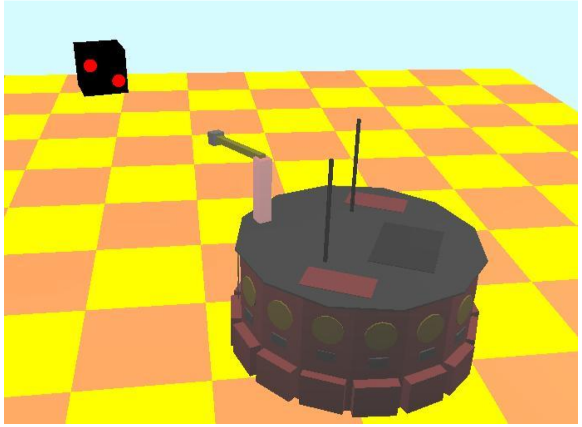
$$x(t) = v_x \bullet (t - 1.5) + 10.0$$
$$y(t) = v_y \bullet (t - 1.5) + 20.0$$

$$20.0 = v_x \bullet (3.5 - 1.5) + 10.0$$
$$50.0 = v_y \bullet (3.5 - 1.5) + 20.0$$

$$v_x = 5.0 \text{ cm/s}$$
$$v_y = 15.0 \text{ cm/s}$$

Equation of the robot's trajectory is:

$$x(t) = 5.0 \bullet (t - 1.5) + 10.0$$
$$y(t) = 15.0 \bullet (t - 1.5) + 20.0$$



Equations of Trajectory of 2D Linear Motion

- Case 2: Constant acceleration (i.e., Rest to Velocity)

$$x(t) - x_0 = \frac{1}{2} a_x \bullet (t - t_0)^2 + v_x \bullet (t - t_0)$$

$$y(t) - y_0 = \frac{1}{2} a_y \bullet (t - t_0)^2 + v_y \bullet (t - t_0)$$

$$a_x = \frac{d^2}{dt^2} x(t)$$

$$a_y = \frac{d^2}{dt^2} y(t)$$

Equivalent Axis of Rotation :

$$\vec{r}\Big|_{t_1,t_2} = \begin{bmatrix} c_{32} - c_{23} \\ c_{13} - c_{31} \\ c_{21} - c_{12} \end{bmatrix}$$

Between Two Adjacent Poses

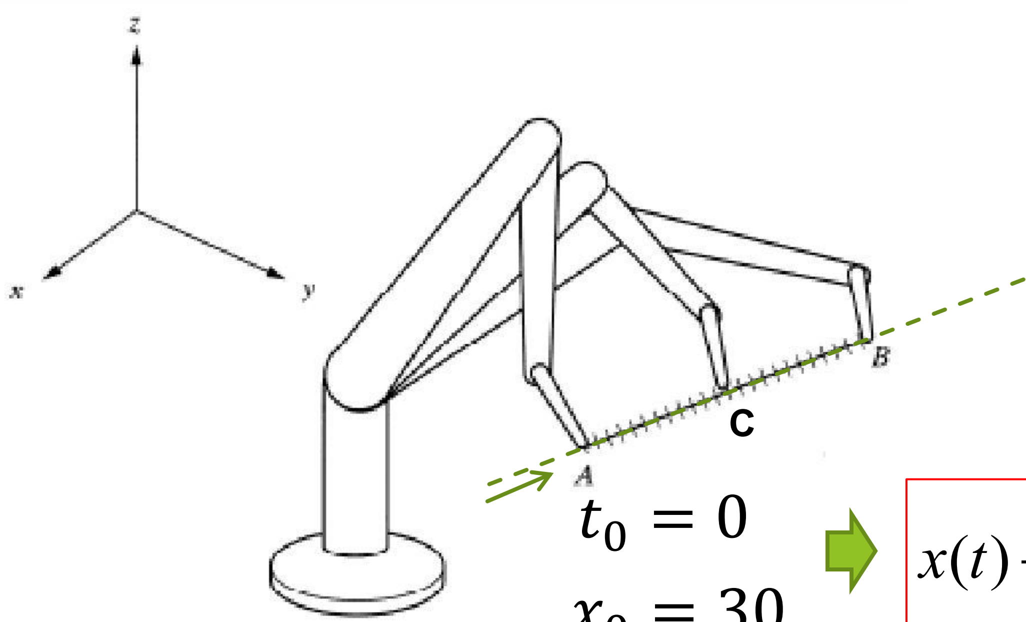
Equivalent Angle of Rotation :

$$\theta_{t_1,t_2} = \arccos\left(\frac{c_{11} + c_{22} + c_{33} - 1}{2}\right)$$

$$\theta(t) - 0 = \frac{1}{2} \alpha \times (t - t_1)^2$$

Example of Arm Manipulator

- A robot manipulator moves its tool tip with a constant acceleration. We start to observe the motion when the tool tip passes through point A(30.0,30.0,30.0) (cm). Then, we notice that the tool tip passes though point C(0.0, 50.0, 35.0) (cm) at t=2.5 seconds. And, its reaches point B(-30, 70.0, 40.0) (cm) at t=3.5 seconds. What is the equation of the trajectory of the tool tip's X coordinate?



$$t_0 = 0$$
$$x_0 = 30$$

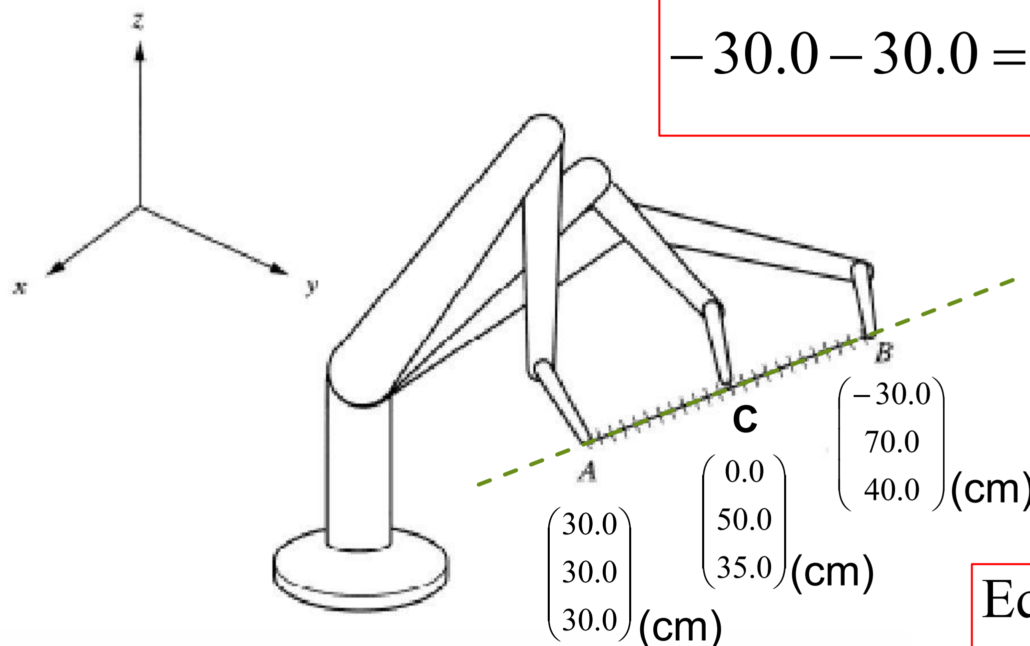
$$x(t) - x_0 = \frac{1}{2} a_x \bullet (t - t_0)^2 + v_x \bullet (t - t_0)$$

Solution

- Answer:

$$0.0-30.0=\frac{1}{2}a_x\bullet(2.5-0.0)^2+v_x\bullet(2.5-0.0)$$

$$-30.0-30.0=\frac{1}{2}a_x\bullet(3.5-0.0)^2+v_x\bullet(3.5-0.0)$$



$$a_x=-10.28\ \mathrm{cm/s^2}$$
$$v_x=0.86\ \mathrm{cm/s}$$

Equation of X coodinate's trajectory is :

$$x(t)=-5.14t^2+0.86t+30.0\ \mathrm{(cm)}$$

Equations of Trajectory of 3D Linear Motion

- Case 3: Variable acceleration (i.e., Rest to Velocity)

$$x(t) = a_x t^3 + b_x t^2 + c_x t + d_x$$
$$y(t) = a_y t^3 + b_y t^2 + c_y t + d_y$$
$$z(t) = a_z t^3 + b_z t^2 + c_z t + d_z$$

Equivalent Axis of Rotation :

$$\vec{r}\big|_{t_1,t_2} = \begin{bmatrix} c_{32} - c_{23} \\ c_{13} - c_{31} \\ c_{21} - c_{12} \end{bmatrix}$$

Between Two Adjacent Poses

Equivalent Angle of Rotation :

$$\theta_{t_1,t_2} = \arccos\left(\frac{c_{11} + c_{22} + c_{33} - 1}{2}\right)$$

$$\theta(t) - 0 = a_3 t^3 + a_2 t^2 + a_1 t + a_0$$

Example of Arm Manipulator

- A robot manipulator moves its tool tip along a straight line. The equations of its tool tip are in the form of polynomials of order 3.

$$x(t) = a_x t^3 + b_x t^2 + c_x t + d_x$$
$$y(t) = a_y t^3 + b_y t^2 + c_y t + d_y$$
$$z(t) = a_z t^3 + b_z t^2 + c_z t + d_z$$

- What are the velocities and accelerations of the tool tip?
- Answer:

$$v_x(t) = \frac{d}{dt} x(t) = 3a_x t^2 + 2b_x t + c_x$$
$$v_y(t) = \frac{d}{dt} y(t) = 3a_y t^2 + 2b_y t + c_y$$
$$v_z(t) = \frac{d}{dt} z(t) = 3a_z t^2 + 2b_z t + c_z$$

$$a_x(t) = \frac{d^2}{dt^2} x(t) = 6a_x t + 2b_x$$
$$a_y(t) = \frac{d^2}{dt^2} y(t) = 6a_y t + 2b_y$$
$$a_z(t) = \frac{d^2}{dt^2} z(t) = 6a_z t + 2b_z$$

Equations of Trajectory of Curved Motion

- Case 1: Time Functions of Polynomials of Degree 2 (i.e., Rest to Velocity)

$$x(t) = a_x t^2 + b_x t + c_x$$
$$y(t) = a_y t^2 + b_y t + c_y$$
$$z(t) = a_z t^2 + b_z t + c_z$$

$$P = At^2 + Bt + C$$

Equivalent Axis of Rotation :

$$\vec{r}\big|_{t_1,t_2} = \begin{bmatrix} c_{32} - c_{23} \\ c_{13} - c_{31} \\ c_{21} - c_{12} \end{bmatrix}$$

Between Two Adjacent Poses

Equivalent Angle of Rotation :

$$\theta_{t_1,t_2} = \arccos\left(\frac{c_{11} + c_{22} + c_{33} - 1}{2}\right)$$

$$\theta(t) - 0 = a_2 t^2 + a_1 t + a_0$$

Equations of Trajectory of Curved Motion

- Case 2: Time Functions of Polynomials of Degree 3 (i.e., Rest to Velocity)

$$x(t) = a_x t^3 + b_x t^2 + c_x t + d_x$$
$$y(t) = a_y t^3 + b_y t^2 + c_y t + d_y$$
$$z(t) = a_z t^3 + b_z t^2 + c_z t + d_z$$

$$P = At^3 + Bt^2 + Ct + D$$

Equivalent Axis of Rotation:

$$\vec{r}\Big|_{t_1,t_2} = \begin{bmatrix} c_{32} - c_{23} \\ c_{13} - c_{31} \\ c_{21} - c_{12} \end{bmatrix}$$

Between Two Adjacent Poses

Equivalent Angle of Rotation:

$$\theta_{t_1,t_2} = \arccos\left(\frac{c_{11} + c_{22} + c_{33} - 1}{2}\right)$$

$$\theta(t) - 0 = a_3 t^3 + a_2 t^2 + a_1 t + a_0$$

Equations of Trajectory of Curved Motion

- Case 3: Time Functions of Polynomials of Degree 5 (i.e., Rest to Velocity)

$$x(t) = a_x t^5 + b_x t^4 + c_x t^3 + d_x t^2 + e_x t + f_x$$
$$y(t) = a_y t^5 + b_y t^4 + c_y t^3 + d_y t^2 + e_y t + f_y$$
$$z(t) = a_z t^5 + b_z t^4 + c_z t^3 + d_z t^2 + e_z t + f_z$$

$$P = At^5 + Bt^4 + Ct^3 + Dt^2 + Et + F$$

Equivalent Axis of Rotation :

$$\vec{r}\Big|_{t_1,t_2} = \begin{bmatrix} c_{32} - c_{23} \\ c_{13} - c_{31} \\ c_{21} - c_{12} \end{bmatrix}$$

Between Two Adjacent Poses

Equivalent Angle of Rotation :

$$\theta_{t_1,t_2} = \arccos\left(\frac{c_{11} + c_{22} + c_{33} - 1}{2}\right)$$

$$\theta(t) - 0 = a_5 t^5 + a_4 t^4 + a_3 t^3 + a_2 t^2 + a_1 t + a_0$$

Outline of Lecture 5

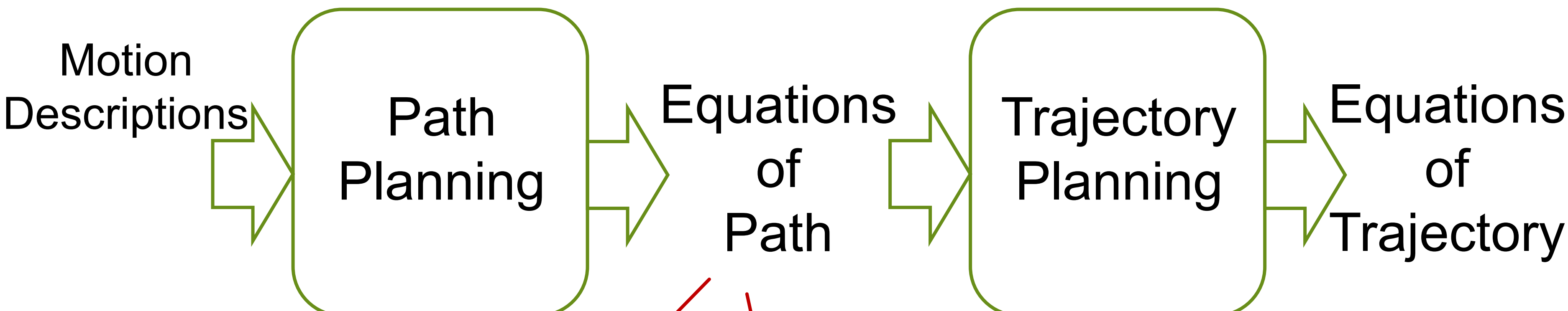
Start-Move-Stop Motions

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

- Purpose of Trajectory Planning
- Input to Trajectory Planning
- Output from Trajectory Planning
- Process of Autonomous Trajectory Planning

Input to Trajectory Planning

- The input to trajectory planning is a set of equations of given paths (spatial constraints) and time constraints in the form of duration.

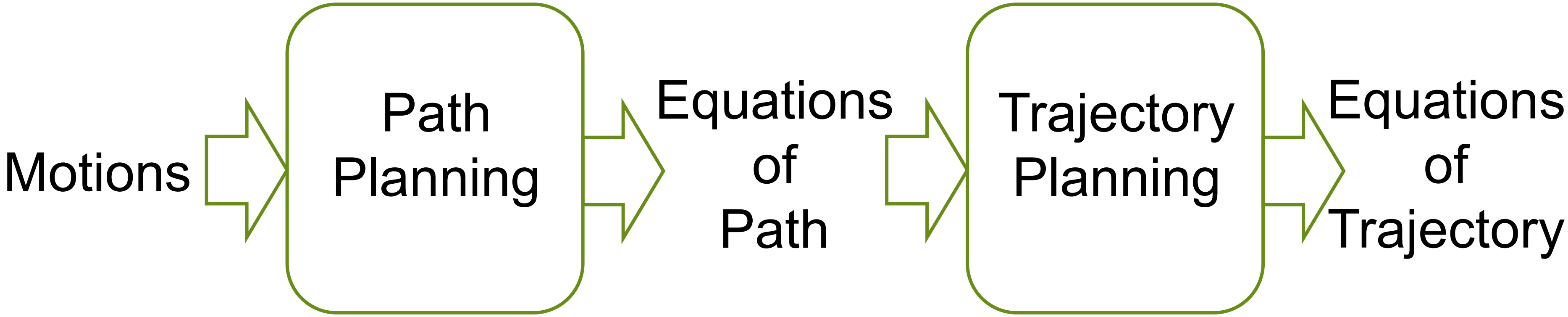


$$y = ax^3 + bx^2 + cx + d$$

$$z = ex^3 + fx^2 + gx + h$$

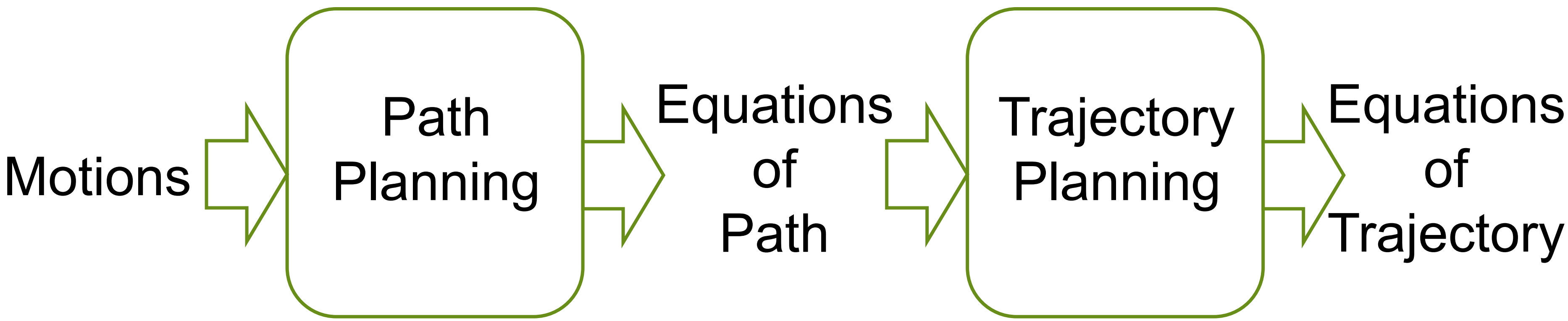
Spatial Constraints

- Linear Paths
- Circular Paths
- Curved Paths



Time Constraints

- Constant accelerations which are zero
- Constant accelerations which are positive
- Constant accelerations which are negative



- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Integration of Spatial and Time Constraints into Library Functions

- MoveTo(Position, Duration, Type)
- MoveWith(Displacement, Duration, Type)
- RotateTo(Angular Position, Duration, Type)
- RotateWith(Angular Displacement, Duration, Type)

Outline of Lecture 5

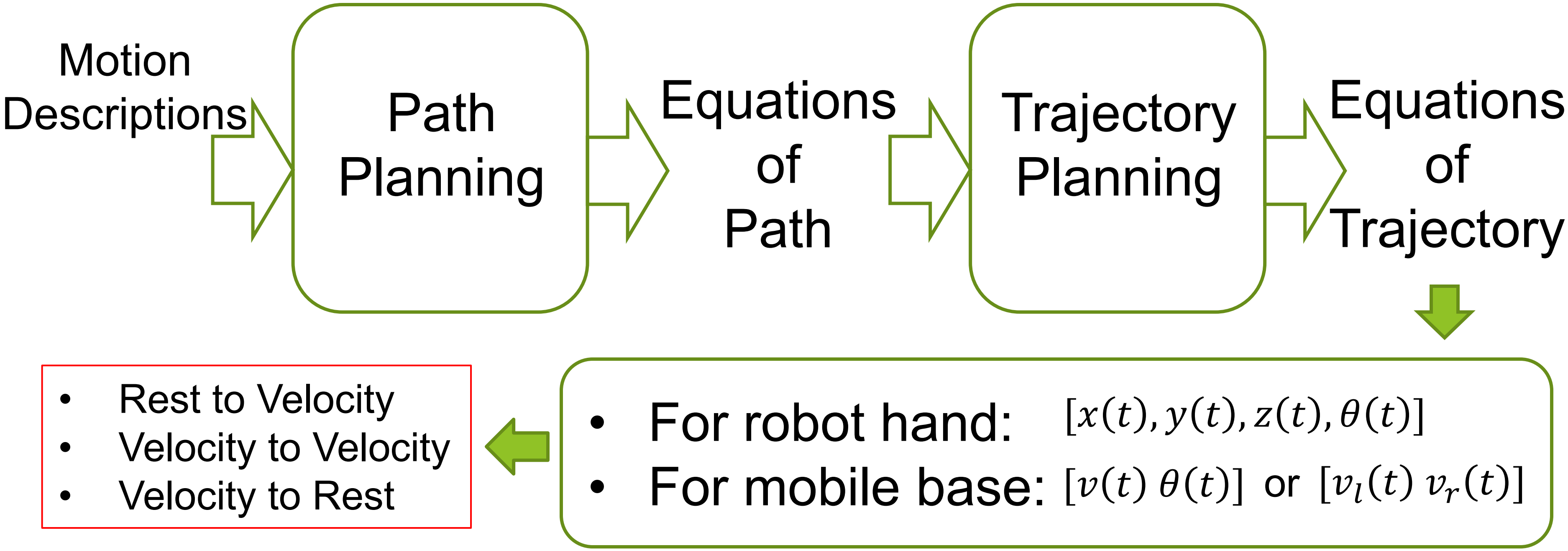
Start-Move-Stop Motions

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

- Purpose of Trajectory Planning
- Input to Trajectory Planning
- Output from Trajectory Planning
- Process of Autonomous Trajectory Planning

Output from Trajectory Planning

- The output of trajectory planning is a set of equations of trajectory with time constraints, which will be the input to robots' control systems.



What are typical types of equations of trajectory?

- Polynomial equations of time of degree 1
- Polynomial equations of time of degree 2
- Polynomial equations of time of degree 3
- Sine and Cosine functions of time, etc.

$$x(t) = a_x t^3 + b_x t^2 + c_x t + d_x$$
$$y(t) = a_y t^3 + b_y t^2 + c_y t + d_y$$
$$z(t) = a_z t^3 + b_z t^2 + c_z t + d_z$$

$$\theta(t) = a_3 t^3 + a_2 t^2 + a_1 t + a_0$$

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

How to determine the coefficients inside the equations of trajectory?

- Use of Continuity in positions
- Use of Continuity in velocities
- Use of Continuity in accelerations

$$x(t) = a_x t^3 + b_x t^2 + c_x t + d_x$$
$$y(t) = a_y t^3 + b_y t^2 + c_y t + d_y$$
$$z(t) = a_z t^3 + b_z t^2 + c_z t + d_z$$

$$\theta(t) = a_3 t^3 + a_2 t^2 + a_1 t + a_0$$

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Use of Continuity in Positions

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

$$x(t) = a_x t^3 + b_x t^2 + c_x t + d_x$$
$$y(t) = a_y t^3 + b_y t^2 + c_y t + d_y$$
$$z(t) = a_z t^3 + b_z t^2 + c_z t + d_z$$

Coordinates at Known Waypoints

$$x(t) = a_x t^5 + b_x t^4 + c_x t^3 + d_x t^2 + e_x t + f_x$$
$$y(t) = a_y t^5 + b_y t^4 + c_y t^3 + d_y t^2 + e_y t + f_y$$
$$z(t) = a_z t^5 + b_z t^4 + c_z t^3 + d_z t^2 + e_z t + f_z$$

Coordinates at Known Waypoints

Use of Continuity in Velocities

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

$$\dot{x}(t) = 3a_x t^2 + 2b_x t + c_x$$
$$\dot{y}(t) = 3a_y t^2 + 2b_y t + c_y$$
$$\dot{z}(t) = 3a_z t^2 + 2b_z t + c_z$$

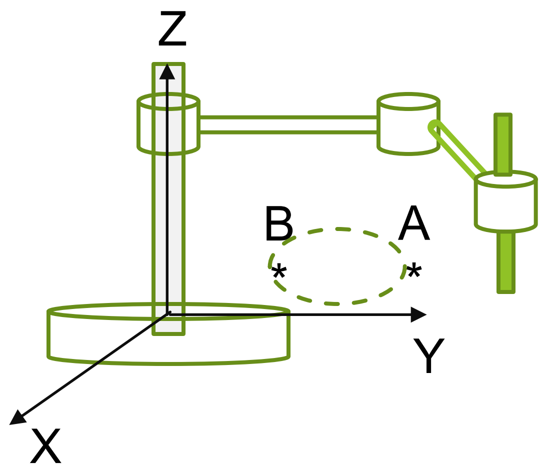
Velocities at Known Waypoints

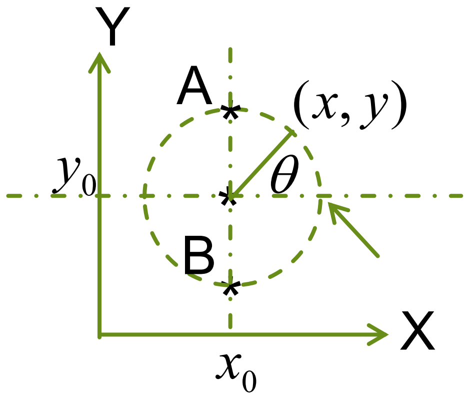
$$\dot{x}(t) = 5a_x t^4 + 4b_x t^3 + 3c_x t^2 + 2d_x t + e_x$$
$$\dot{y}(t) = 5a_y t^4 + 4b_y t^3 + 3c_y t^2 + 2d_y t + e_y$$
$$\dot{z}(t) = 5a_z t^4 + 4b_z t^3 + 3c_z t^2 + 2d_z t + e_z$$

Velocities at Known Waypoints

Example of Continuous Motion

- A SCARA robot moves its tool tip from point A to point B, and from point B back to point A, continuously by following a circle with a radius of R. And, the circle is inside a horizontal plane. The centre of the circle is at position (x0, y0, z0). If the tool tip's motion is a uniform circular motion with a circular speed of v0, what are the equations of the tool tip's trajectory?





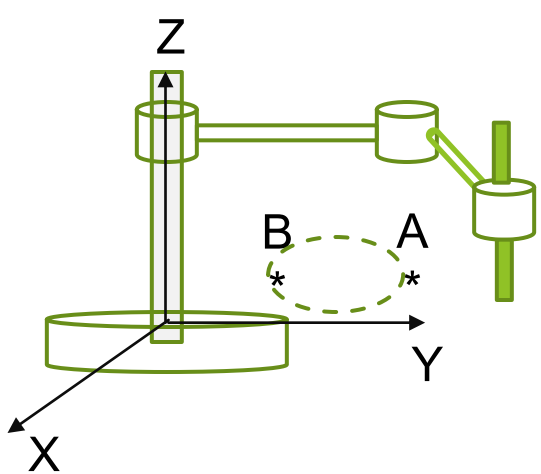
$$\omega = \frac{v}{R}$$

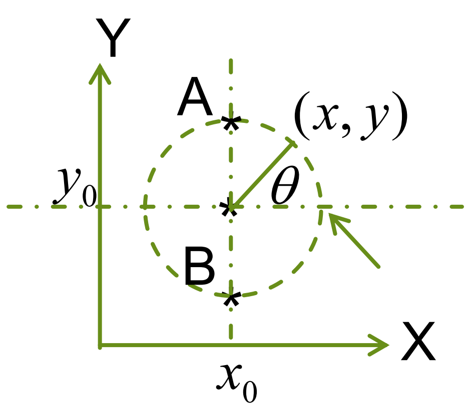
$$\theta = \omega t$$

Solution

- Analysis:
  - The tool tip's motion is an uniform circular motion.
  - The tangential acceleration is zero.
  - The centre of the circle is at (x0, y0, z0).
  - The circular speed is v0.
  - The rotated angle about the circle's centre and the angular velocity are:

$$\theta = \frac{\pi}{2} - \omega \bullet t \ \text{ and } \ \omega = \frac{v_0}{R}$$



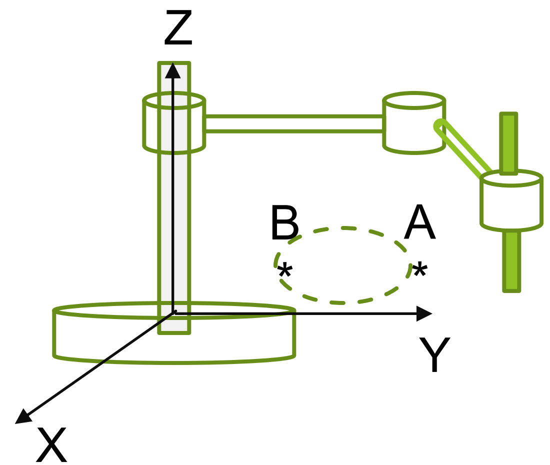


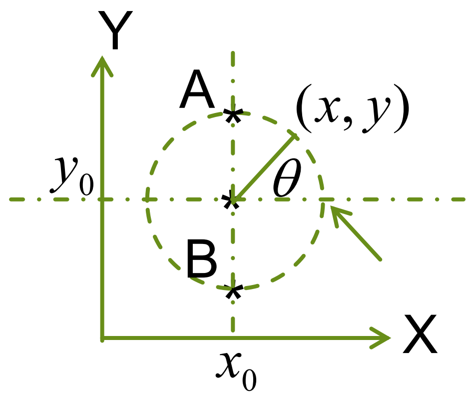
Solution (continued)

- Equations of trajectory along the circle are:

$$\theta = \frac{\pi}{2} - \omega \bullet t \text{ and } \omega = \frac{v_0}{R}$$

$$x(t) = x_0 + R\cos(\frac{\pi}{2} - \omega \bullet t) = x_0 + R\cos(\frac{\pi}{2} - v_0 t/R)$$
$$y(t) = y_0 + R\sin(\frac{\pi}{2} - \omega \bullet t) = y_0 + R\sin(\frac{\pi}{2} - v_0 t/R)$$
$$z(t) = z_0$$



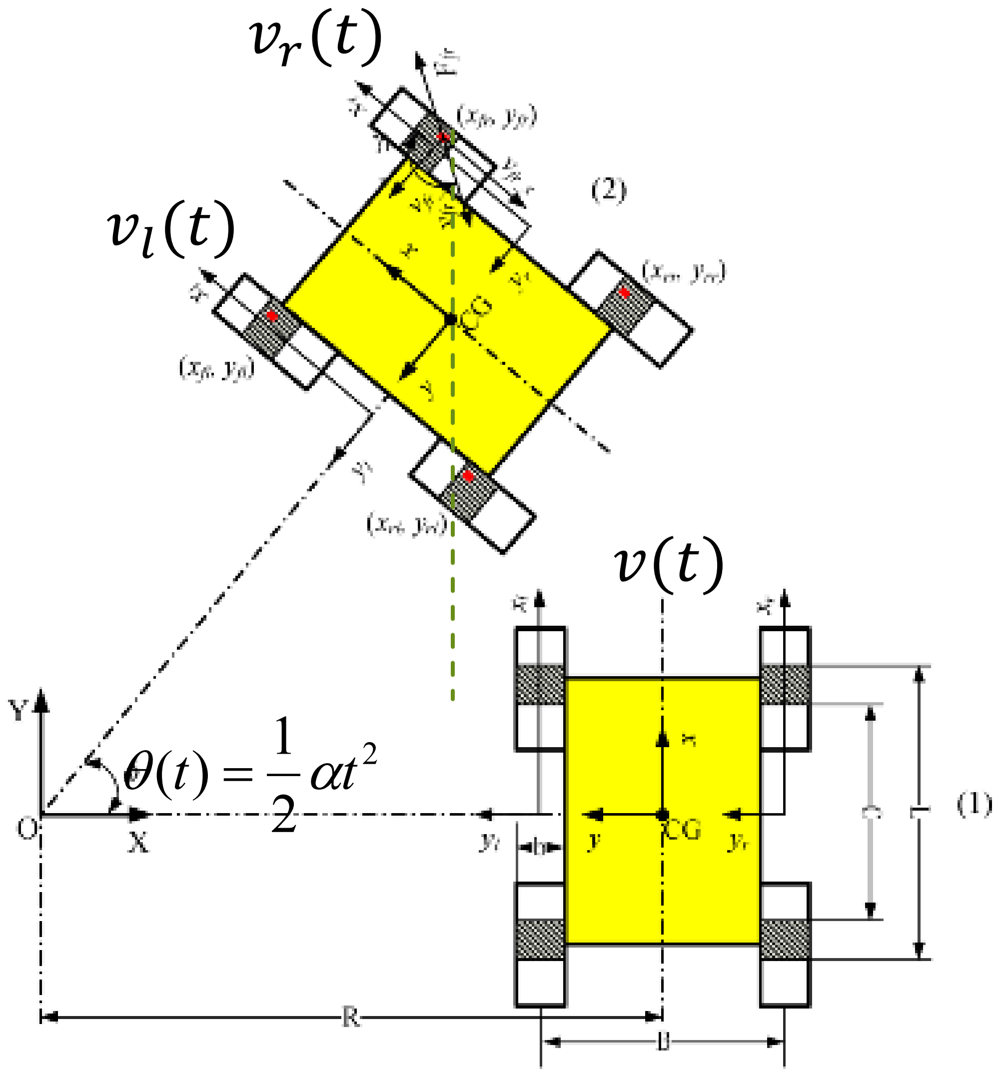


These are desired positions of robot arm's tool tip.

How to enable robots to do Start-Move-Stop motions?

Example of Continuous Motion

- A mobile robot is at rest when its position is at (R, 0) (m). The mobile robot starts to move along a circle. The radius of the circle followed by the mobile robot's CG is R. Assume that the mobile robot moves with a constant tangential acceleration a_t at the mobile robot's CG.

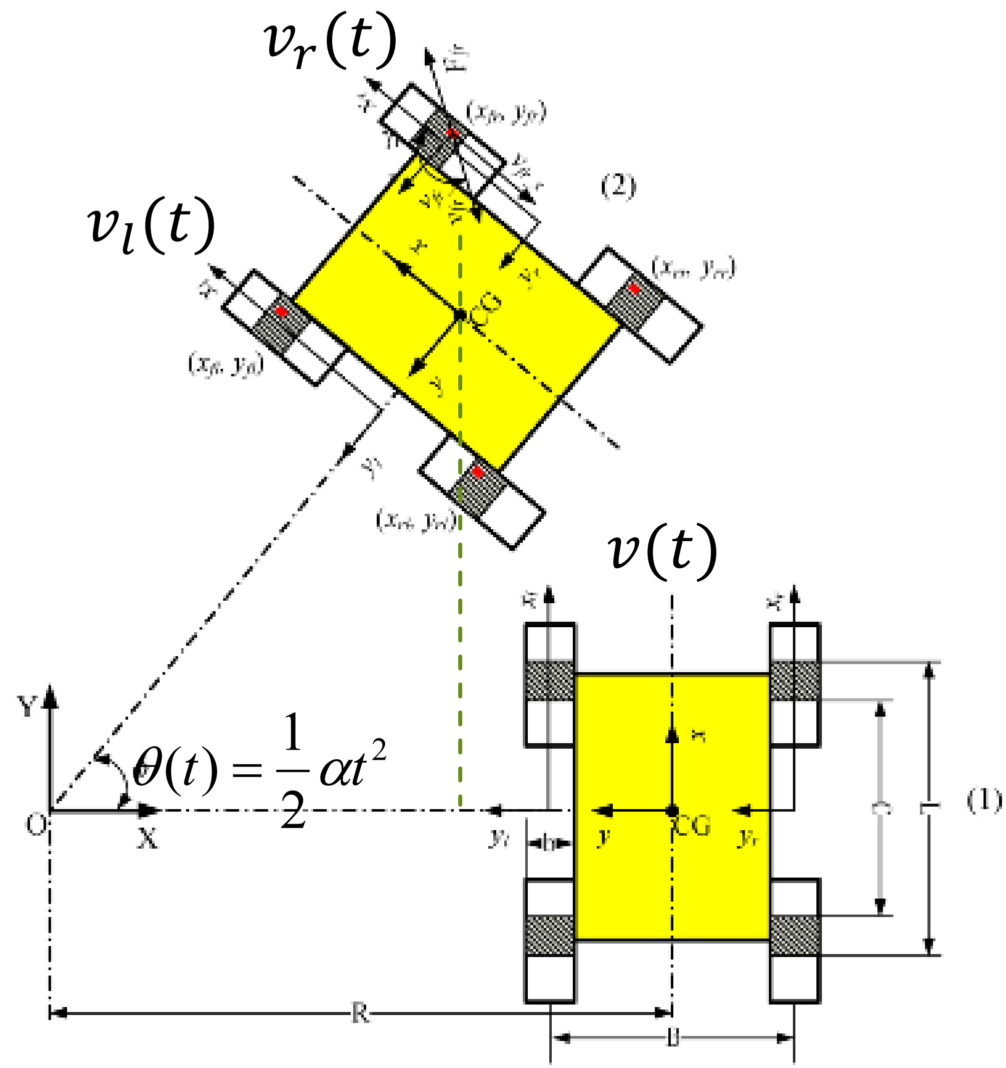


- What are the equations of CG's trajectory?
- What are the equations of left-wheel's trajectory?
- What are the equations of right-wheel's trajectory?

Solution

- Analysis:
  - The initial position of CG is at (R, 0.0) (m)
  - The tangential acceleration (or circular acceleration) is constant
  - Then, the angular acceleration about Z axis is also constant.
  - The radius of the circle followed by CG is R.
  - The CG's angular acceleration is:

$$\alpha = \frac{a_t}{R}$$



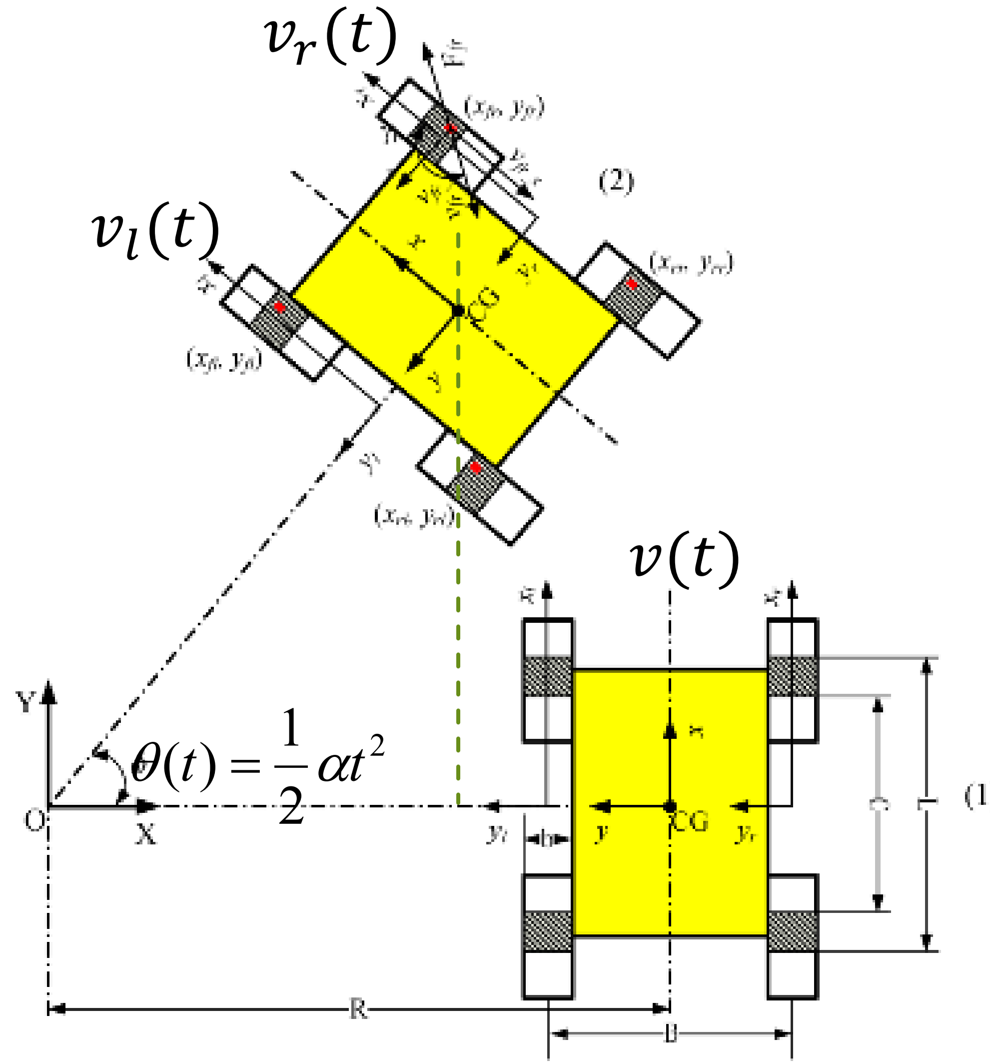
Solution

- Equations of CG's Trajectory:

$$\alpha = \frac{a_t}{R}$$

$$\theta(t) = \frac{1}{2}\alpha t^2$$

$$x(t) = R\cos(\theta(t))$$
$$y(t) = R\sin(\theta(t))$$
$$v(t) = R\alpha t$$



How to enable robots to do Start-Move-Stop motions?

Solution (continued)

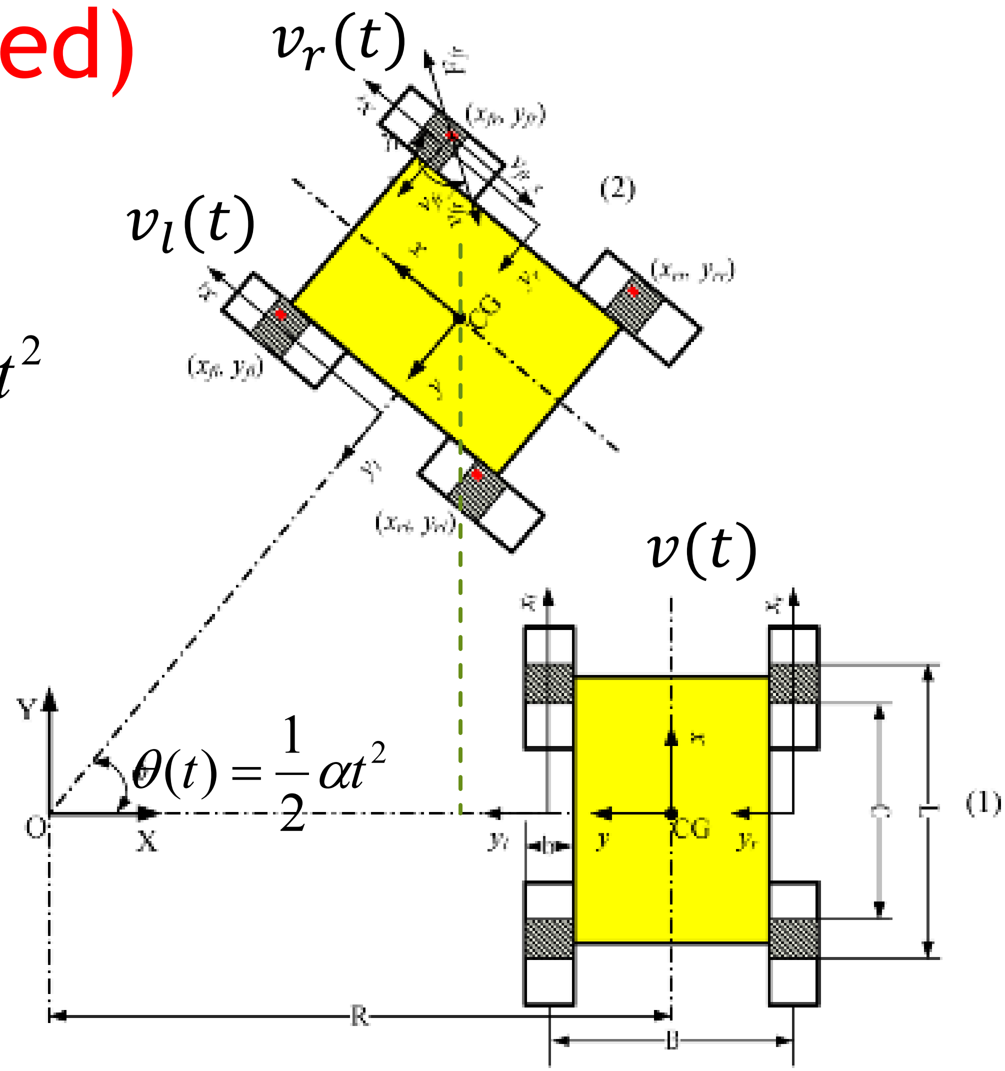
- Equations of left-wheel's Trajectory:

$$\alpha = \frac{a_t}{R} \longrightarrow \theta(t) = \frac{1}{2}\alpha t^2$$

$$x_l(t) = (R - \frac{U}{2}) \cos(\theta(t))$$

$$y_l(t) = (R - \frac{U}{2}) \sin(\theta(t))$$

$$v_l(t) = (R - \frac{U}{2})\alpha t$$



How to enable robots to do Start-Move-Stop motions?

Solution (continued)

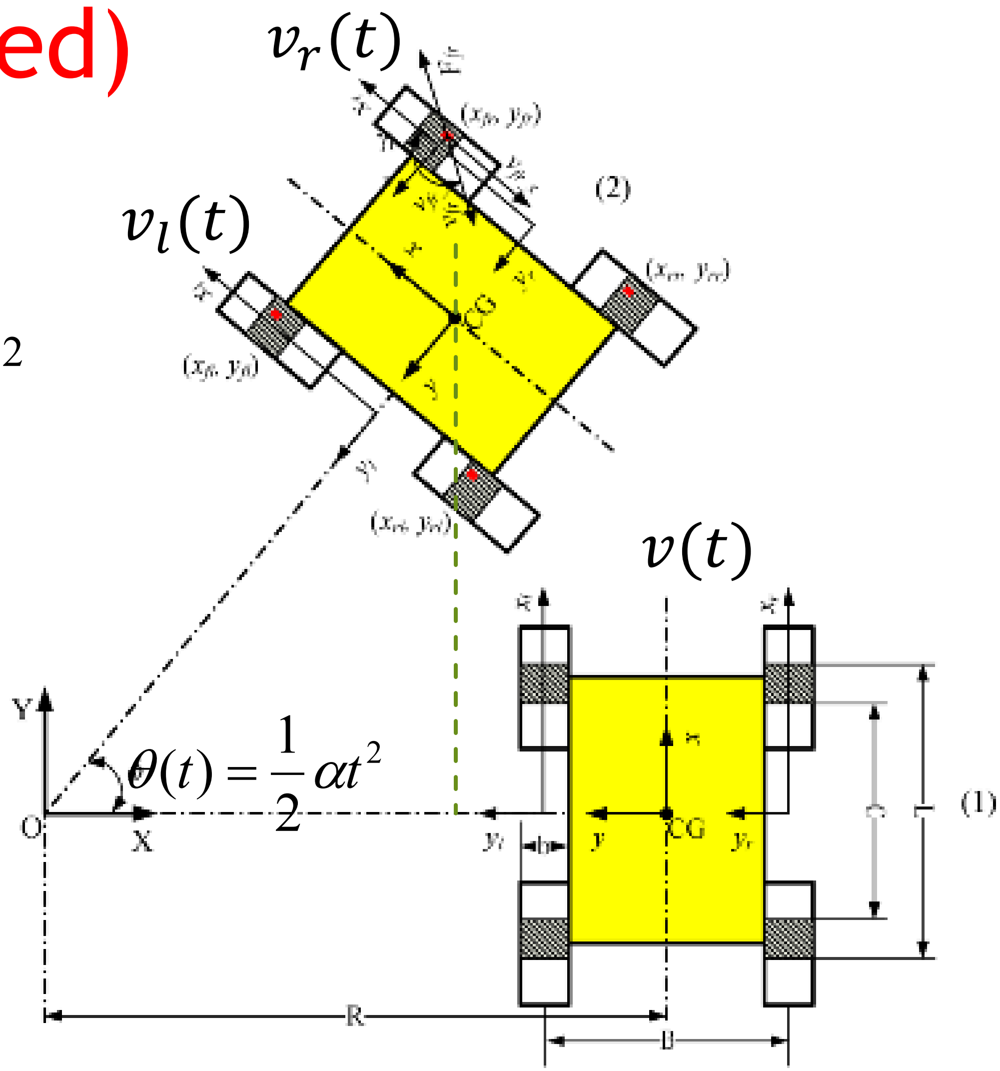
- Equations of right-wheel's Trajectory:

$$\alpha = \frac{a_t}{R} \longrightarrow \theta(t) = \frac{1}{2}\alpha t^2$$

$$x_r(t) = (R + \frac{U}{2}) \cos(\theta(t))$$

$$y_r(t) = (R + \frac{U}{2}) \sin(\theta(t))$$

$$v_r(t) = (R + \frac{U}{2}) \alpha t$$



How to enable robots to do Start-Move-Stop motions?

Outline of Lecture 5

Start-Move-Stop Motions

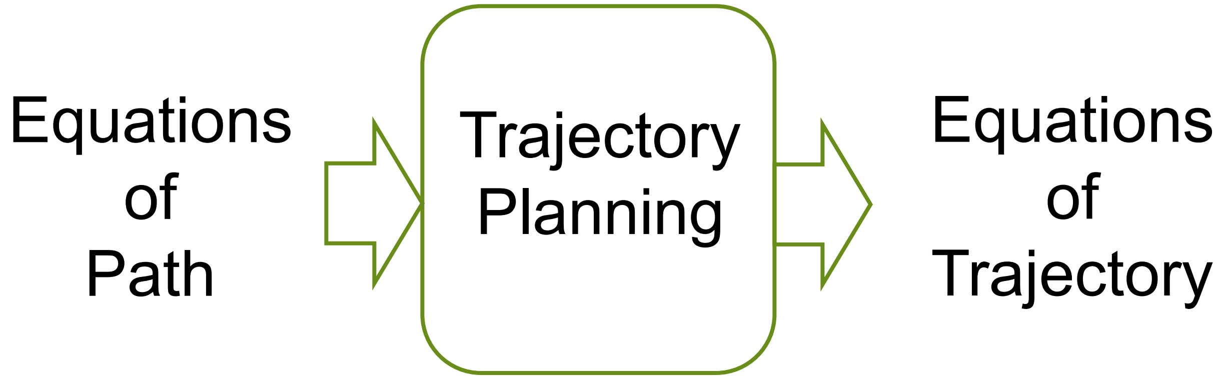
- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

- Purpose of Trajectory Planning
- Input to Trajectory Planning
- Output from Trajectory Planning
- Process of Autonomous Trajectory Planning

How to do trajectory planning?

- Scenario 1: You are the buyers or users
  - Human-Assisted Trajectory Planning
    - Trajectory Planning by Teaching
    - Trajectory Planning by Programming
- Scenario 2: You are the designers of robots
  - Autonomous Trajectory Planning by Robots

    Level 1 of Intelligence Readiness



There are two ways for human operators to assist robots to do trajectory planning

- Trajectory planning assisted by teaching
  - For Robot Hand: $[x(t), y(t), z(t), \theta(t)]$
  - For Mobile Base: $[v(t)\ \theta(t)]$ or $[v_l(t)\ v_r(t)]$

1. Users
2. Designers

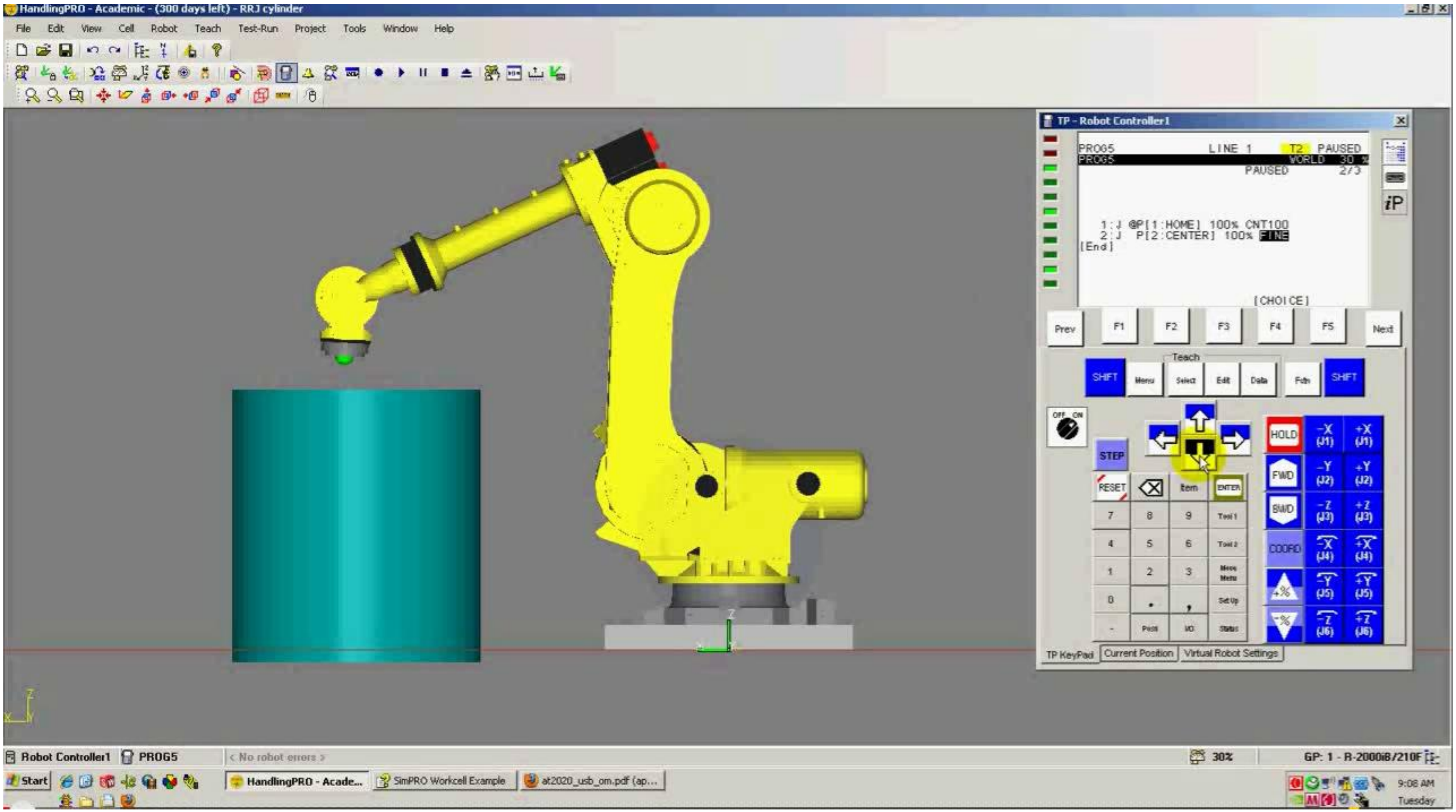
- Trajectory planning assisted by programming
  - For Robot Hand: $[x(t), y(t), z(t), \theta(t)]$
  - For Mobile Base: $[v(t)\ \theta(t)]$ or $[v_l(t)\ v_r(t)]$

1. Users
2. Designers

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Start-Move-Stop Motions

Trajectory Planning by Teaching



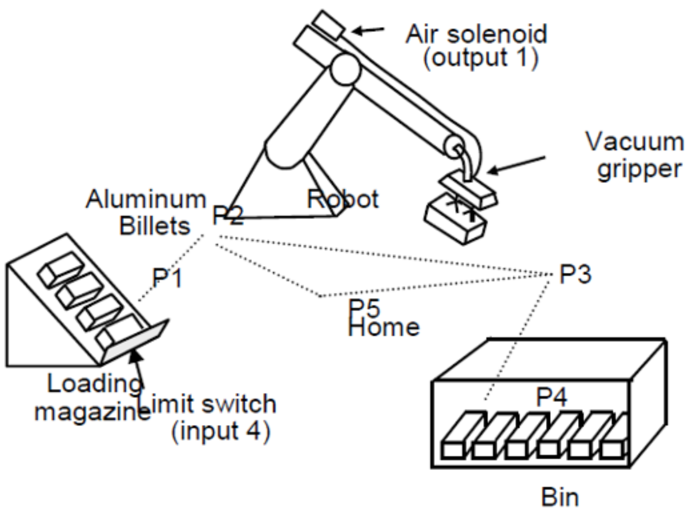
Trajectory Planning by Programming

- Motion Coordinates for Robot Hand:

  $[x(t), y(t), z(t), \theta(t)]$

- Motion Coordinates for Mobile Base:

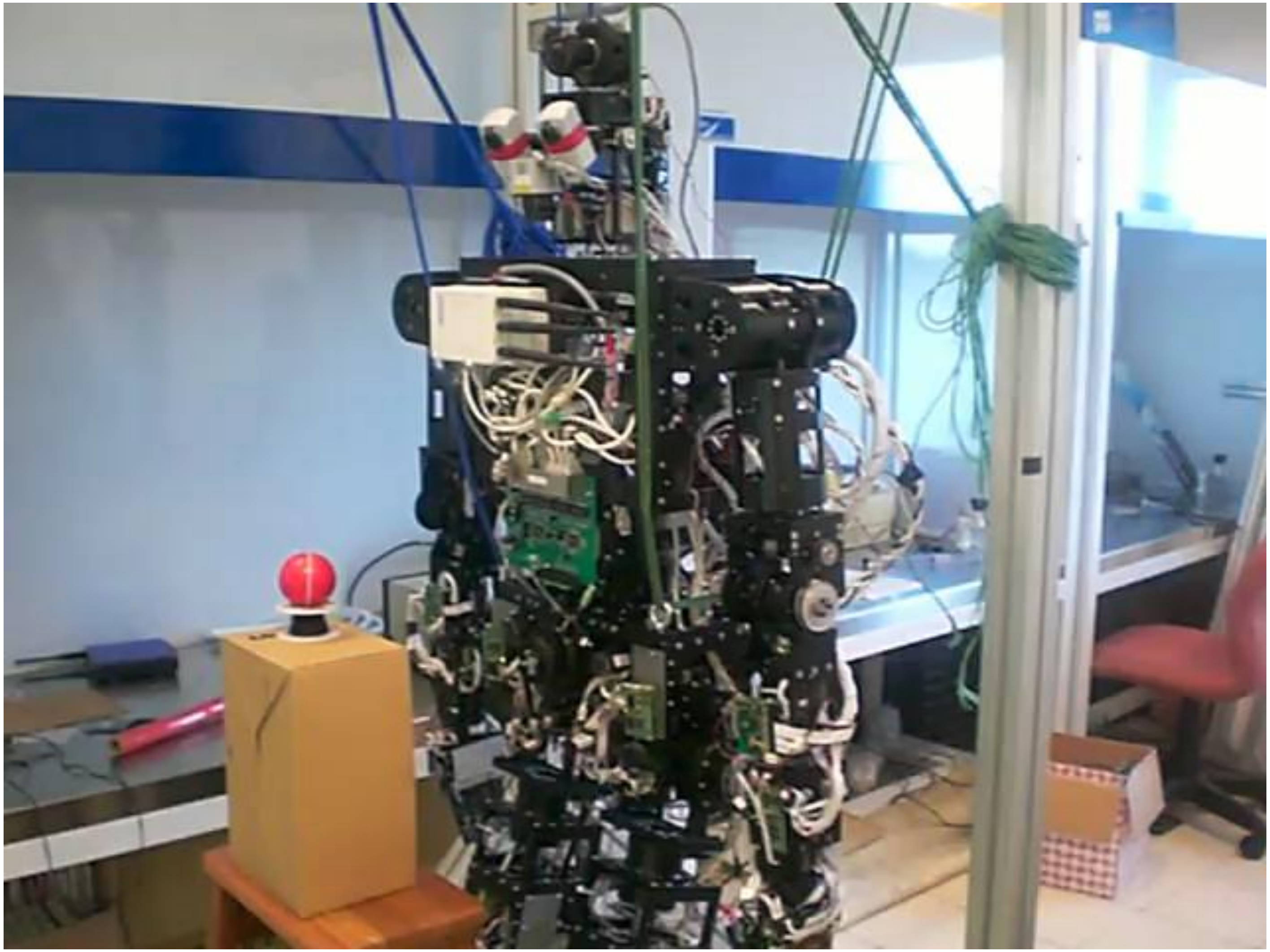
  $[v(t)\ \theta(t)]$ or $[v_l(t)\ v_r(t)]$



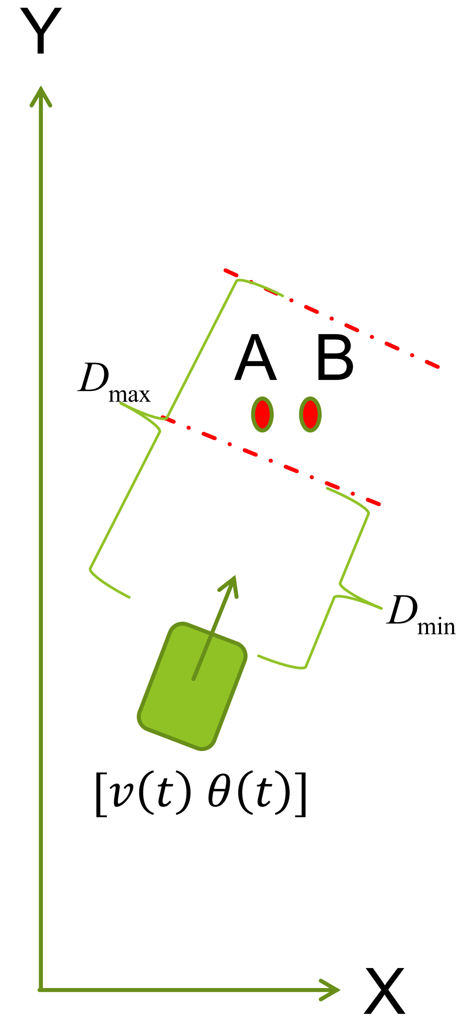
- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

```
Program Execute_Path
BEGIN
    While (Limit-Swith = On)
    BEGIN
        Open()
        MoveTo(P2);
        SpeedTo(0.2);
        MoveTo(P1)
        Close();
        SpeedTo(0.9);
        MoveTo(P2):
        MoveTo(P3);
        SpeedTo(0.2);
        MoveTo(P4);
        Shift_Bin_Left(1);
        Open();
        SpeedTo(0.9);
        MoveTo(P3)
        MoveTo(P5):
    END
END
```

Autonomous Trajectory Planning by Robot



Autonomous Trajectory Planning by Robot

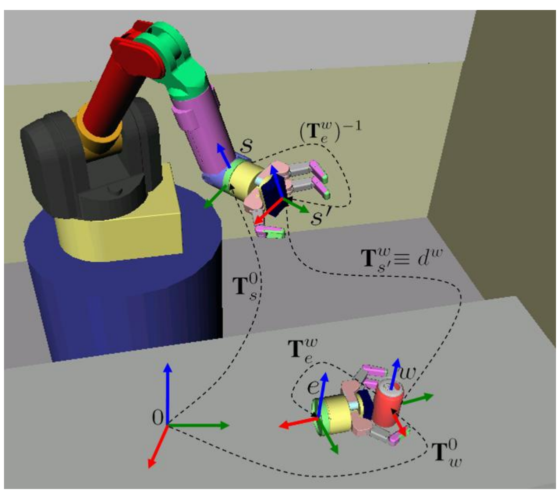


How to determine the equations of trajectory from a given path?

Case for Robot Hand

- For example, a motion may require a robot arm to move its hand from the current pose at rest to a final pose at rest. How to determine:

$$[x(t), y(t), z(t), \theta(t)]$$



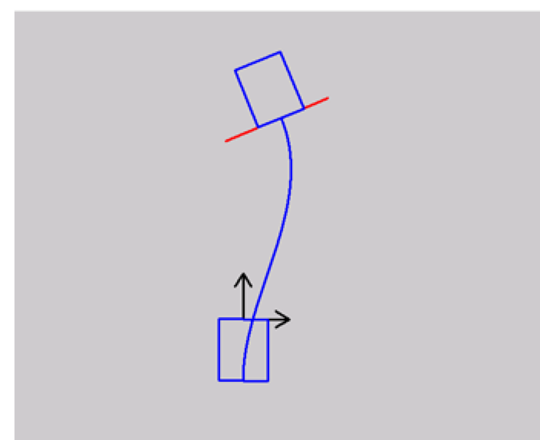
Case for Mobile Base

- For example, a mobile robot may enter a parking lot from its initial pose at rest and stops at the parking lot after the parking is accomplished. How to determine:

$[v(t)\ \theta(t)]$ or $[v_l(t)\ v_r(t)]$

Start-Move-Stop Motions

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest



Designers' Observations from Start-Move-Stop Motions

- Every motion has a duration of speed-up trajectory.
- Every motion has a duration of cruising trajectory.
- Every motion has a duration of slow-down trajectory.
- The transitions among these motions must be continuous.

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Start-Move-Stop Motions

Design Options for Speed-Up Trajectory

- 1: Trajectory of Linear Path with Constant Acceleration
- 2: Trajectory of Linear Path with Variable Acceleration
- 3: Trajectory of Curved Path with Time Constraints

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Start-Move-Stop Motions

Design Options for Cruising Trajectory

- 1: Trajectory of Linear Path with Constant Speed
- 2: Trajectory of Linear Path with Constant Acceleration
- 3: Trajectory of Linear Path with Variable Acceleration
- 4: Trajectory of Curved Path with Time Constraints

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Start-Move-Stop Motions

Design Options for Slow-Down Trajectory

- 1: Trajectory of Linear Path with Constant Deceleration
- 2: Trajectory of Linear Path with Variable Deceleration
- 3: Trajectory of Curved Path with Time Constraints

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Start-Move-Stop Motions

Design Solutions for Trajectory Having Two Motion Phases

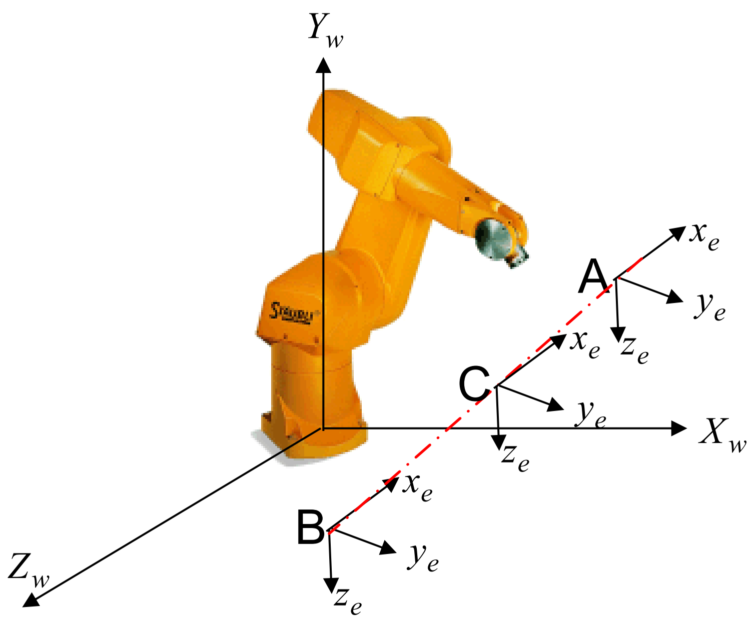
Design ID	First Motion	Second Motion
1	Linear	Linear
2	Linear	Curved
3	Curved	Linear
4	Curved	Curved

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Start-Move-Stop Motions

Design Solution 1

- Linear path + Linear path



From A to C: Speed-up Motion

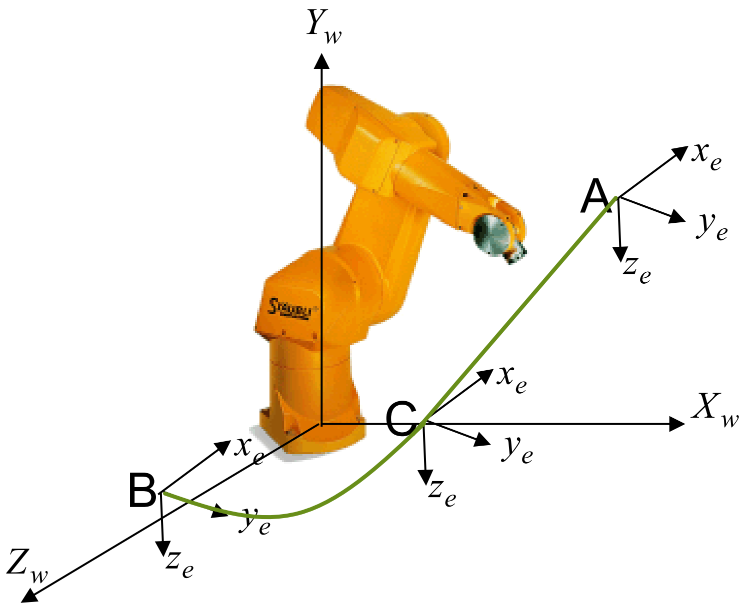
From C to B: Slow-down Motion

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Start-Move-Stop Motions

Design Solution 2

- Linear path + Curved path



From A to C: Speed-up Motion

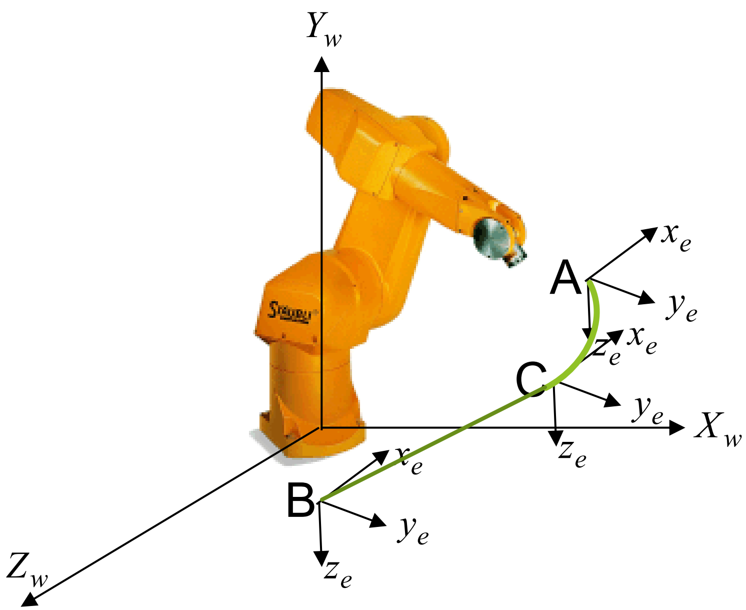
From C to B: Slow-down Motion

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Start-Move-Stop Motions

Solution 3

- Curved path + Linear path



From A to C: Speed-up Motion

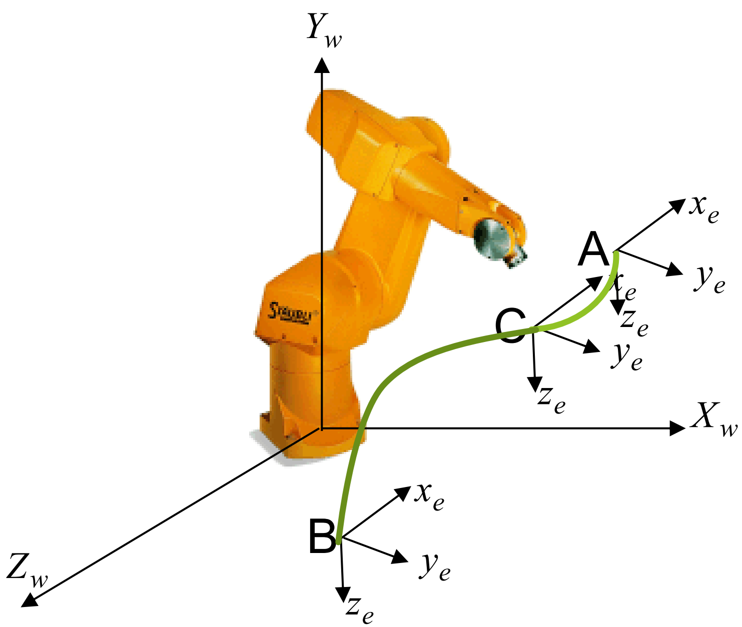
From C to B: Slow-down Motion

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Start-Move-Stop Motions

Solution 4

- Curved path + Curved path



From A to C: Speed-up Motion

From C to B: Slow-down Motion

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Start-Move-Stop Motions

Design Solutions for Trajectory Having Three Motion Phases

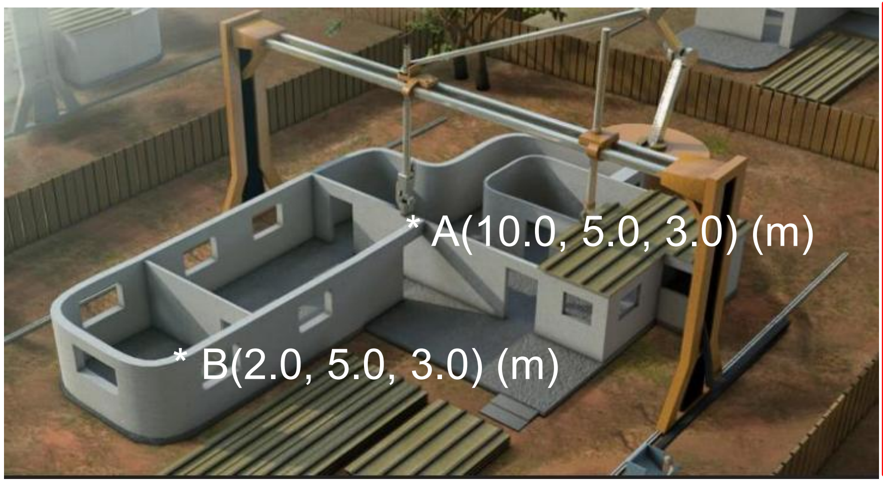
Design ID	First Motion	Second Motion	Third Motion
1	Linear	Linear	Linear
2	Linear	Linear	Curved
3	Linear	Curved	Linear
4	Linear	Curved	Curved
5	Curved	Linear	Linear
…	…	…	…
8	Curved	Curved	Curved

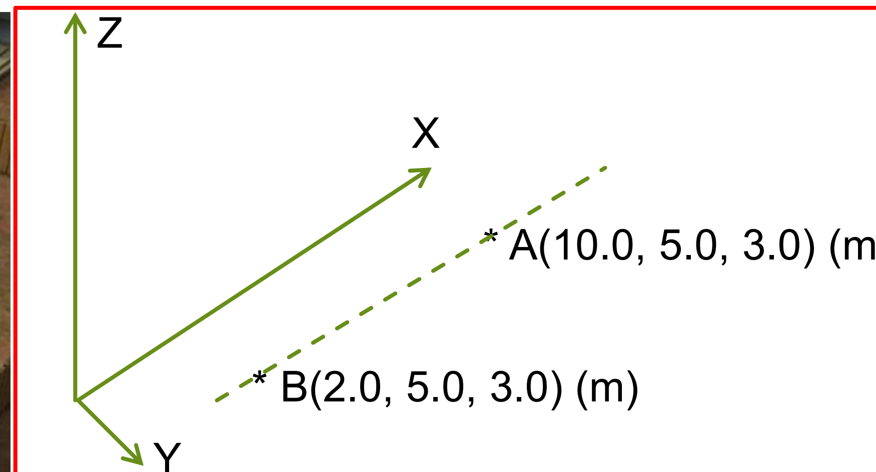
- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Start-Move-Stop Motions

Design Example for Robot Hand's Trajectory

- A gantry robot is doing the construction of a building automatically. At a process, the tool tip is at rest at point A(10.0, 5.0, 3.0)(m). Then, it moves along a straight line, and stops at point B(2.0,5.0, 3.0)(m) within 12.0 seconds. Design a trajectory of the robot's tool tip which enables it to achieve the above motion.





- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

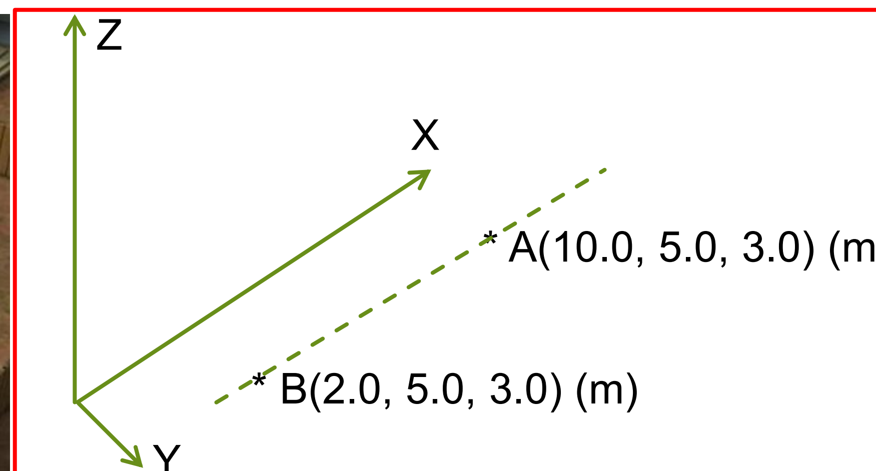
Start-Move-Stop Motions

Solution

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Start-Move-Stop Motions

- Analysis:
  - The tool tip's X coordinate changes while Y and Z coordinates remain unchanged.
  - The displacement of X coordinate is 2.0 – 10.0 = -8.0 (m)
  - The duration of motion is 12.0 seconds

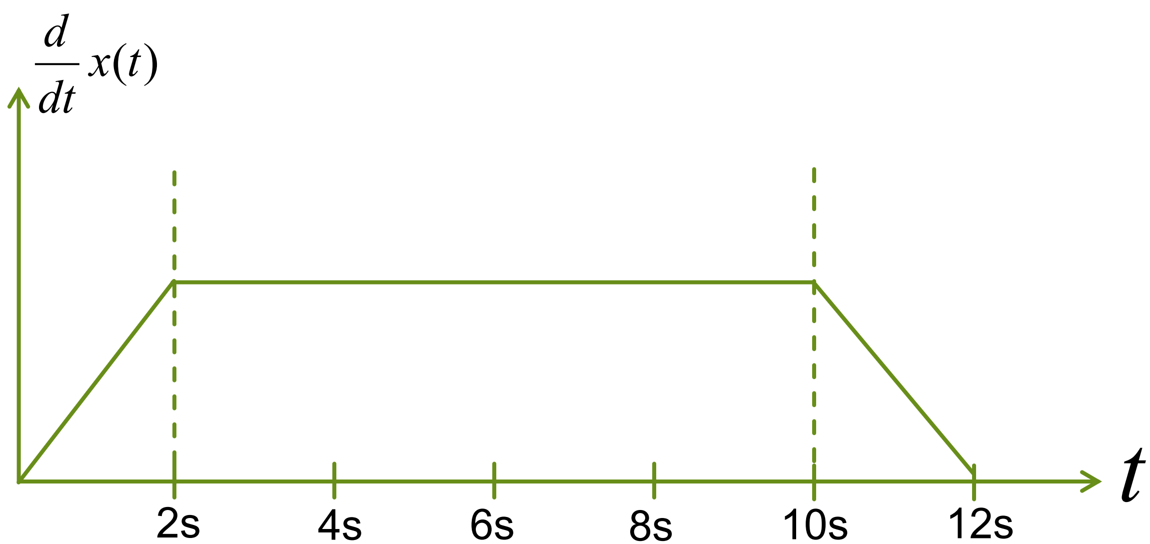


Solution (continued)

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Start-Move-Stop Motions

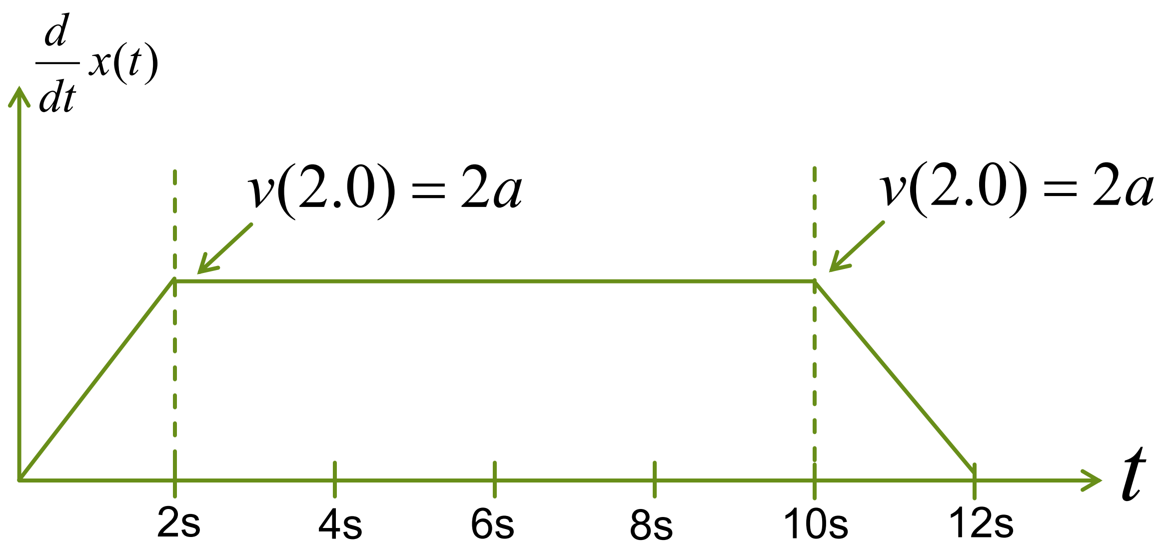
- Design of Trajectory:
  - Speed-up Motion: trajectory of linear motion with a constant acceleration for a duration of 2.0 seconds.
  - Cruising Motion: trajectory of linear motion with a constant velocity for a duration of 8.0 seconds.
  - Slow-down Motion: trajectory of linear motion with a constant deceleration for a duration of 2.0 seconds.



Solution (continued)

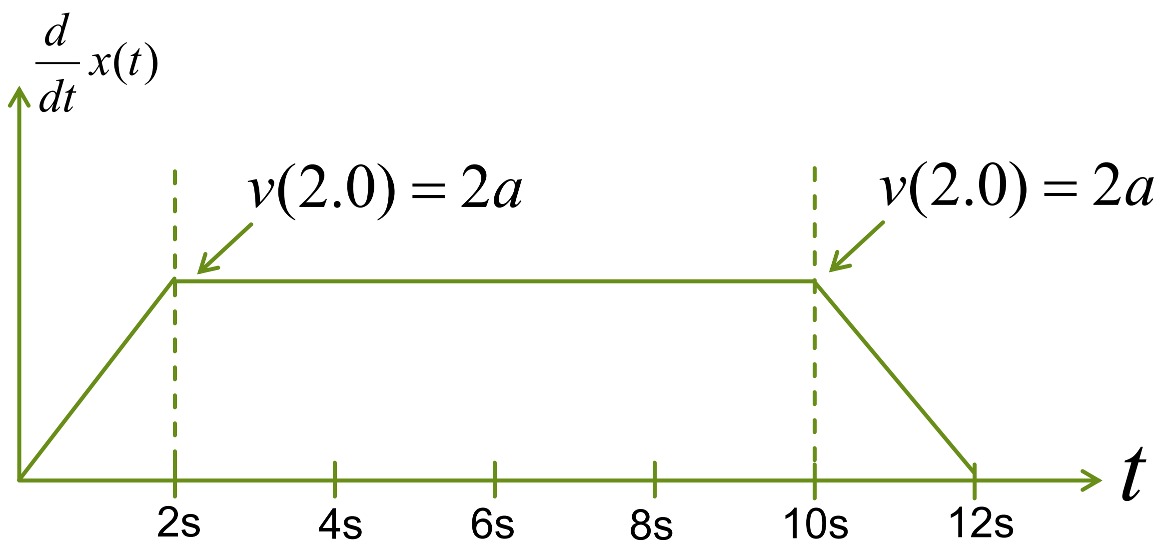
How to determine the unknown a ?

- Equations of Trajectory:
  - From t=0.0 to t=2.0: $$x(t) - x(0.0) = \frac{1}{2}at^2$$
  - From t=2.0 to t=10.0: $$x(t) - x(2.0) = 2a(t - 2.0)$$
  - From t=10.0 to t=12.0: $$x(t) - x(10.0) = 2a(t - 10.0) - \frac{1}{2}a(t - 10.0)^2$$



Solution (continued)

- Constraints for us to determine the unknown value of a:
  - Travelled distance from t=0.0 to t=2.0: $x(2.0)-x(0.0)=2a$
  - Travelled distance from t=2.0 to t=10.0: $x(10.0)-x(2.0)=16.0a$
  - Travelled distance from t=10.0 to t=12.0: $x(12.0)-x(10.0)=2a$



Solution (continued)

- Compute the unknown value of a:
  - Travelled distance from t=0.0 to t=2.0: $x(2.0)-x(0.0)=2.0a$
  - Travelled distance from t=2.0 to t=10.0: $x(10.0)-x(2.0)=16.0a$
  - Travelled distance from t=10.0 to t=12.0: $x(12.0)-x(10.0)=2.0a$

\+

$$x(12.0)-x(0.0)=20.0a$$

$$-8.0 = 20.0a$$

(acceleration toward negative X axis) ⇐ $a = -0.4\ (\mathrm{m/s^2})$

Solution (continued)

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Start-Move-Stop Motions

- Planned Equations of Trajectory:
  - From t=0.0 to t=2.0:

$$x(t) - x(0.0) = -0.2t^2$$

  - From t=2.0 to t=10.0:

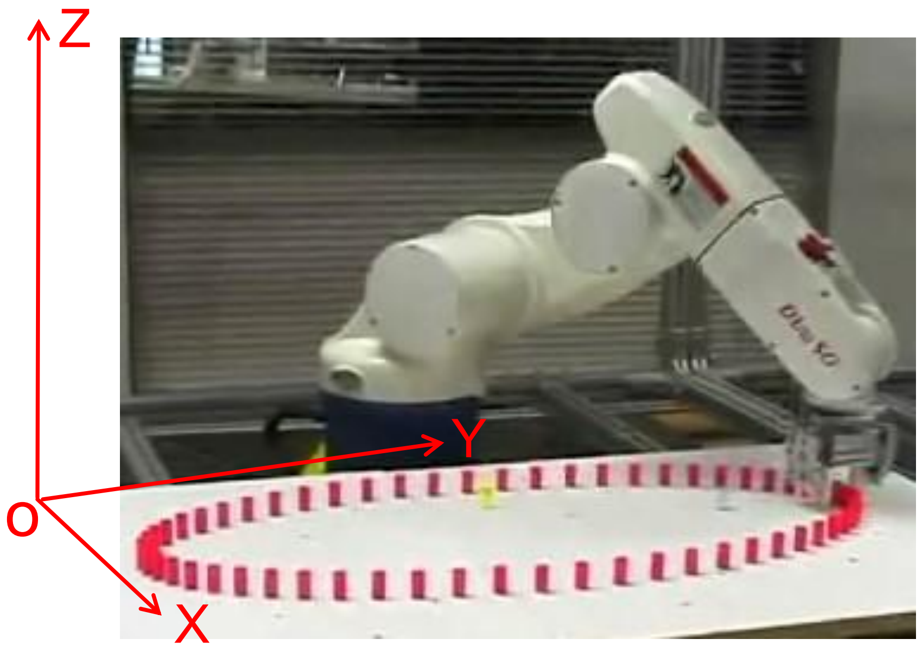
$$x(t) - x(2.0) = -0.8(t - 2.0)$$

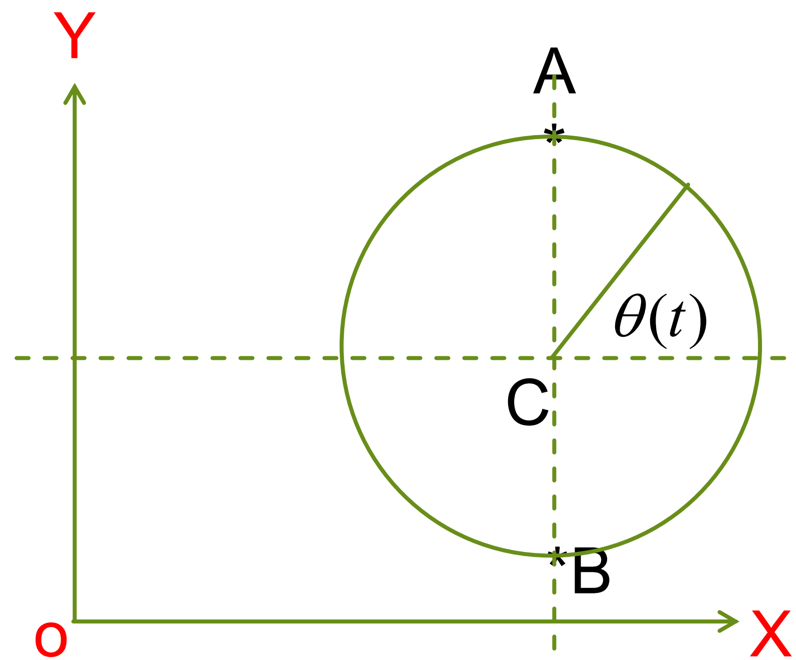
  - From t=10.0 to t=12.0:

$$x(t) - x(10.0) = -0.8(t - 10.0) + 0.2(t - 10.0)^2$$

Design Example of Robot Hand's Trajectory

- A robot manipulator's gripper is at rest at point A(40.0, 35.0, 2.0) (cm). Then, it moves along a circle which is parallel to a table, and stops at point B(40.0, 5.0, 2.0) (cm) within 12.0 second. The radius of the circle is 15.0 (cm). Design a trajectory of the robot's gripper, which enables it to achieve the above motion. (NOTE: just design the trajectory of the gripper's position)



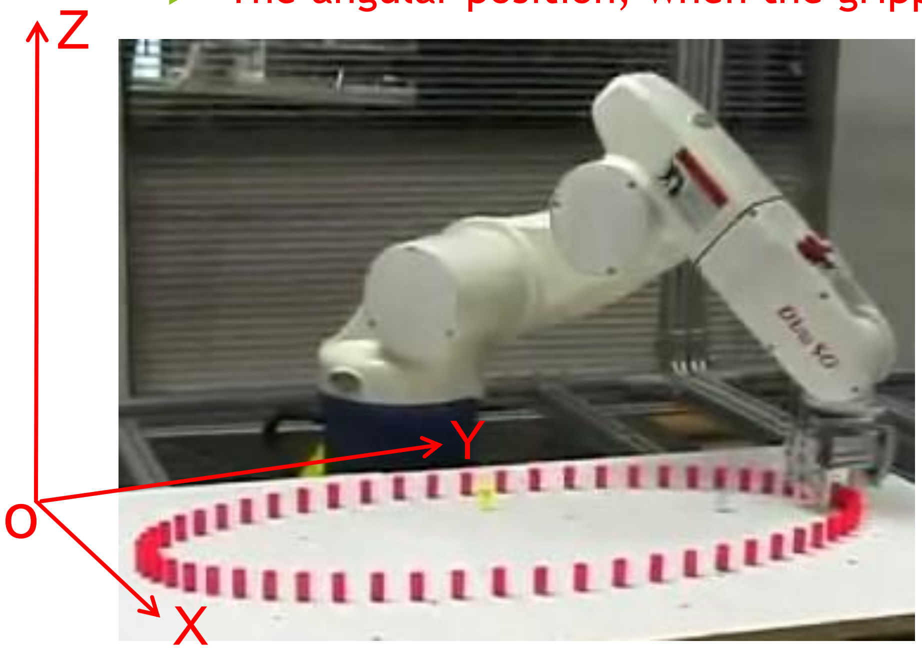


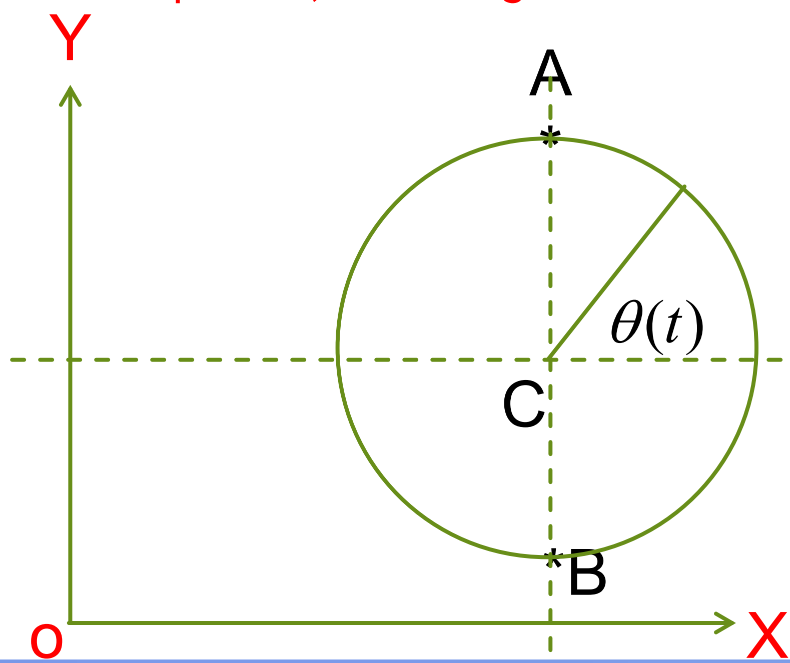
Solution

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Start-Move-Stop Motions

- Analysis:
  - Only Z coordinate remains unchanged
  - The duration of motion is 12.0 seconds
  - The centre of the circle is at C(40.0, 20.0, 2.0) (cm)
  - The radius of the circle is 15.0 (cm)
  - The angular position, when the gripper reaches point B, is -90 degrees



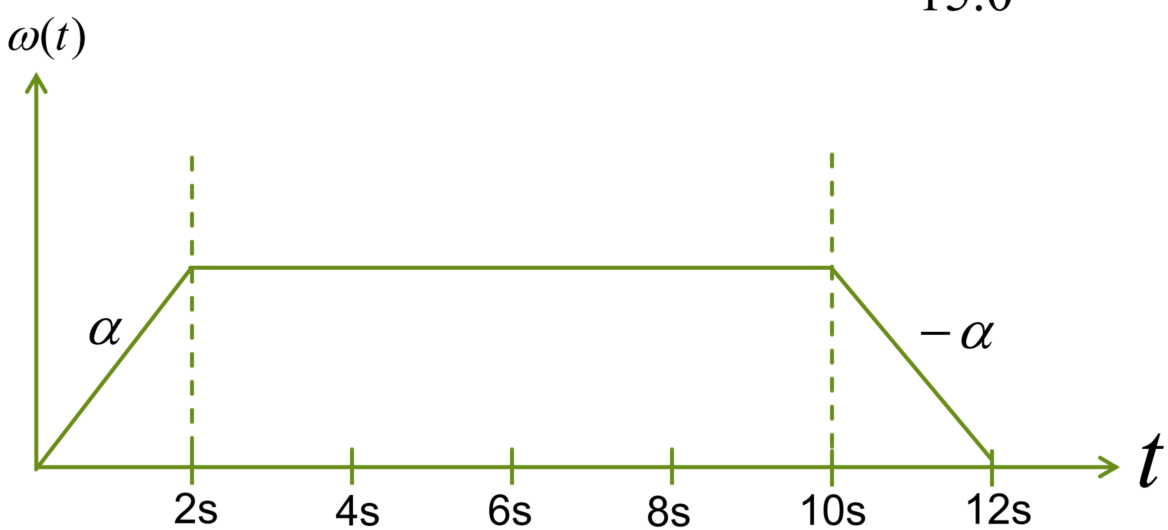


Solution (continued)

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Start-Move-Stop Motions

- Design of Trajectory:
  - Speed-up Motion: trajectory of curved motion with a constant tangential acceleration for a duration of 2.0 seconds.
  - Cruising Motion: trajectory of curved motion with a constant circular velocity for a duration of 8.0 seconds.
  - Slow-down Motion: trajectory of curved motion with a constant deceleration for a duration of 2.0 seconds.
  - The corresponding angular acceleration is: $\alpha = \frac{a_t}{15.0} = \text{constant}$



Solution (continued)

- Equations of Trajectory for Coordinate x(t):
  - From t=0.0 to t=2.0:

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Start-Move-Stop Motions

$$\omega(2.0) = -2\alpha$$

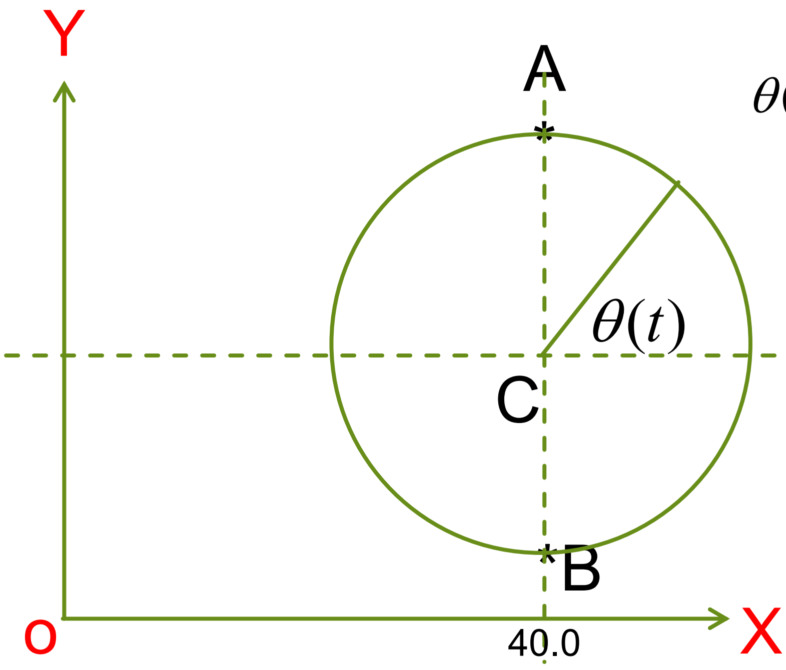
$$\theta(0.0) = \frac{\pi}{2}$$

$$\theta(2.0) = \frac{\pi}{2} - 2\alpha$$

$$\theta(t) = \frac{\pi}{2} - \frac{1}{2}\alpha t^2$$

$$\omega(t) = -\alpha t$$

$$x(t) - 40.0 = 15.0\cos(\theta(t))$$



Solution (continued)

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Start-Move-Stop Motions

- Equations of Trajectory for Coordinate x(t):
  - From t=2.0 to t=10.0 with constant angular velocity:

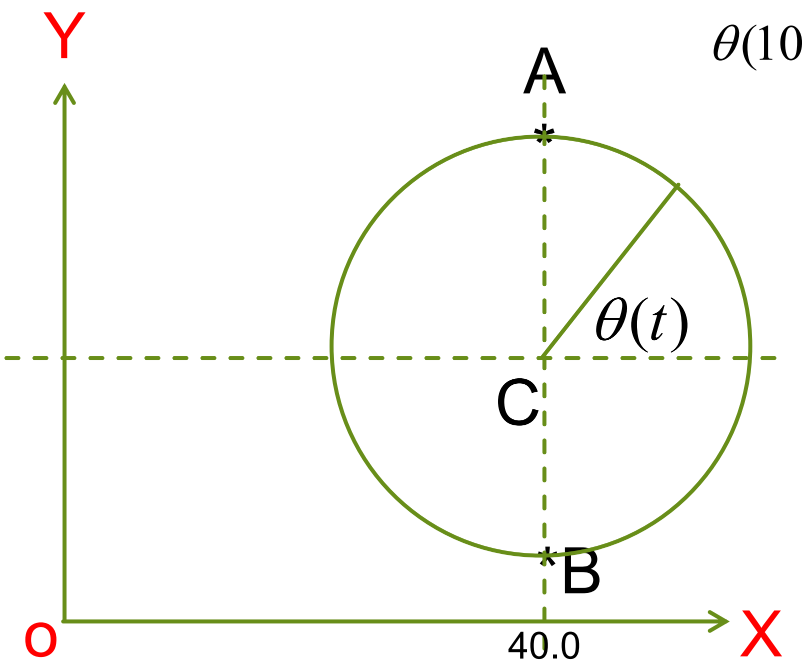
$$\omega(10.0) == \omega(2.0) = -2\alpha \qquad \theta(2.0) = \frac{\pi}{2} - 2\alpha$$

$$\theta(10.0) = \frac{\pi}{2} - 18\alpha$$

$$\theta(t) - \theta(2) = \omega(2)(t - 2.0)$$

$$\theta(t) = (\frac{\pi}{2} - 2\alpha) - 2\alpha(t - 2.0)$$

$$x(t) - 40.0 = 15.0\cos(\theta(t))$$



Solution (continued)

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Start-Move-Stop Motions

- Equations of Trajectory for Coordinate x(t):
  - From t=10.0 to t=12.0:

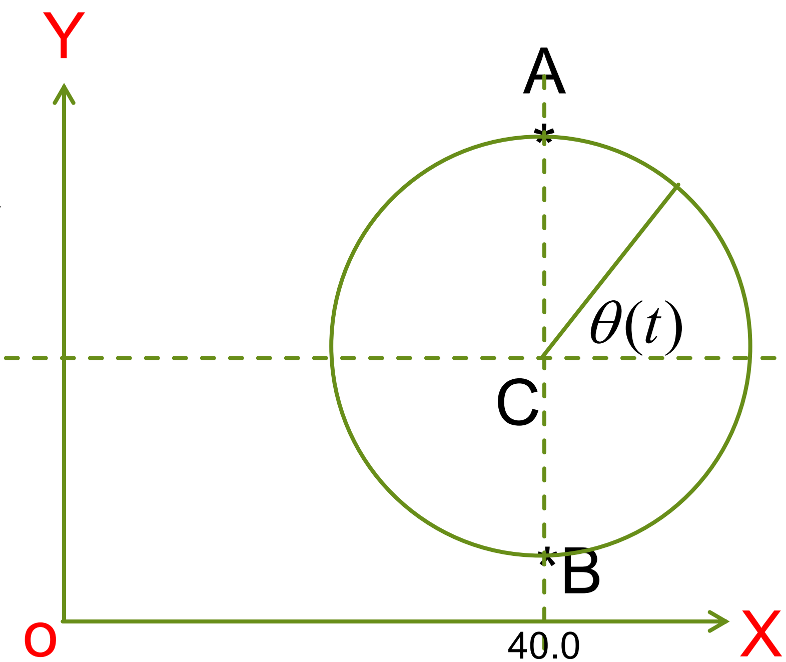
$$\theta(10.0) = \frac{\pi}{2} - 18\alpha \qquad \theta(12.0) = \frac{\pi}{2} - 20\alpha = -\frac{\pi}{2}$$

$$\alpha = \frac{\pi}{20}$$

$$\theta(t) - \theta(10) = \omega(10)(t-10.0) + \frac{1}{2}\alpha(t-10.0)^2$$

$$\theta(t) = \frac{\pi}{2} - 18\alpha - 2\alpha(t-10.0) + \frac{1}{2}\alpha(t-10.0)^2$$

$$x(t) - 40.0 = 15.0\cos(\theta(t))$$



Solution (continued)

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Start-Move-Stop Motions

- Equations of Trajectory for Coordinate y(t):
  - From t=0.0 to t=2.0:

$$\omega(2.0) = -2\alpha$$

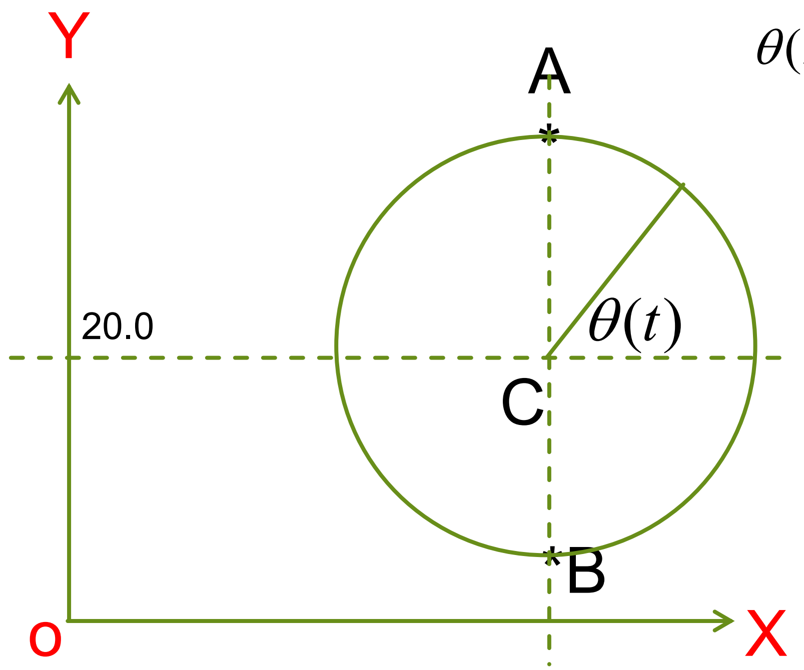
$$\theta(0.0) = \frac{\pi}{2}$$

$$\theta(2.0) = \frac{\pi}{2} - 2\alpha$$

$$\theta(t) = \frac{\pi}{2} - \frac{1}{2}\alpha t^2$$

$$\omega(t) = -\alpha t$$

$$y(t) - 20.0 = 15.0\sin(\theta(t))$$



Solution (continued)

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Start-Move-Stop Motions

- Equations of Trajectory for Coordinate y(t):
  - From t=2.0 to t=10.0:

$$\omega(10.0) = -2\alpha$$

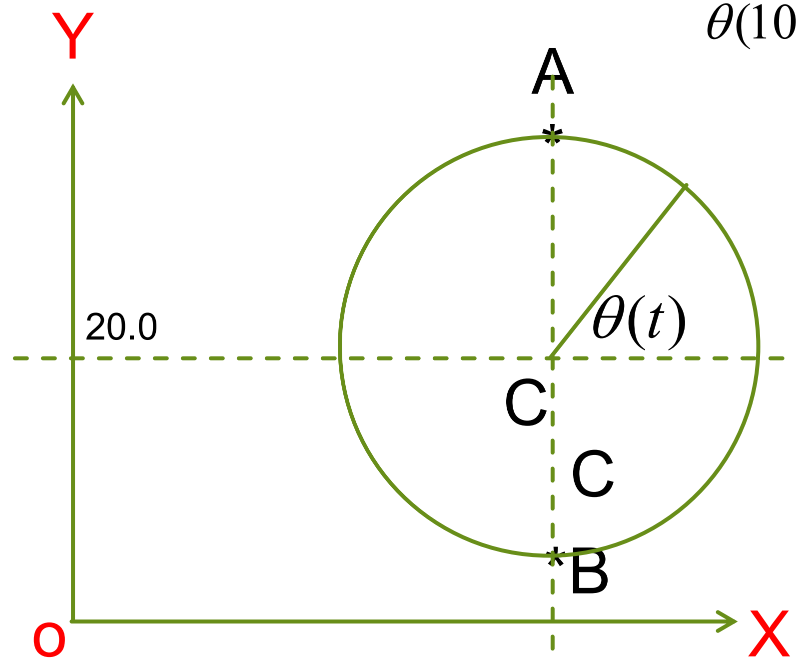
$$\theta(2.0) = \frac{\pi}{2} - 2\alpha$$

$$\theta(10.0) = \frac{\pi}{2} - 18\alpha$$

$$\theta(t) - \theta(2) = \omega(2)(t - 2.0)$$

$$\theta(t) = (\frac{\pi}{2} - 2\alpha) - 2\alpha(t - 2.0)$$

$$y(t) - 20.0 = 15.0\sin(\theta(t))$$



Solution (continued)

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Start-Move-Stop Motions

- Equations of Trajectory for Coordinate y(t):
  - From t=10.0 to t=12.0:

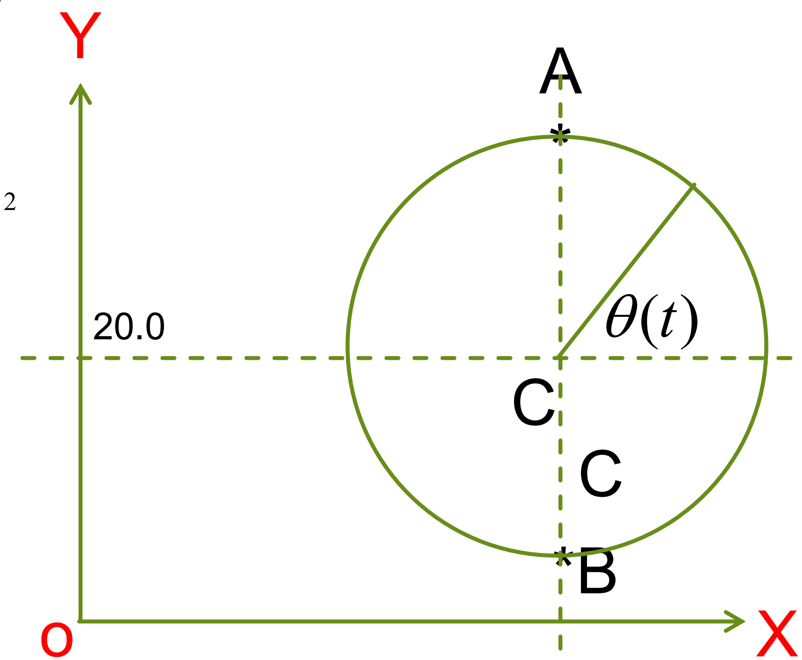
$$\theta(10.0)=\frac{\pi}{2}-18\alpha \qquad \theta(12.0)=\frac{\pi}{2}-20\alpha=-\frac{\pi}{2}$$

$$\alpha=\frac{\pi}{20}$$

$$\theta(t)-\theta(10)=\omega(10)(t-10.0)+\frac{1}{2}\alpha(t-10.0)^2$$

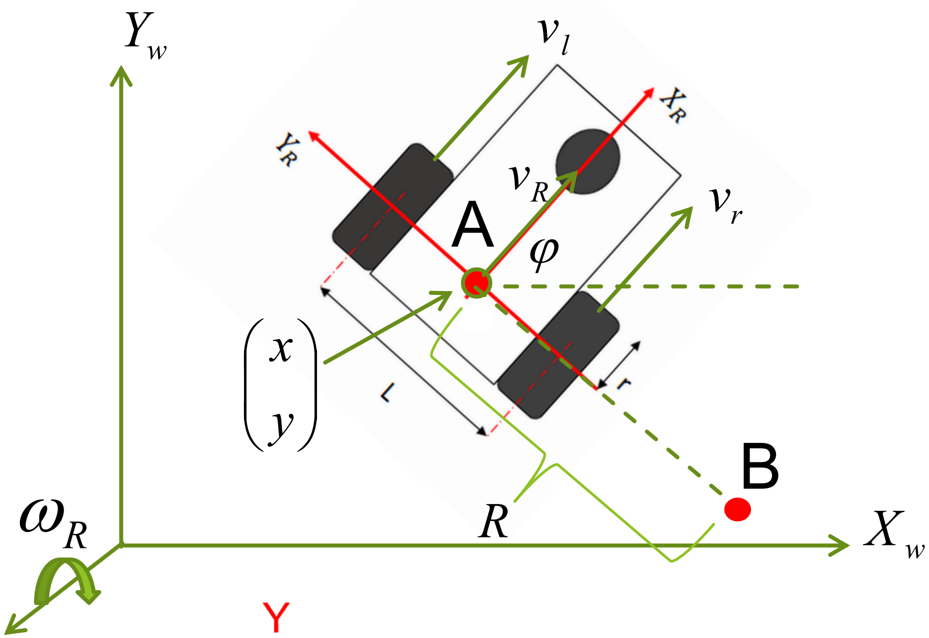
$$\theta(t)=\frac{\pi}{2}-18\alpha-2\alpha(t-10.0)+\frac{1}{2}\alpha(t-10.0)^2$$

$$y(t)-20.0=15.0\sin(\theta(t))$$



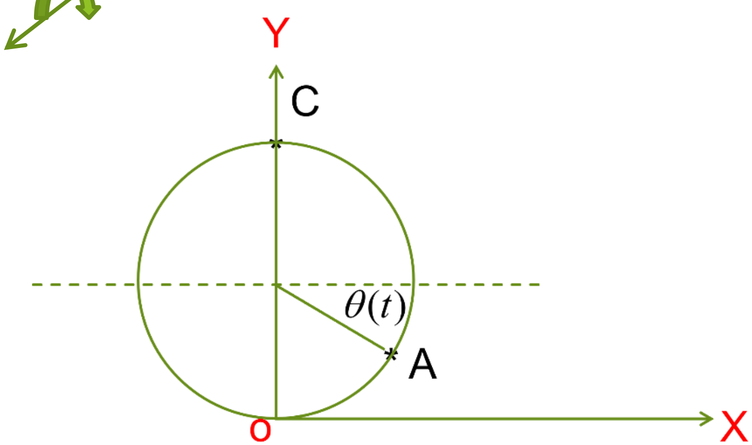
Design Example of Mobile Base's Trajectory

- Refer to the figure given. A mobile robot has a pair of differential wheels. The initial pose of the mobile robot is at (0, 0) (cm) with the orientation of zero degree. Assume that the width of the mobile robot is 50.0 cm and that the radius of the wheels is 20.0cm. Design the equations of point A's trajectory so that the mobile robot will take 12.0 seconds to reach and stop at the final pose which is at (0, 300) (cm) with the orientation of -180 degrees.



- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Start-Move-Stop Motions

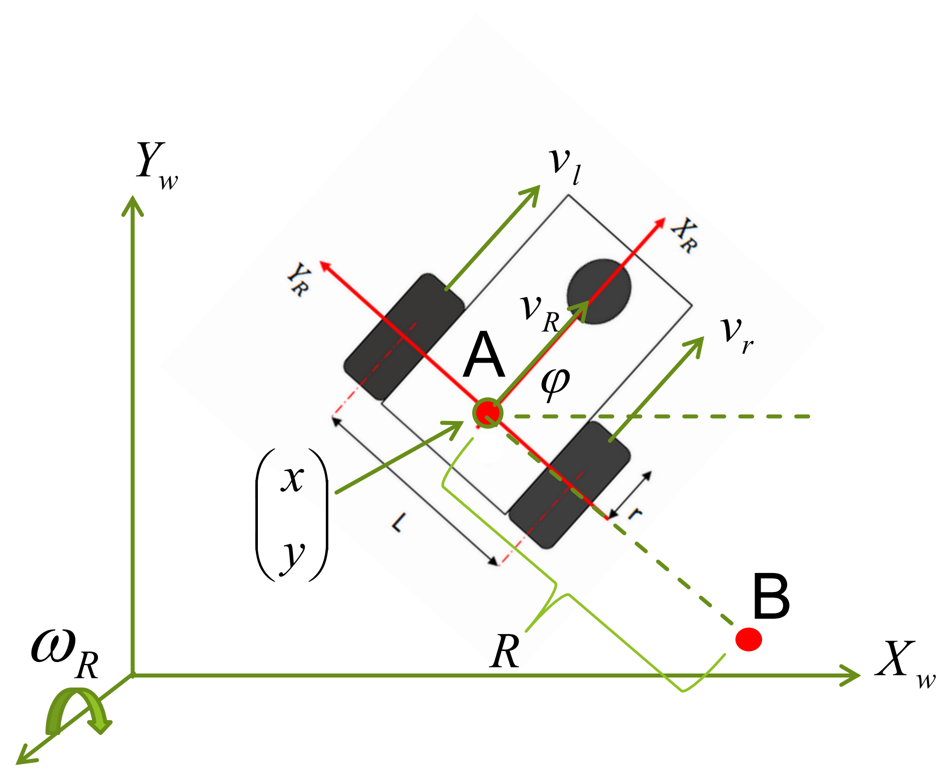


Solution

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Start-Move-Stop Motions

- Analysis:
  - The mobile robot follows a circular path.
  - The radius of the circular path is 150.0 cm
  - The duration of motion is 12.0 seconds

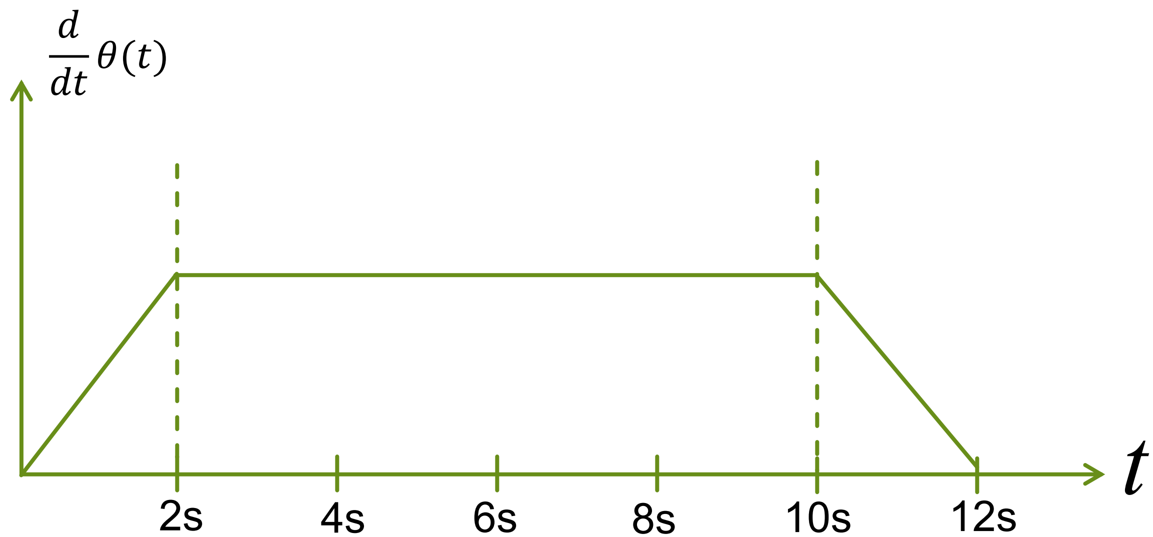


Solution (continued)

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Start-Move-Stop Motions

- Design of Trajectory:
  - Speed-up Motion: trajectory of circular motion with a constant acceleration for a duration of 2.0 seconds.
  - Cruising Motion: trajectory of circular motion with a constant velocity for a duration of 8.0 seconds.
  - Slow-down Motion: trajectory of circular motion with a constant deceleration for a duration of 2.0 seconds.



Solution (continued)

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Start-Move-Stop Motions

- Equations of Trajectory During Speed-up Phase [0, 2] s:

$$\theta(0) = -\frac{\pi}{2}$$

$$\omega_R(0) = 0$$

$$\theta(t) = -\frac{\pi}{2} + \frac{1}{2}\alpha t^2$$

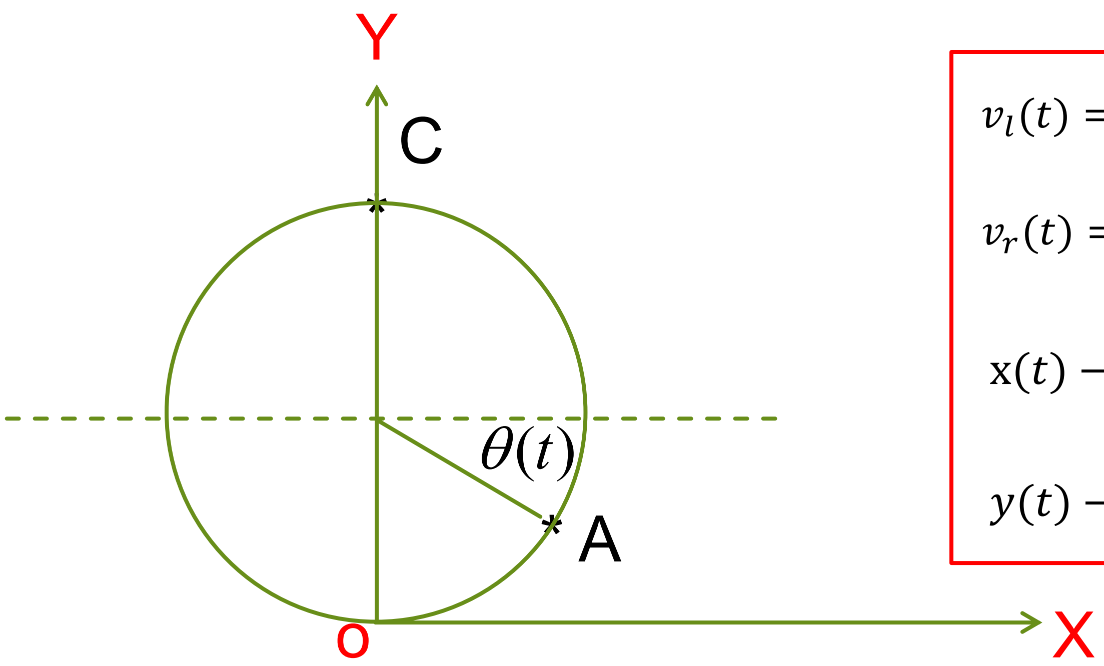
$$\omega_R(t) = \omega_R(0) + \alpha t$$

$$v_l(t) = (150.0 - \frac{L}{2}) \times \omega_R(t)$$

$$v_r(t) = (150.0 + \frac{L}{2}) \times \omega_R(t)$$

$$x(t) - 0.0 = 150.0\, cos(\theta(t))$$

$$y(t) - 150.0 = 150.0 \sin(\theta(t))$$



Solution (continued)

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Start-Move-Stop Motions

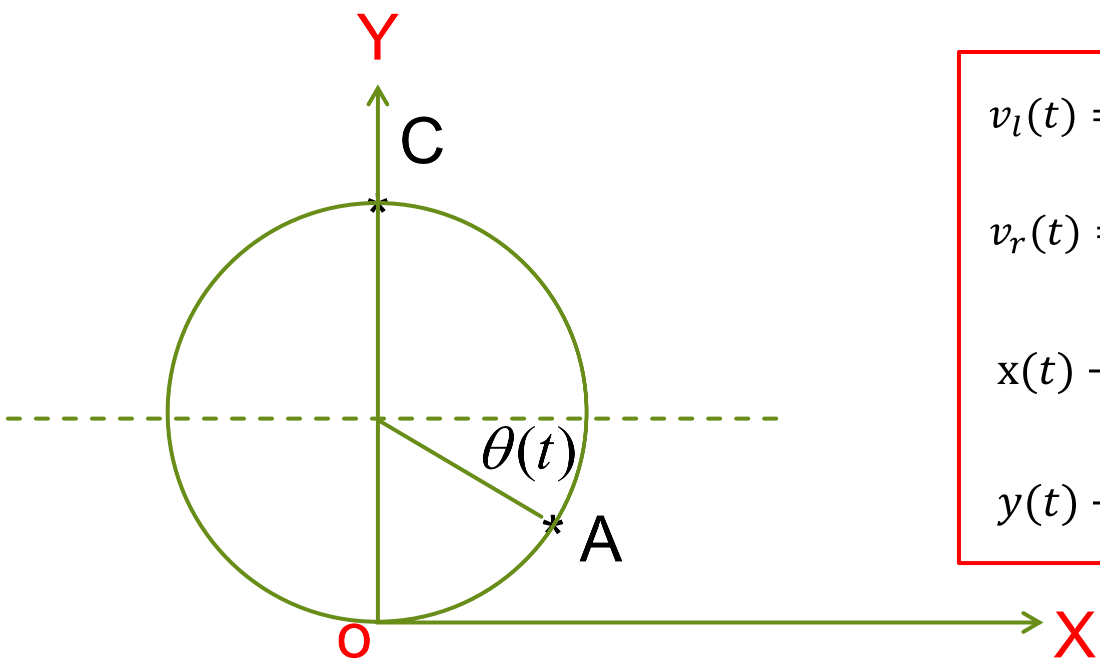
- Equations of Trajectory During Cruising Phase [2, 10] s:

$$\theta(2) = -\frac{\pi}{2} + 2\alpha$$

$$\theta(t) = \theta(2) + 2\alpha(t-2)$$

$$\omega_R(2) = 2\alpha$$

$$\omega_R(t) = 2\alpha$$



$$v_l(t) = (150.0 - \frac{L}{2}) \times \omega_R(t)$$

$$v_r(t) = (150.0 + \frac{L}{2}) \times \omega_R(t)$$

$$x(t) - 0.0 = 150.0\, cos(\theta(t))$$

$$y(t) - 150.0 = 150.0 \sin(\theta(t))$$

Solution (continued)

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Start-Move-Stop Motions

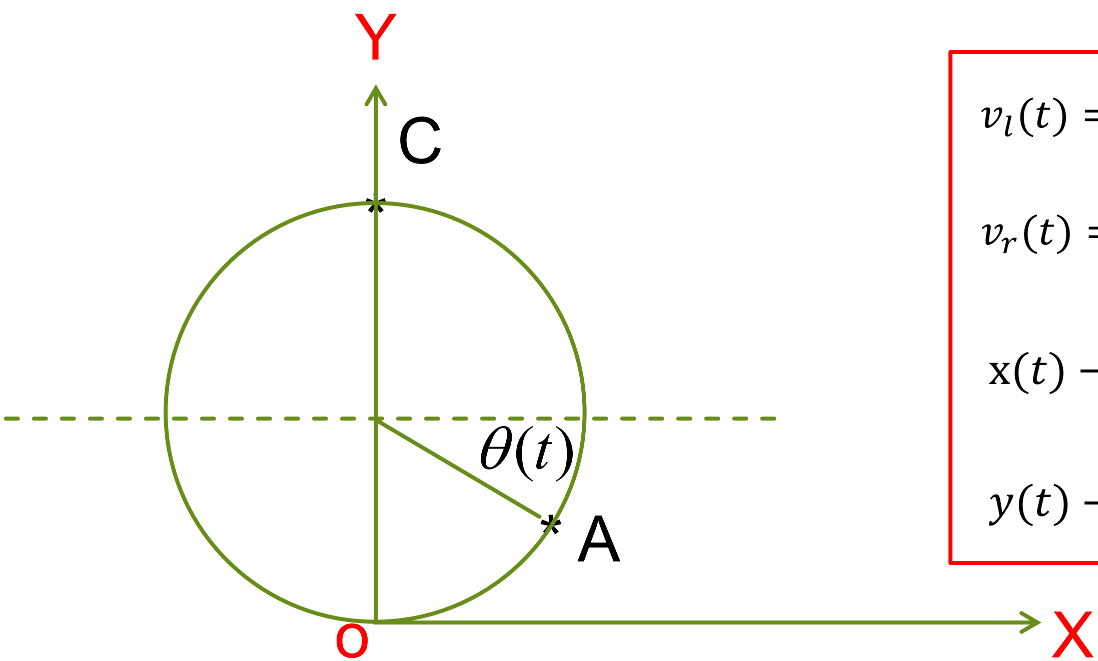
- Equations of Trajectory During Slow-down Phase [10, 12] s:

$$\theta(10) = -\frac{\pi}{2} + 2\alpha + 16\alpha$$

$$\omega_R(10) = 2\alpha$$

$$\theta(t) = \theta(10) + 2\alpha(t-10) - \frac{1}{2}\alpha(t-10)^2$$

$$\omega_R(t) = \omega_R(10) - \alpha \times (t-10)$$



$$v_l(t) = (150.0 - \frac{L}{2}) \times \omega_R(t)$$

$$v_r(t) = (150.0 + \frac{L}{2}) \times \omega_R(t)$$

$$x(t) - 0.0 = 150.0\cos(\theta(t))$$

$$y(t) - 150.0 = 150.0\sin(\theta(t))$$

Solution (continued)

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

Start-Move-Stop Motions

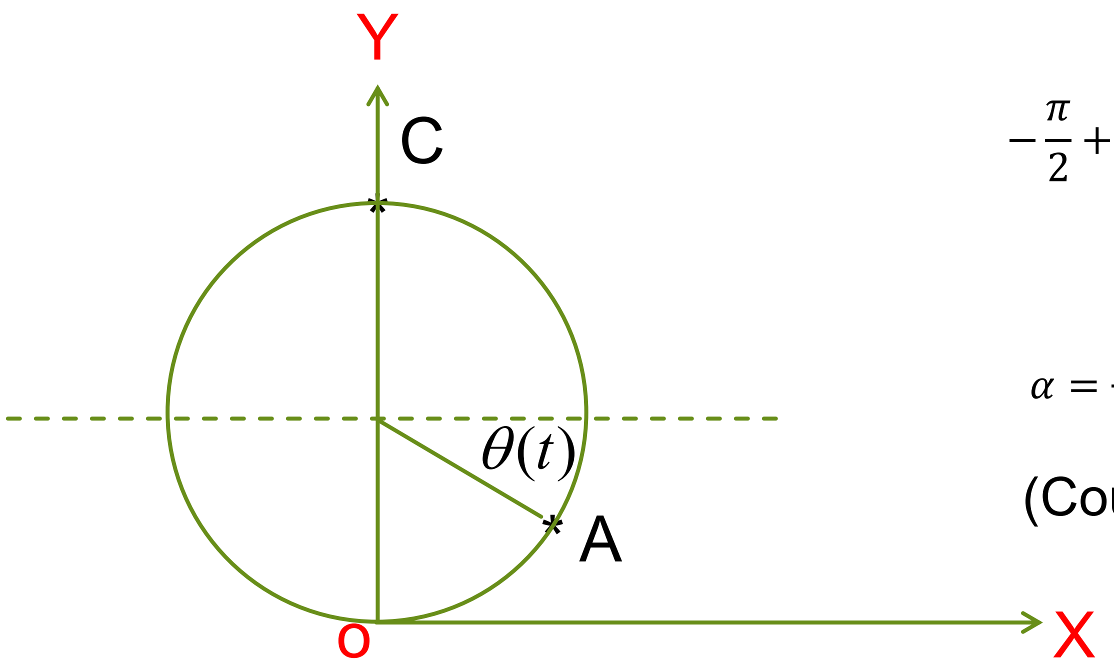
- Determination of the value of acceleration/deceleration a:

$$\theta(t) = \theta(10) + 2\alpha(t-10) - \frac{1}{2}\alpha(t-10)^2$$

$$\theta(12) = -\frac{\pi}{2} + 2\alpha + 16\alpha + 4\alpha - 2\alpha$$

$$\omega_R(12) = 0$$

$$\theta(12) = \frac{\pi}{2}$$



$$-\frac{\pi}{2} + 2\alpha + 16\alpha + 4\alpha - 2\alpha = \frac{\pi}{2}$$

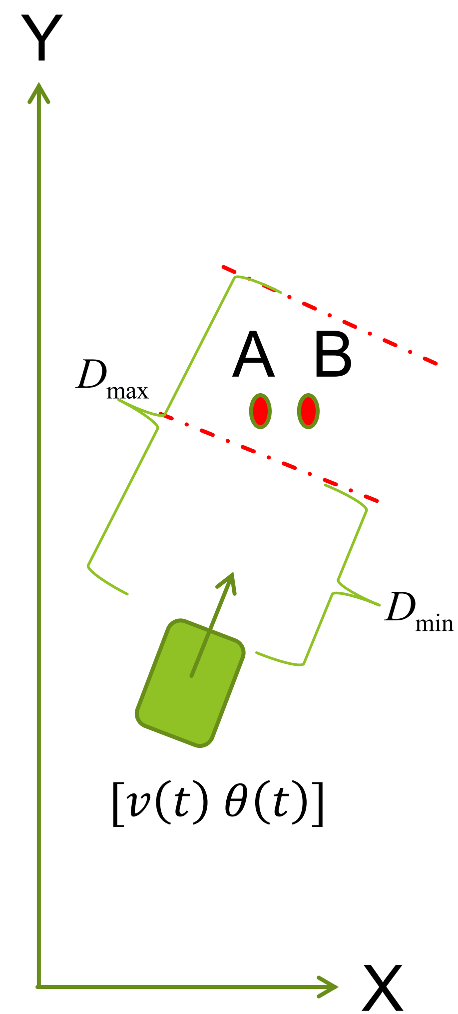
$$\alpha = \frac{\pi}{20}$$

(Counterclockwise Acceleration)

Result

Vision-Guided Planning of Trajectory

- A vehicle is following a moving target. The vehicle's vision system can measure the distances to points A and B of the moving target. What should be the trajectories of the vehicle?



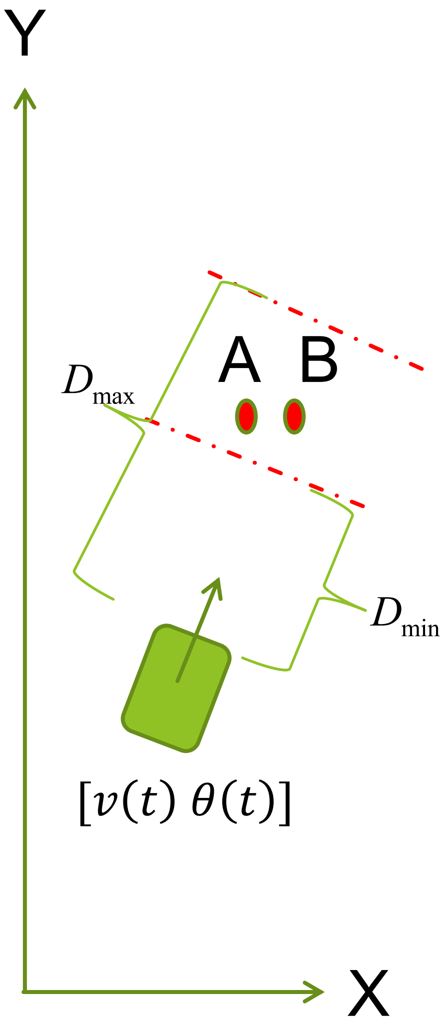
- Answer:
  - Equation of velocity:

$$v(t_{k+1}) = \begin{cases} v(t_k) + \Delta v & \text{if } \mathrm{d} > \mathrm{D}_{\max} \\ v(t_k) & \text{if } \mathrm{D}_{\min} > \mathrm{d} > \mathrm{D}_{\max} \\ v(t_k) - \Delta v & \text{if } \mathrm{d} < \mathrm{D}_{\min} \end{cases}$$

  - Equation of steering:

$$\theta(t_{k+1}) = \begin{cases} \theta(t_k) + \Delta\theta & \text{if } d_A - d_B < -\varepsilon \\ \theta(t_k) & \text{if } |d_A - d_B| < \varepsilon \\ \theta(t_k) - \Delta\theta & \text{if } d_A - d_B > +\varepsilon \end{cases}$$

Result



Summary of Lecture 5

Start-Move-Stop Motions

- Rest to Velocity
- Velocity to Velocity
- Velocity to Rest

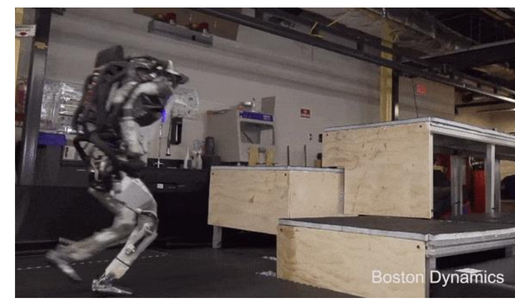
- Purpose of Trajectory Planning
- Input to Trajectory Planning
- Output from Trajectory Planning
- Process of Autonomous Trajectory Planning

Summary of Module 3

Conceptual World

- Task Planning

- Action Planning



- Motion Planning

Physical World

- Path Planning

- Trajectory Planning

"Ask not what your country can do for you – ask what you can do for your country," - John F. Kennedy

"Do not think that you are needy – think that you are needed in the world", - Manis Friedman

"Study will make you knowledgeable, resourceful, and hence more needed", - Xie Ming

Thank You for Listening!